IEEE Trial-Use Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions

Sponsor

Power System Instrumentation and Measurements Committee of the IEEE Power Engineering Society

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Abstract: This is a trial-use standard for definitions used for measurement of electric power quantities under sinusoidal, nonsinusoidal, balanced, or unbalanced conditions. It lists the mathematical expressions that were used in the past, as well as new expressions, and explains the features of the new definitions.
Keywords: active power, apparent power, nonactive power, power factor, reactive power, total harmonic distortion

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Introduction

(This introduction is not part of IEEE Std 1459-2000, IEEE Trial-Use Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions.)

The definitions for active, reactive, and apparent powers that are currently used are based on the knowledge developed and agreed upon during the 1940s. Such definitions served the industry well, as long as the current and voltage waveforms remained nearly sinusoidal.

Important changes have occurred in the last 50 years. The new environment is conditioned by the following facts:

a) Power electronics equipment, such as Adjustable Speed Drives, Controlled Rectifiers, Cycloconverters, Electronically Ballasted Lamps, Arc and Induction Furnaces, and clusters of Personal Computers, represent major nonlinear and parametric loads proliferating among industrial and commercial customers. Such loads have the potential to create a host of disturbances for the utility and the end-user's equipment. The main problems stem from the flow of nonactive energy caused by harmonic currents and voltages.

b) New definitions of powers have been discussed in the last 30 years in the engineering literature (Filipski [B6]). The mechanism of electric energy flow for nonsinusoidal and/or unbalanced conditions is well understood today.

c) The traditional instrumentation designed for the sinusoidal 60/50 Hz waveform is prone to significant errors when the current and the voltage waveforms are distorted (Filipski [B6]).

d) Microprocessors and minicomputers enable today's manufacturers of electrical instruments to construct new, accurate, and versatile metering equipment that is capable of measuring electrical quantities defined by means of advanced mathematical models.

e) There is a need to quantify correctly the distortions caused by the nonlinear and parametric loads, and to apply a fair distribution of the financial burden required to maintain the quality of electric service.

This trial-use standard lists new definitions of powers needed for the following particular situations:

— When the voltage and current waveforms are nonsinusoidal.

— When the load is unbalanced or the supplying voltages are asymmetrical.

— When the energy dissipated in the neutral path due to zero-sequence current components has economical significance.

The new definitions were developed to give guidance with respect to the quantities that should be measured or monitored for revenue purposes, engineering economic decisions, and determination of major harmonic polluters. The following important electrical quantities are recognized by this trial-use standard:

1) The power frequency (60/50 Hz or fundamental) apparent, active, and reactive powers. These three basic quantities are the quintessence of the power flow in electric networks. They define the product generated, transmitted, distributed, and sold by the electric utilities and bought by the end-users. This is the electric energy transmitted by the 60/50 Hz electromagnetic field. In poly-phase systems the power frequency positive-sequence powers are the important dominant quantities. The power frequency positive-sequence power factor is a key value that helps determine and adjust the flow of power frequency positive-sequence reactive power. The fundamental positive-sequence reactive power is of utmost importance in power systems; it governs the fundamental voltage magnitude and its distribution along the feeders, and affects electromechanical stability as well as the energy loss.

2) The effective apparent power in three-phase systems, $S_e = 3V_eI_e$, where V_e and I_e are the equivalent voltage and current. In sinusoidal and balanced situations, S_e is equal to the conventional apparent power $S = 3V_{\ell n}I = \sqrt{3}V_{\ell\ell}I$, where $V_{\ell n}$ and $V_{\ell\ell}$ are the line-to-neutral and the line-to-line voltage, respectively. For sinusoidal unbalanced or for nonsinusoidal balanced or unbalanced situations, S_e allows rational and correct computation of the power factor. This quantity was proposed in 1922 by the German engineer F. Buchholz [B1] and in 1933 was explained by the American engineer W. M. Goodhue [B7].

3) The non-60 Hz or nonfundamental apparent power, S_N (for brevity, 50 Hz power is not always mentioned). This power quantifies the overall amount of harmonic pollution delivered or absorbed by a load. It also quantifies the required capacity of dynamic compensators or active filters when used for nonfundamental compensation alone.

4) Current distortion power, D_I, identifies the segment of nonfundamental nonactive power due to current distortion. This is usually the dominant component of S_N.

5) Voltage distortion power, D_V, separates the nonfundamental nonactive power component due to voltage distortion.

6) Apparent harmonic power, S_H, indicates the level of apparent power due to harmonic voltages and currents alone. This is the smallest component of S_N and includes the harmonic active power P_H.

To avoid confusion, it was decided not to add new units. The use of the watts (W) for instantaneous and active powers, volt-amperes (VA) for apparent powers, and varistor (var) for all the nonactive powers, maintains the distinct separation among these three major types of powers.

There is not yet available a generalized power theory that can provide a simultaneous common base for

— Energy billing

— Evaluation of electric energy quality

— Detection of the major sources of waveform distortion

— Theoretical calculations for the design of mitigation equipment such as active filters or dynamic-compensators

This trial-use standard is meant to provide definitions extended from the well-established concepts. It is meant to serve the user who wants to measure and design instrumentation for energy and power quantification. It is not meant to help in the design of real-time control of dynamic compensators or for diagnosis instrumentation used to pinpoint to a specific type of annoying event or harmonic.

To the working group's knowledge, no commercially available instruments are fully capable of quantifying S_e and S_N according to the definitions given in this standard. These definitions are meant to serve as a guideline and a useful benchmark for future developments.

Publication of this trial-use standard for comment and criticism has been approved by the Institute of Electrical and Electronics Engineers. Trial-use standards are effective for 24 months from the date of publication. Comments for revision will be accepted for 18 months after publication. Suggestions for revision should be directed to the Secretary, IEEE-SA Standards Board, 445 Hoes Lane, P.O. Box 1331, Piscataway, NJ 08855-1331, and should be received no later than 21 November 2001. It is expected that following the 24-month period, this trial-use standard, revised as necessary, shall be submitted to the IEEE-SA Standards Board for approval as a full-use standard.

Participants

At the time this trial-use standard was completed, the Working Group on Nonsinusoidal Situations had the following membership:

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When the IEEE-SA Standards Board approved this standard on 30 January 2000, it had the following membership:

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Contents

IEEE Trial-Use Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions

1. Overview

This trial-use standard is divided into three clauses. Clause 1 lists the scope of this document. Clause 2 lists references to other standards that are useful in applying this trial-use standard. Clause 3 provides the definitions, among which there are several new expressions.

The preferred mathematical expressions recommended for the instrumentation design are marked with a $\|$ sign. The additional expressions are meant to reinforce the theoretical approach and facilitate a better understanding of the explained concepts.

1.1 Scope

This is a trial-use standard for definitions used for measurement of electric power quantities under sinusoidal, nonsinusoidal, balanced, or unbalanced conditions. It lists the mathematical expressions that were used in the past, as well as new expressions, and explains the features of the new definitions.

1.2 Purpose

This trial-use standard is meant to provide organizations with criteria for designing and using metering instrumentation.

2. References

This trial-use standard shall be used in conjunction with the following publications. If the following publications are superseded by an approved revision, the revision shall apply.

DIN 40110-1997, Quantities Used in Alternating Current Theory.[1]

IEEE Std 280-1985 (Reaff 1997), IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering.[2]

ISO 31-5:1992, Quantities and Units—Part 5: Electricity and Magnetism.[3]

3. Definitions

Mathematical expressions that are considered appropriate for instrumentation design are marked with the sign ‖. When the sign ‖ appears on the right side, it means that the last expression that is listed is favored. Each descriptor of a power type is followed by its measurement unit in parentheses.

3.1 Single-Phase

3.1.1 Single-Phase sinusoidal

A sinusoidal voltage source

$$v = \sqrt{2}V\sin(\omega t)$$

supplying a linear load, will produce a sinusoidal current of

$$i = \sqrt{2}I\sin(\omega t - \theta)$$

where

V is the rms value of the voltage (V)
I is the rms value of the current (A)
ω is the angular frequency $2\pi f$ (rad/s)
f is the frequency (Hz)
θ is the phase angle (rad)
t is the time (s)

[1]DIN publications are available from the Deutsches Institut für Normung, Burggrafenstrasse 6, Postfach 1107, 12623 Berlin 30, Germany (011 49 30 260 1362).

[2]IEEE publications are available from the Institute of Electrical and Electronics Engineers, 445 Hoes Lane, P.O. Box 1331, Piscataway, NJ 08855-1331, USA (http://standards.ieee.org/).

[3]ISO publications are available from the ISO Central Secretariat, Case Postale 56, 1 rue de Varembé, CH-1211, Genève 20, Switzerland/Suisse (http://www.iso.ch/). ISO publications are also available in the United States from the Sales Department, American National Standards Institute, 11 West 42nd Street, 13th Floor, New York, NY 10036, USA (http://www.ansi.org/).

3.1.1.1 Instantaneous power (W)

The instantaneous power p is given by

$$\| \; p = vi$$

$$p = p_a + p_q$$

where

$$p_a = VI\cos\theta[1 - \cos(2\omega t)] = P[1 - \cos(2\omega t)]; \qquad \mathrm{P} = VI\cos\theta$$
$$p_q = -VI\sin\theta\sin(2\omega t) = -Q\sin(2\omega t); \qquad Q = VI\sin\theta$$

NOTES

1—The instantaneous power is produced by the active component of the current, i.e., the component that is in phase with the voltage. It is the rate of flow of the energy

$$w_a = \int p_a dt = Pt - \frac{P}{2\omega}\sin(2\omega t)$$

This energy flows unidirectionally from the source to the load. Its rate of flow is not negative, $p_a \geq 0$.

2—The instantaneous power p_q is produced by the reactive component of the current, i.e., the component that is in quadrature with the voltage. It is the rate of flow of the energy

$$w_q = \int p_q dt = \frac{Q}{2\omega}\cos(2\omega t)$$

This type of energy oscillates between the source and inductances, capacitances, and moving masses pertaining to electromechanical systems (motor and generator rotors, plungers, and armatures). The average value of this rate of flow is zero, and the net transfer of energy to the load is nil.

3.1.1.2 Active power (W)

The active power P is the mean value of the instantaneous power during the observation time interval τ to $\tau + kT$

$$\| \; P = \frac{1}{kT}\int_{\tau}^{\tau + kT} p\,dt$$

where

$T = 1/f$	is the cycle (s),
k	is an integer number,
τ	is the moment when the measurement starts.

$$\| \; P = VI\cos\theta$$

3.1.1.3 Reactive power (var)

The reactive power Q is the amplitude of the oscillating instantaneous power p_q.

$$Q = \frac{1}{2\pi}\oint v\,di = \frac{-1}{2\pi}\oint i\,dv = \frac{1}{kT}\int_{\tau}^{\tau + kT} v\frac{di}{dt}dt = \frac{-1}{kT}\int_{\tau}^{\tau + kT} i\frac{dv}{dt}dt = \frac{-\omega}{kT}\int_{\tau}^{\tau + kT} v\left[\int i\,dt\right]dt$$

$$\| \; Q = \frac{\omega}{kT}\int_{\tau}^{\tau + kT} i\left[\int v\,dt\right]dt$$

$$Q = VI\sin\theta$$

NOTE— If the load is inductive, then $Q > 0$. If the load is capacitive, then $Q < 0$.

3.1.1.4 Apparent power (VA)

The apparent power S is the product of the root-mean-square (rms) voltage and the root-mean-square (rms) current.

$$\| \; S = VI$$

$$S = \sqrt{P^2 + Q^2}$$

NOTE—Instantaneous power p follows a sinusoidal oscillation with a frequency $2f = 2\omega/2\pi$ biased by the active power P. The amplitude of the sinusoidal oscillation is the apparent power S.

3.1.1.5 Power factor

$$\| \; P_F = \frac{P}{S}$$

3.1.1.6 Complex power (VA)

$$\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^* = P + jQ$$

where

$\boldsymbol{V} = V\angle 0°$ is the voltage phasor,

$\boldsymbol{I}^* = I\angle\theta$ is the conjugated current phasor.

This expression stems from the power triangle, $S, P, Q,$ and is useful in power flow studies. Figure 1 summarizes the conventional power flow directions as interpreted in literature (Stevens [B12][4]).

3.1.2 Single-Phase nonsinusoidal

For steady-state conditions a nonsinusoidal instantaneous voltage or current has two distinctive components: the power system frequency components v_1 and i_1, and the remaining terms v_H and i_H that contains all integer and noninteger number harmonics.

$$v_1 = \sqrt{2}V_1 \sin(\omega t - \alpha_1)$$

$$i_1 = \sqrt{2}I_1 \sin(\omega t - \beta_1)$$

$$v_H = \sqrt{2}\sum_{h \neq 1} V_h \sin(h\omega t - \alpha_h)$$

$$i_H = \sqrt{2}\sum_{h \neq 1} I_h \sin(h\omega t - \beta_h)$$

[4]The numbers in brackets correspond to those of the bibliography in Annex C.

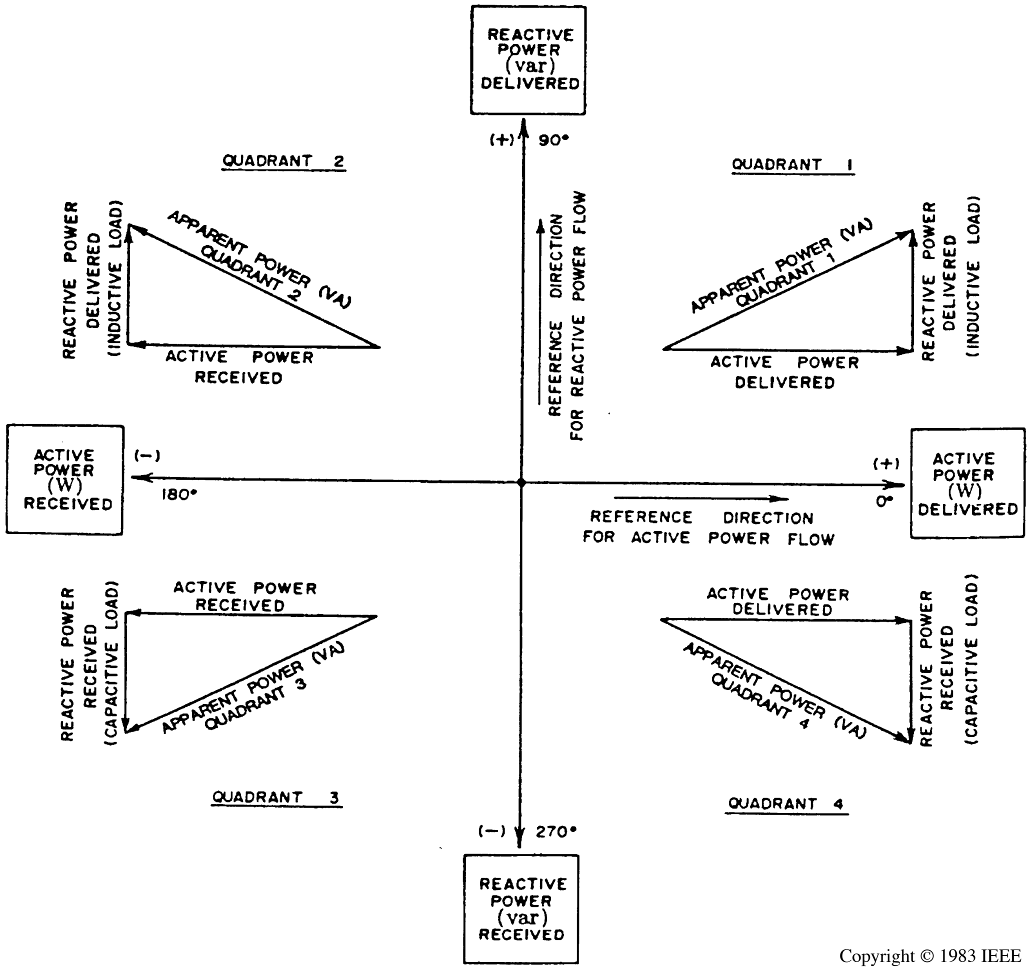


Figure 1—Four-quadrant power flow directions (see [B12])

The corresponding rms values squared are as follows:

$$V^2 = \frac{1}{kT}\int_{\tau}^{\tau+kT} v^2 dt = V_1^2 + V_H^2$$

$$I^2 = \frac{1}{kT}\int_{\tau}^{\tau+kT} i^2 dt = I_1^2 + I_H^2$$

where

$$V_H^2 = \sum_{h \neq 1} V_h^2 = V^2 - V_1^2 \quad \|$$

$$I_H^2 = \sum_{h \neq 1} I_h^2 = I^2 - I_1^2 \quad \|$$

NOTE—The direct voltage and the direct current terms V_0 and I_0, obtained for $h = 0$, must be included in V_H and I_H. They correspond to a hypothetical $\alpha_0 = \beta_0 = -45°$; $(\sin(-\alpha_0) = \sin(-\beta_0) = \sin 45° = 1/\sqrt{2})$. Significant dc components are rarely present in ac power systems; however, traces of dc are not uncommon.

3.1.2.1 Total harmonic distortion

The overall deviation of a distorted wave from its fundamental can be estimated with the help of the total harmonic distortion. The total harmonic distortion of the voltage is as follows:

$$\| \; THD_v = \frac{V_H}{V_1} = \sqrt{\left(\frac{V}{V_1}\right)^2 - 1}$$

The total harmonic distortion of the current is as follows:

$$\| \; THD_I = \frac{I_H}{I_1} = \sqrt{\left(\frac{I}{I_1}\right)^2 - 1}$$

3.1.2.2 Instantaneous power (W)

$$p = vi$$

$$p = p_a + p_q$$

where

$$p_a = \sum_h V_h I_h \cos\theta_h [1 - \cos(2h\omega t)]$$

is a term that contains all the components that have non-zero average value, and

$$p_q = \sum_h V_h I_h \sin\theta_h \sin(2h\omega t) + \sum_{\substack{m \neq n \\ m,\, n \neq 1}} 2V_m I_n \sin(m\omega t + \alpha_m)\sin(n\omega t + \beta_n)$$

is a term that does not contribute to the net transfer of energy, i.e., its average value is nil.

The angle $\theta_h = \beta_h - \alpha_h$ is the phase angle between the phasors V_h and I_h.

3.1.2.3 Active power (W)

$$\| \; P = \frac{1}{kT}\int_{\tau}^{\tau + kT} p\,dt$$

$$P = P_1 + P_H$$

3.1.2.4 Fundamental or 60 Hz active power (W)

$$\| \; P_1 = \frac{1}{kT}\int_{\tau}^{\tau + kT} v_1 i_1\,dt = V_1 I_1 \cos\theta_1$$

3.1.2.5 Harmonic active power (W)

$$P_H = \sum_{h \neq 1} V_h I_h \cos\theta_h = P - P_1 \; \|$$

NOTE—For ac motors, which make up the vast majority of loads, the harmonic active power is not a useful power. Consequently, it is useful to separate the fundamental active power P_1 from the harmonic active power P_H.

3.1.2.6 Fundamental reactive power (var)

$$\| Q_1 = \frac{\omega_1}{kT}\int_{\tau}^{\tau + kT} i_1\left[\int v_1 dt\right]dt$$

$$= V_1 I_1 \sin\theta_1$$

3.1.2.7 Budeanu's reactive power (var)

$$Q_B = \sum_h V_h I_h \sin\theta_h$$

$$Q_B = Q_1 + Q_{BH}$$

where

$$\mathrm{Q_{BH}} = \sum_{h \neq 1} V_h I_h \sin\theta_h$$

NOTE—The usefulness of Q_B for quantifying the flow of harmonic nonactive power has been questioned by many engineers (Czarnecki [B2], Lyon [B10]). Field measurements and simulations (Pretorius, van Wyk, and Swart [B11]) prove that in many situations $Q_{BH} < 0$, thus leading to situations where $Q_B < Q_1$.

3.1.2.8 Apparent power (VA)

$$\| S = VI$$

NOTE—An important practical property of S is that the power loss ΔP, in the feeder that supplies the apparent power S, is a nearly linear function of S^2 (Emanuel [B4]).

$$\Delta P = \frac{r_e}{V^2}S^2 + \frac{V^2}{R}$$

where

R is an equivalent shunt resistance, representing transformer core losses and cable dielectric losses,
r_e is the effective Thevenin resistance. Theoretically r_e can be obtained from the equivalence of losses as follows:

$$r_e I^2 = r_{dc}\sum_h K_{sh} I_h^2$$

where

K_{sh} is the skin effect coefficient for the h harmonic,
r_{dc} is the Thevenin dc resistance (Ω).

3.1.2.9 Fundamental or 60/50 Hz apparent power (VA)

Fundamental apparent power S_1 and its components P_1 and Q_1 are the actual quantities that help define the rate of flow of the electromagnetic field energy associated with the 60/50 Hz voltage and current. This is a product of high interest for both the utility and the end-user.

$$S_1 = V_1 I_1$$

$$S_1^2 = P_1^2 + Q_1^2$$

3.1.2.10 Nonfundamental apparent power (VA)

The separation of the rms current and voltage into fundamental and harmonic terms (see 3.1.2) resolves the apparent power in the following manner (Emanuel [B5]):

$$S^2 = (VI)^2 = (V_1^2 + V_H^2)(I_1^2 + I_H^2) = (V_1 I_1)^2 + (V_1 I_H)^2 + (V_H I_1)^2 + (V_H I_H)^2 = (S_1^2 + S_N^2)$$

$$S_N = \sqrt{S^2 - S_1^2}$$

is the nonfundamental apparent power, and is resolved in the following three distinctive terms:

$$S_N^2 = D_1^2 + D_v^2 + S_H^2$$

3.1.2.11 Current distortion power (var)

$$D_I = V_1 I_H = S_1 (THD_I)$$

3.1.2.12 Voltage distortion power (var)

$$D_v = V_H I_1 = S_1 (THD_V)$$

3.1.2.13 Harmonic apparent power (VA)

$$S_H = V_H I_H = S_1 (THD_1)(THD_V)$$

$$S_H = \sqrt{P_H^2 + D_H^2}$$

3.1.2.14 Harmonic distortion Power (var)

$$D_H = \sqrt{S_H^2 - P_H^2}$$

NOTE—In practical power systems, $THD_V < THD_I$ and S_N can be computed using the following expression (Emanuel [B5]):

$$S_N \approx S_1 \sqrt{(THD_I)^2 + (THD_V)^2}$$

When $THD_V \leq 5\%$ and $THD_I \leq 200\%$, this expression yields an error less than 0.15%.

For $THD_V < 5\%$ and $THD_I > 40\%$, an error less than 1.00% is obtained using the following expression (Emanuel [B5]):

$$S_N \approx S_1(THD_I)$$

3.1.2.15 Nonactive power (var)

$$N = \sqrt{S^2 - P^2}$$

This power lumps together both fundamental and nonfundamental nonactive components. In the past, this power was called *fictitious power*.

3.1.2.16 Budeanu's distortion power (var)

This power results from the resolution of S using Budeanu's reactive power Q_B (see 3.1.2.7) that leads to the following:

$$S^2 = P^2 + Q_B^2 + D_B^2$$

hence,

$$D_B = \sqrt{S^2 - P^2 - Q_B^2}$$

NOTE—This distortion power is affected by the deficiency of Q_B (Pretorius, van Wyk, and Swart [B11]).

3.1.2.17 Fundamental or 60/50 Hz power factor

$$P_{F1} = \cos\theta_1 = \frac{P_1}{S_1}$$

This ratio helps evaluate separately the fundamental power flow conditions. It can be called the *fundamental power factor* or *60/50 Hz power factor*. It is also often referred to as the *displacement power factor.*

3.1.2.18 Power factor

$$P_F = \frac{P}{S}$$

$$P_F = \frac{P}{S} = \frac{P_1 + P_H}{\sqrt{S_1^2 + S_N^2}} = \frac{(P_1/S_1)[1 + (P_H/P_1)]}{\sqrt{1 + (S_N/S_1)^2}} = \frac{[1 + (P_H/P_1)]P_{F1}}{\sqrt{1 + THD_I^2 + THD_V^2 + (THD_I THD_V)^2}}$$

NOTES

1—The apparent power S can be viewed as the maximum active power that can be transmitted to a load while keeping its load voltage V constant and line losses constant. The result is that for a given S and V, maximum utilization of the line is obtained when $P = S$; hence, the ratio P/S is a utilization factor indicator.

2—The overall degree of harmonic injection produced by a large nonlinear load, or by a group of loads or consumers, can be estimated from the ratio S_N/S_1. The effectiveness of harmonic filters also can be evaluated from such a measurement. The measurements of S_1, P_1, P_{F1}, or Q_1 help establish the characteristics of the fundamental power flow.

3—In most common practical situations, $P_H \ll P_1$. It is difficult to measure correctly the higher-order components of P_H with most metering instrumentation. Thus, one cannot rely on measurements of P_H components when making technical decisions regarding harmonics compensation, energy tariffs, or to quantify the detrimental effects made by a nonlinear or parametric load to a particular power system (Emanuel [B5]; IEEE [B9]; Swart, van Wyk, and Case [B13]).

4—When $THD_V < 5\%$ and $THD_I > 40\%$, it is convenient to use the following expression:

$$P_F \approx \frac{1}{\sqrt{1 + THD_I^2}} P_{F1}$$

5— In typical nonsinusoidal situations, $D_I > D_V > S_H > P_H$.

The definitions presented in 3.1.2.8 to 3.1.2.17 are summarized in Table 1.

Table 1—Summary and grouping of the quantities in single-phase systems with nonsinusoidal waveforms

Quantity or indicator	Combined	60 Hz powers (fundamental)	Non-60 Hz powers (nonfundamental)
Apparent	S (VA)	S_1 (VA)	S_N (VA) S_H (VA)
Active	P (W)	P_1 (W)	P_H (W)
Nonactive	N (var)	Q_1 (var)	D_I D_V D_H (var)
Line utilization	$P_F = P/S$	$P_{F1} = P_1/S_1$	—
Harmonic pollution	—	—	S_N/S_1

3.2 Three-Phase systems

3.2.1 Three-Phase sinusoidal balanced

In this case, the line-to-neutral voltages are as follows:

$$v_a = \sqrt{2} V_{\ell n} \sin(\omega t)$$

$$v_b = \sqrt{2} V_{\ell n} \sin(\omega t - 120°)$$

$$v_c = \sqrt{2} V_{\ell n} \sin(\omega t + 120°)$$

The line currents have similar expressions. They are as follows:

$$i_a = \sqrt{2}I\sin(\omega t - \theta)$$

$$i_b = \sqrt{2}I\sin(\omega t - \theta - 120°)$$

$$i_c = \sqrt{2}I\sin(\omega t - \theta + 120°)$$

NOTES

1—Perfectly sinusoidal and balanced three-phase, low-voltage systems are uncommon. Only under laboratory conditions, using low distortion power amplifiers, is it possible to work with ac power sources with $THD_V < 0.1\%$ and voltage unbalance $V^-/V^+ < 0.1\%$. Practical low-voltage systems will rarely operate with $THD_V < 1\%$ and $V^-/V^+ < 0.4\%$, where V^+ and V^- are the positive- and negative-sequence voltages, see 3.2.2.2.1.

2— In the case of a three-wire system, the line-to-neutral voltages are defined assuming an artificial neutral node.

3.2.1.1 Instantaneous power (W)

$$\| \; p = v_a i_a + v_b i_b + v_c i_c = P$$

3.2.1.2 Active power (W)

$$\| \; P = \frac{1}{kT}\int_{\tau}^{\tau + kT} p\,dt$$

$$P = 3V_{\ell n}I\cos\theta = \sqrt{3}V_{\ell\ell}I\cos\theta$$

where

$V_{\ell n}$ is line-to neutral voltage,

$V_{\ell\ell}$ is line-to-line voltage.

3.2.1.3 Reactive power (var)

$$Q = 3V_{\ell n}I\sin\theta = \sqrt{3}V_{\ell\ell}I\sin\theta$$

$$|Q| = \sqrt{S^2 - P^2} \; \|$$

3.2.1.4 Apparent power (VA)

$$\| \; S = 3V_{\ell n}I = \sqrt{3}V_{\ell\ell}I$$

3.2.1.5 Power factor

$$\| \; P_F = \frac{P}{S}$$

3.2.2 Three-Phase sinusoidal unbalanced

In this case, the three current phasors, I_a, I_b, and I_c, do not have equal magnitudes, nor are they shifted exactly with respect to each other. Load unbalance leads to asymmetrical currents that in turn can cause voltage asymmetry. There are situations when the three voltage phasors are not symmetrical. This leads to asymmetrical currents even when the load is perfectly balanced.

The line-to-neutral voltages are as follows:

$$v_a = \sqrt{2}V_{a\ell n}\sin(\omega t + \alpha_a)$$

$$v_b = \sqrt{2}V_{b\ell n}\sin(\omega t + \alpha_b - 120°)$$

$$v_c = \sqrt{2}V_{c\ell n}\sin(\omega t + \alpha_c + 120°)$$

The line currents have similar expressions. They are as follows:

$$i_a = \sqrt{2}I_a\sin(\omega t - \beta_a)$$

$$i_b = \sqrt{2}I_b\sin(\omega t - \beta_b - 120°)$$

$$i_c = \sqrt{2}I_c\sin(\omega t - \beta_c + 120°)$$

NOTE—In the case of three-wire systems, the line-to-neutral voltages are defined assuming an artificial neutral node, which can be obtained with the help of three identical resistances connected in Y.

3.2.2.1 Instantaneous power (W)

$$p = v_a i_a + v_b i_b + v_c i_c$$

for three-wire systems where $i_a + i_b + i_c = 0$, and

$$p = v_{ab}i_a + v_{cb}i_c = v_{ac}i_a + v_{bc}i_b = v_{ba}i_b + v_{ca}i_c$$

where v_{ab}, v_{bc}, and v_{ca} are the instantaneous line-to-line voltages.

3.2.2.2 Active power (W)

$$P = \frac{I}{kT}\int_{\tau}^{\tau + kT} p\,dt$$

$$P = P_a + P_b + P_c$$

where

$$P_a = \frac{1}{Kt}\int_{\tau}^{\tau + kT} v_a i_a dt = V_{a\ell n}I_a\cos\theta_a; \qquad \theta_a = \alpha_a + \beta_a$$

$$P_b = \frac{1}{Kt}\int_{\tau}^{\tau+kT} v_b i_b dt = V_{b\ell n} I_b \cos\theta_b; \qquad \theta_b = \alpha_b + \beta_b$$

$$P_c = \frac{1}{Kt}\int_{\tau}^{\tau+kT} v_c i_c dt = V_{c\ell n} I_c \cos\theta_c; \qquad \theta_c = \alpha_c + \beta_c$$

P_a, P_b, and P_c are phase active powers.

3.2.2.2.1 Positive-, negative-, and zero-sequence active powers (W)

In some situations the use of symmetrical components may be helpful. The symmetrical voltage components V^+, V^-, V^0 and current components I^+, I^-, I^0 with the respective phase angles θ^+, θ^-, θ^0 yield the following three active power components:

The positive-sequence active power

$$P^+ = 3V_{\ell n}^+ I^+ \cos\theta^+$$

The negative-sequence active power

$$P^- = 3V_{\ell n}^- I^- \cos\theta^-$$

The zero-sequence active power

$$P^0 = 3V_{\ell n}^0 I^0 \cos\theta^0$$

The total active power is

$$P = P^+ + P^- + P^0$$

3.2.2.3 Reactive power (var)

Per phase reactive powers are defined with the help of the following expressions:

$$Q_a = \frac{\omega}{kT}\int_{\tau}^{\tau+kT} i_a\left[\int v_a dt\right]dt = V_{a\ell n} I_a \sin\theta_a$$

$$Q_b = \frac{\omega}{kT}\int_{\tau}^{\tau+kT} i_b\left[\int v_b dt\right]dt = V_{b\ell n} I_b \sin\theta_b$$

$$Q_c = \frac{\omega}{kT}\int_{\tau}^{\tau+kT} i_c\left[\int v_c dt\right]dt = V_{c\ell n} I_c \sin\theta_c$$

For the vector apparent power S_V (see 3.2.2.6) the total reactive power Q is as follows:

$$Q = Q_a + Q_b + Q_c$$

NOTE—The above expression of Q cannot be used in conjunction with the arithmetic apparent power S_A, defined in 3.2.2.5.

3.2.2.3.1 Positive-, negative-, and zero-sequence reactive powers (var)

In some situations the use of symmetrical components may be helpful. The three reactive powers are as follows:

The positive-sequence reactive power

$$Q^+ = 3V_{\ell n}^+ I^+ \sin\theta^+$$

The negative-sequence reactive power

$$Q^- = 3V_{\ell n}^- I^- \sin\theta^-$$

The zero-sequence reactive power

$$Q^0 = 3V_{\ell n}^0 I^0 \sin\theta^0$$

The total reactive power is

$$Q = Q^+ + Q^- + Q^0$$

3.2.2.4 Phase apparent powers (VA)

$$S_a = V_{a\ell n} I_a\ ; \qquad S_b = V_{b\ell n} I_b\ ; \qquad S_c = V_{c\ell n} I_c$$

$$S_a^2 = P_a^2 + Q_a^2; \qquad S_b^2 = P_b^2 + Q_b^2 \qquad S_c^2 = P_c^2 + Q_c^2$$

3.2.2.5 Arithmetic apparent power (VA)

$$S_A = S_a + S_b + S_c$$

NOTE—The arithmetic apparent power cannot be resolved according to 3.1.1.4,

$$S_A \neq \sqrt{P^2 + Q^2}$$

where

$$P = P_a + P_b + P_c$$
$$Q = Q_a + Q_b + Q_c$$

3.2.2.6 Vector apparent power (VA)

$$S_V = \sqrt{P^2 + Q^2}$$

$$S_V = |P_a + P_b + P_c + j(Q_a + Q_b + Q_c)| = |P + jQ|$$

$$S_V = |P^+ + P^- + P^0 + j(Q^+ + Q^- + Q^0)|$$

A geometrical interpretation of S_V is presented in Figure 2.

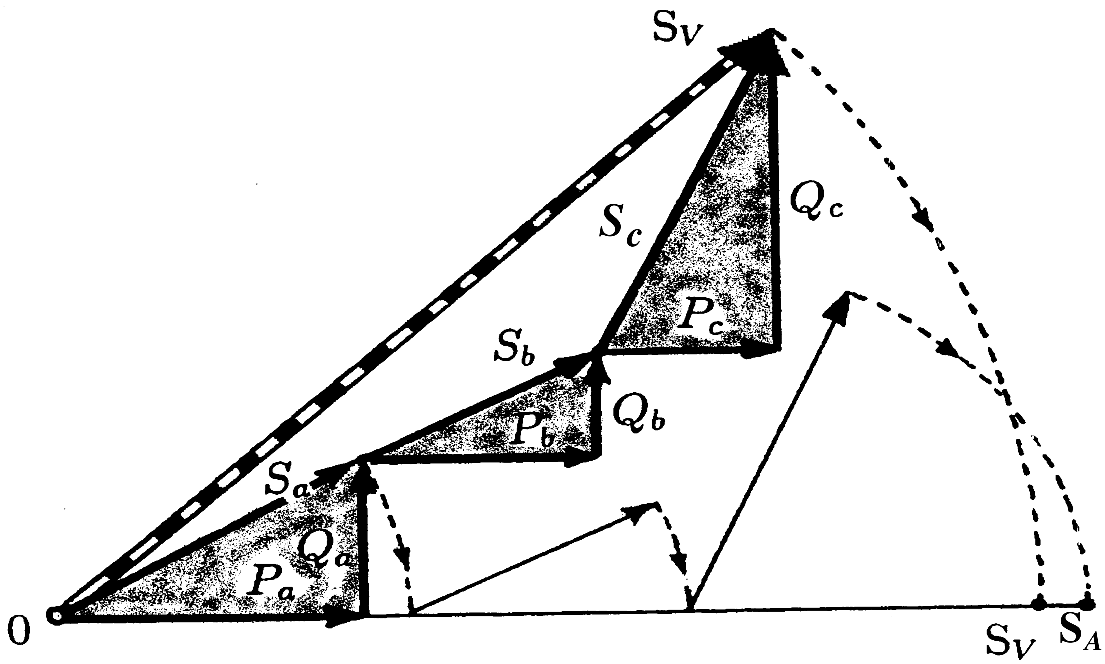


Figure 2—Arithmetic and vector apparent powers: sinusoidal situation

3.2.2.6.1 Positive-, negative-, and zero-sequence apparent powers (VA)

$$S^+ = |\mathbf{S}^+| = |P^+ + jQ^+|$$

$$S^- = |\mathbf{S}^-| = |P^- + jQ^-|$$

$$S^0 = |\mathbf{S}^0| = |P^0 + jQ^0|$$

$$S_V = |\mathbf{S}^+ + \mathbf{S}^- + \mathbf{S}^0|$$

$$S_A \neq S^+ + S^- + S^0$$

3.2.2.7 Vector power factor and arithmetic power factor

$$P_{FV} = \frac{P}{S_V}$$

$$P_{FA} = \frac{P}{S_A}$$

NOTE—A three-phase line supplying one or more customers should be viewed as one single path, one entity that transmits the electric energy to locations where it is converted into other forms of energy. It is wrong to view each phase as an independent energy route. In poly-phase systems, the meaning of *power factor* as a utilization indicator is retained (see 3.1.2.18). Unity power factor means minimum possible line losses for a given total active power transmitted. The following example helps clarify certain limitations pertinent to the old apparent power definitions S_A and S_V.

EXAMPLE:

A four-wire, three-phase system, Figure 3(a), supplies a resistance R connected between phases a and b. The active power dissipated by R is as follows:

$$P_R = \frac{3V_{\ell n}^2}{R}$$

and assume each line has the resistance r that results in a line current $I = \sqrt{3}V_{\ell n}/R$, which is causing the following power loss:

$$\Delta P = 6r\left(\frac{V_{\ell n}}{R}\right)^2$$

Now let us assume a second system with a perfectly balanced three-phase load, Figure 3(b), consisting of three resistances R_B connected in Y. This system dissipates the same power as the unbalanced one; hence,

$$P_{RB} = 3\frac{V_{\ell n}^2}{R_B} = P_R$$

$R_B = R$ results, and the line power loss for the balanced system is as follows:

$$\Delta P_B = 3r\left(\frac{V_{\ell n}}{R}\right)^2 = 0.5\Delta P$$

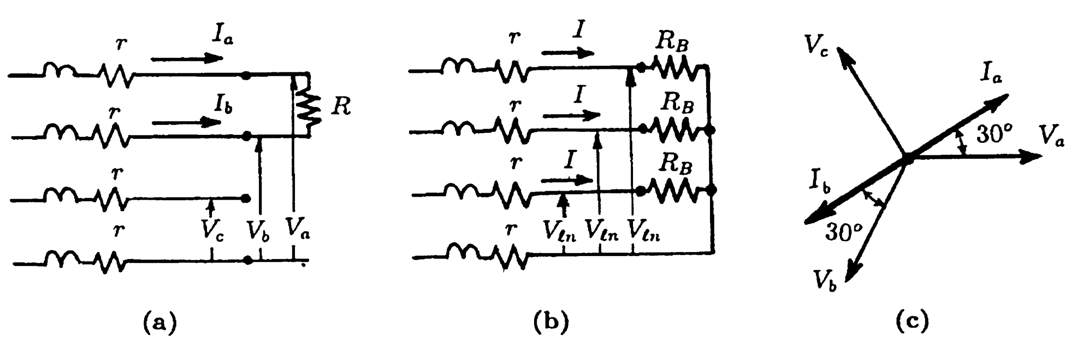


Figure 3—Unbalanced System: (a) actual circuit; (b) balanced equivalent circuit; (c) phasor diagram

The power loss dissipated in the unbalanced system is twice the power loss in the balanced one. This observation leads to the conclusion that the unbalanced system has $P_F < 1$. The balanced system operates with minimum possible losses for a given load voltage and active power, hence its power factor is unity.

For the unbalanced system, the arithmetic and vector apparent powers have the following components [see phasor diagram in Figure 3(c)]:

$$P_a = V_a I_a \cos(30°) = \frac{\sqrt{3}}{2} V_{\ell n} I; \quad Q_a = V_a I_a \sin 30° = \frac{1}{2} V_{\ell n} I \quad S_a = V_a I_a = V_{\ell n} I$$

$$P_b = V_b I_b \cos(-30°) = \frac{\sqrt{3}}{2} V_{\ell n} I; \quad Q_b = V_b I_b \sin(-30°) = \frac{-1}{2} V_{\ell n} I; \quad S_b = V_b I_b = V_{\ell n} I$$

$$P_c = Q_c = S_c = 0$$

The total active power is

$$P = P_a + P_b = \sqrt{3} V_{\ell n} I = \frac{3 V_{\ell n}^2}{R}$$

The total reactive power is

$$Q = Q_a + Q_b + Q_c = 0$$

The vector apparent power is

$$S_V = P$$

The arithmetic apparent power is

$$S_A = S_a + S_b + S_c = 2 V_{\ell n} I = 2\sqrt{3} \frac{V_{\ell n}^2}{R}$$

The power factor computed for the unbalanced system using S_V gives $P_{FV} = P/S_V = 1.0$. The power factor computed with S_A gives $P_{FA} = P/S_A = \sqrt{3}/2 = 0.866$.

If the unbalanced load consists of a resistance connected between line and neutral, then $S_a = S_b = P$ and $P_{FA} = P_{FV} = 1.0$.

These results indicate that both the arithmetic and the vector apparent powers do not measure or compute power factor correctly for unbalanced loads. As a rule, $P_{FA} \le P_{FV}$.

3.2.2.8 Effective apparent power (VA)

This concept assumes a virtual balanced circuit that has exactly the same power losses as the actual unbalanced circuit. This equivalence leads to the definition of an effective line current I_e and an effective line-to-neutral voltage V_e (Depenbrock [B3], Emanuel [B5]).

For a four-wire system, the balance of power loss is expressed in the following way:

$$r(I_a^2 + I_b^2 + I_c^2 + I_n^2) = 3rI_e^2$$

$$\frac{V_a^2 + V_b^2 + V_c^2}{R} + \frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{3R} = 3\frac{V_e^2}{R} + \frac{9V_e^2}{3R}$$

where

I_n	is the neutral rms current
r	is the line resistance assumed to be equal to the neutral wire (or return path) resistance,
R	is the equivalent line-to-neutral shunt resistance, also assumed to be 1/3 of the equivalent line-to-line shunt resistance.

From the above equations, the equivalent current and voltage for a four-wire system is obtained.

$$\| I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2 + I_n^2}{3}} = \sqrt{(I^+)^2 + (I^-)^2 + 4(I^0)^2}$$

and

$$\| V_e \sqrt{\frac{1}{18}[3(V_a^2 + V_b^2 + V_c^2) + V_{ab}^2 + V_{bc}^2 + V_{ca}^2]} = \sqrt{(V^+)^2 + (V^-)^2 + \frac{(V^0)^2}{2}}$$

For practical situations where the differences between α_a, α_b, and α_c do not exceed ±10° and the differences among the line-to-neutral voltages remain within the range of ±10%, the following simplified expression can be used:

$$\| V_e \approx \sqrt{\frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{9}} = \sqrt{(V^+)^2 + (V^-)^2}$$

The error caused by this simplified expression is less than 0.2% for the above conditions.

In the same manner, the equivalent current and voltage for a three-wire system can be found by using

$$r(I_a^2 + I_b^2 + I_c^2) = 3rI_e^2$$

$$\frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{3R} = \frac{9V_e^2}{3R}$$

From these equations, the following results:

$$\| I_e = \sqrt{\frac{(I_a^2 + I_b^2 + I_c^2)}{3}} = \sqrt{(I^+)^2 + (I^-)^2}$$

and

$$\| V_e = \sqrt{\frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{9}} = \sqrt{(V^+)^2 + (V^-)^2}$$

The effective apparent power (Buchholz [B1], Goodhue [B7]) is as follows:

$$\| S_e = 3 V_e I_e$$

3.2.2.9 Effective power factor

$$\| P_{Fe} = P / S_e$$

NOTES

1—Applying the concept of S_e to the unbalanced circuit described in the example given in 3.2.2.7, results in the following:

$$V_e = V_{\ell n}; \qquad I_e = \sqrt{\frac{I_a^2 + I_b^2}{3}} = \frac{\sqrt{2} V_{\ell n}}{R}$$

$$S_e = 3\sqrt{2}\frac{V_{\ell n}^2}{R}; \qquad P = \frac{3 V_{\ell n}^2}{R}$$

Hence the power factor is as follows:

$$P_{Fe} = \frac{P}{S_e} = \frac{1}{\sqrt{2}} = 0.707 < P_{FA} < P_{FV}$$

2—When the system is balanced, then

$$V_a = V_b = V_c = V_{\ell n} = V_e$$

$$I_a = I_b = I_c = I$$

$$I_n = 0$$

and

$$S_V = S_A = S_e$$

3—When the system is unbalanced, then

$$S_V \le S_A \le S_e$$

and

$$P_{Fe} \le P_{FA} \le P_{FV}$$

4—Both the vector and the arithmetic apparent powers do not satisfy the linearity requirement of system power loss versus the apparent power squared (Emanuel [B4]).

3.2.2.10 Positive-Sequence power factor

$$\| P_F^+ = P^+/S^+$$

This index has the same significance as the fundamental power factor P_{F1} (see 3.1.2.17). It helps evaluate the positive-sequence power flow conditions.

3.2.2.11 Effective apparent power resolution for three-phase unbalanced sinusoidal systems

$$S_e^2 = (S^+)^2 + (S_U)^2$$

where

$$S^+ = 3V_{\ell n}^+ I^+$$

$$(S^+)^2 = (P^+)^2 + (Q^+)^2$$

3.2.2.12 Unbalance power

$S_U = \sqrt{S_e^2 - (S^+)^2}$ evaluates the unbalance of the system. It should not be confused with the voltage unbalance. It reflects both the load unbalance and voltage asymmetry.

3.2.3 Three-Phase nonsinusoidal balanced systems

The line-to-neutral voltages are as follows:

$$v_a = \sqrt{2}V_1 \sin\omega t + \sqrt{2}\sum_{h \neq 1} V_h \sin(h\omega t + \alpha_h)$$

$$v_b = \sqrt{2}V_1 \sin(\omega t - 120°) + \sqrt{2}\sum_{h \neq 1} V_h \sin(h\omega t + \alpha_h - 120° h)$$

$$v_c = \sqrt{2}V_1 \sin(\omega t + 120°) + \sqrt{2}\sum_{h \neq 1} V_h \sin(h\omega t + \alpha_h + 120° h)$$

The line currents have similar expressions. They are as follows:

$$i_a = \sqrt{2}I_1 \sin(\omega t + \beta_1) + \sqrt{2}\sum_{h \neq 1} I_h \sin(h\omega t - \beta_h)$$

$$i_b = \sqrt{2}I_1 \sin(\omega t + \beta_1 - 120°) + \sqrt{2}\sum_{h \neq 1} I_h \sin(h\omega t - \beta_h - 120° h)$$

$$i_c = \sqrt{2}I_1 \sin(\omega t + \beta_1 + 120°) + \sqrt{2}\sum_{h \neq 1} I_h \sin(h\omega t - \beta_h + 120° h)$$

NOTES

1—In this case, $S_a = S_b = S_c$, $P_a = P_b = P_c$, $Q_{Ba} = Q_{Bb} = Q_{Bc,}$ and $D_a = D_b = D_c$.

2—When triplen harmonics are present, in spite of the fact that the load is perfectly balanced, the neutral current is not nil.

$$i_n = i_a + i_b + i_c = 3 \sum_{h = 0, 3, 6, \ldots} \sqrt{2} I_h \sin(h\omega t - \beta_h)$$

$$I_n = \sqrt{\sum_{h = 0, 3, 6, \ldots} I_h^2}$$

The above equation illustrates the fact that such a system has the potential to produce significant additional power loss in the neutral wire and ground path. This situation should be reflected in the P_F expression.

3—The positive-sequence triplen harmonic voltages that contribute to the rms value of $V_{\ell n}$ cancel each other and do not appear in $V_{\ell\ell}$:

$$v_{ab} = v_a - v_b = \sqrt{3}\sqrt{2} V_1 \sin(\omega t + 30°) + \sqrt{3}\sqrt{2} \sum_{h \neq 0, 3, 6, 9, \ldots} V_h \sin(h\omega t + \alpha_h + 30° h)$$

This means that

$$\sqrt{3} V_{\ell\ell} I \leq 3 V_{\ell n} I$$

The expression $S \approx \sqrt{3} V_{\ell\ell} I$ yields an error less than 0.33% when the rms value of all the triplen harmonics voltage is

$$\sqrt{\sum_{h = 0, 3, 6, \ldots} V_h^2} < 0.08 V_{\ell n}$$

These observations lead to the conclusion that for three-phase systems with nonsinusoidal wave forms the effective apparent power S_e and its components offer an improved set of definitions to better evaluate the power flow conditions (see 3.2.3.2).

3.2.3.1 Apparent power with Budeanu's Resolution

$$S = 3 V_{\ell n} I = \sqrt{P^2 + Q_B^2 + D_B^2}$$

where

$P = P_1 + P_H$ is the active power (W)

where

$$P_1 = 3 V_1 I_1 \cos\beta_1$$

$$P_H = 3 \sum_{h \neq 1} V_h I_h \cos\theta_h$$

$$\theta_h = \alpha_h - \beta_h$$

$Q_B = Q_1 + Q_{BH}$ is the Budeanu's reactive power (var)

where

$$Q_1 = 3V_1 I_1 \sin(-\beta_1)$$

$$Q_H = 3\sum_{h \neq 1} V_h I_h \sin\theta_h$$

$D_B = \sqrt{S^2 - P^2 - Q_B^2}$ is the Budeanu's distortion power (var).

The reactive power Q_B has a drawback that is explained in 3.1.2.7.

3.2.3.2 Effective apparent power (VA)

$$\| S_e = 3V_e I_e$$

For a four-wire balanced system,

$$V_e = V_{\ell n}$$

and

$$I_e = \sqrt{\frac{3I^2 + I_n^2}{3}} = \sqrt{I^2 + \sum_{h = 0, 3, 6, \ldots} I_h^2}$$

For a three-wire system,

$$V_e = V_{\ell\ell}/\sqrt{3}$$

$$I_e = I$$

and

$$S_e = S = \sqrt{3}V_{\ell\ell}I$$

NOTE—In a four-wire system, the apparent power $S < S_e$ and $P_F = P/S > P_{Fe} = P/S_e$.

The detailed resolution of S_e into practical components is presented in 3.2.4.3.

3.2.4 Three-Phase nonsinusoidal and unbalanced systems

This subclause covers the most general case. It deals with all the situations presented in the previous clauses.

3.2.4.1 Arithmetic apparent power (with Budeanu's Resolution) (VA)

This definition is an extension of Budeanu's apparent power resolution for single-phase systems. For each phase, a per phase apparent power is identifiable as follows:

$$S_a = \sqrt{P_a^2 + Q_{Ba}^2 + D_{Ba}^2}$$

$$S_b = \sqrt{P_b^2 + Q_{Bb}^2 + D_{Bb}^2}$$

$$S_c = \sqrt{P_c^2 + Q_{Bc}^2 + D_{Bc}^2}$$

From the above equations, the following arithmetic apparent power is obtained:

$$S_A = S_a + S_b + S_c$$

NOTE—The power factor $P_{FA} = P/S_A$ maintains the significance previously explained. However the major drawback of this definition stems from the difference between the quantities $[Q_{Ba} + Q_{Bb} + Q_{Bc}]^2 + [D_{Ba} + D_{Bb} + D_{Bc}]^2$ and $S_A^2 - P^2$ (see Figure 4).

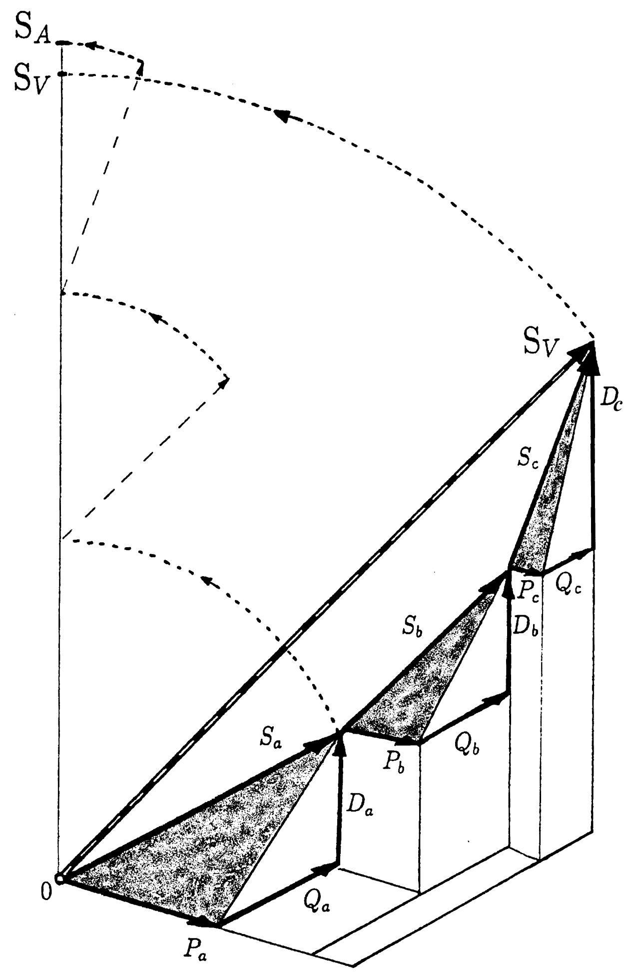


Figure 4—Arithmetic S_A, and Vector S_V, apparent powers: unbalanced nonsinusoidal conditions

3.2.4.2 Vector apparent power (VA), with Budeanu's Resolution

Using the same notations as in 3.2.4.1 applied to 3.2.2.5 results in the following:

$$S_V = \sqrt{P^2 + Q_B^2 + D_B^2}$$

where

$$P = P_a + P_b + P_c$$

$$Q_B = Q_{Ba} + Q_{Bb} + Q_{Bc}$$

$$D_B = D_{Ba} + D_{Bb} + D_{Bc}$$

NOTE—While this expression is free of the drawback discussed in the previous note, the problems with the Budeanu's reactive power also affect this apparent power resolution. Moreover, the fact that no flow direction can be assigned to D_B, limits the usefulness of this definition even more.

3.2.4.3 The effective apparent power and its resolution

In the past, S_e was divided into active power P and nonactive power N as follows:

$$S_e^2 = P^2 + N^2$$

This approach, however, does not separate out the positive-sequence fundamental powers. The approach used in 3.1.2.8 to 3.1.2.14 and 3.2.2.8 can be expanded for this situation. The rms effective current and voltage are divided into two components—the fundamental and the nonfundamental (Emanuel [B5], IEEE [B9]).

$$I_e = \sqrt{I_{e1}^2 + I_{eH}^2}$$

$$V_e = \sqrt{V_{e1}^2 + V_{eH}^2}$$

where for a four-wire system:

$$\| I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2 + I_n^2}{3}}$$

$$\| I_{e1} = \sqrt{\frac{I_{a1}^2 + I_{b1}^2 + I_{c1}^2 + I_{n1}^2}{3}}$$

$$I_{eH} = \sqrt{\frac{I_{aH}^2 + I_{bH}^2 + I_{cH}^2 + I_{nH}^2}{3}} = \sqrt{I_e^2 + I_{e1}^2} \,\|$$

$$\| V_e = \sqrt{\frac{1}{18}[3(V_a^2 + V_b^2 + V_c^2) + V_{ab}^2 + V_{bc}^2 + V_{ca}^2]}$$

$$V_{e1} = \sqrt{\frac{1}{18}[3(V_{a1}^2 + V_{b1}^2 + V_{c1}^2) + V_{ab1}^2 + V_{bc1}^2 + V_{ca1}^2]}$$

$$V_{eH} = \sqrt{\frac{1}{18}[3(V_{aH}^2 + V_{bH}^2 + V_{cH}^2) + V_{abH}^2 + V_{bcH}^2 + V_{caH}^2]} = \sqrt{V_e^2 - V_{e1}^2}$$

For three-wire systems, $I_{n1} = I_{nH} = 0$ and the expressions become simpler.

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2}{3}}$$

$$I_{e1} = \sqrt{\frac{I_{a1}^2 + I_{b1}^2 + I_{c1}^2}{3}}$$

$$I_{eH} = \sqrt{\frac{I_{aH}^2 + I_{bH}^2 + I_{cH}^2}{3}} = \sqrt{I_e^2 + I_{e1}^2}$$

$$V_e = \sqrt{\frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{9}}$$

$$V_{e1} = \sqrt{\frac{V_{ab1}^2 + V_{bc1}^2 + V_{ca1}^2}{9}}$$

$$V_{eH} = \sqrt{\frac{V_{abH}^2 + V_{bcH}^2 + V_{caH}^2}{9}} = \sqrt{V_e^2 - V_{e1}^2}$$

The resolution of $S_e = 3V_eI_e$ is implemented in the manner shown in 3.1.2.8 to 3.1.2.14.

$$S_e^2 = S_{e1}^2 + S_{eN}^2$$

where

$S_{e1} = 3V_{e1}I_{e1}$ is the fundamental effective apparent power,

S_{eN} is the nonfundamental effective apparent power. The resolution of S_{eN} is identical to the resolution of S_N given in 3.1.2.10.

$$S_{eN}^2 = S_e^2 - S_{e1}^2 = D_{e1}^2 + D_{eV}^2 + S_{eH}^2$$

The current distortion power, voltage distortion power, and harmonic apparent power are as follows:

$$D_{e1} = 3V_{e1}I_{eH}$$

$$D_{eV} = 3V_{eH}I_{e1}$$

$$S_{eH} = 3V_{eH}I_{eH}$$

and

$$D_{eH} = \sqrt{S_{eH}^2 - P_{eH}^2}$$

By defining the equivalent total harmonic distortions as follows:

$$THD_{eV} = \frac{V_{eH}}{V_{e1}}$$

$$THD_{eI} = \frac{I_{eH}}{I_{e1}}$$

practical expressions, identical to those found in 3.1.2.10 through 3.1.2.14, for the nonfundamental apparent power S_{eN} and its components D_{eI}, D_{eV}, and S_{eH} are obtained.

$$S_{eN} = S_{e1}\sqrt{THD_{eI}^2 + THD_{eV}^2 + (THD_{eI}THD_{eV})^2}$$

$$D_{eI} = S_{e1}(THD_I)$$

$$D_{eV} = S_{e1}(THD_V)$$

$$S_{eH} = S_{e1}(THD_I)(THD_V)$$

For systems with $THD_{eV} \leq 5\%$ and $THD_{eI} \geq 40\%$, the following approximation is recommended [B9]:

$$S_{eN} \approx S_{e1}(THD_{eI})$$

The load unbalance can be evaluated using the following fundamental unbalanced power:

$$S_{U1} = \sqrt{S_{e1}^2 - (S_1^+)^2}$$

where

S_1^+ is the fundamental positive-sequence apparent power (VA). This important apparent power contains the following components:

$P_1^+ = 3V_1^+I_1^+\cos\theta_1^+$ is the fundamental active power (W), and

$Q_1^+ = 3V_1^+I_1^+\sin\theta_1^+$ is the fundamental reactive power (var).

Together they result in

$$S_1^+ = \sqrt{(P_1^+)^2 + (Q_1^+)^2}$$

and the fundamental or the 60/50 Hz positive-sequence power factor

$$\| P_{F1}^+ = \frac{P_1^+}{S_1^+}$$

that plays the same significant role that the fundamental power factor has in nonsinusoidal single-phase systems.

The power factor is

$$\| P_F = \frac{P}{S_e}$$

The most important definitions are summarized in Table 2.

Table 2—Summary and grouping of quantities for three-phase systems with nonsinusoidal waveforms

Quantity or indicator	Combined	60 Hz powers (fundamental)	Non-60 Hz powers (nonfundamental)
Apparent	S_e (VA)	S_{e1} S_1^+ S_{1U} (VA)	S_{eN} S_{eH} (VA)
Active	P (W)	P_1^+ (W)	P_H (W)
Nonactive	N (var)	Q_1^+ (var)	D_{eI} D_{eV} D_{eH} (var)
Line utilization	$P_F = P/S_e$	$P_{F1}^+ = P_1^+/S_1^+$	—
Harmonic pollution	—	—	S_{eN}/S_{e1}
Load unbalance	—	S_{1U}/S_1^+	—

This table lists the three basic powers: apparent, active, and nonactive. The columns are partitioned into three groups—the combined powers, the 60 Hz powers (fundamental powers), and the non-60 Hz powers (nonfundamental powers). The last three rows give the indices: power factors (i.e., line utilization factor), harmonic pollution factor, and load unbalance factor.

Annex A

(informative)

Theoretical examples

A.1 Single-Phase nonsinusoidal

The circuit used for this example is presented in Figure A.1(a). Voltage *v* and current *i* waveforms are presented in Figure A.1(b). Normalized harmonic voltages and currents are summarized in Table A.1.

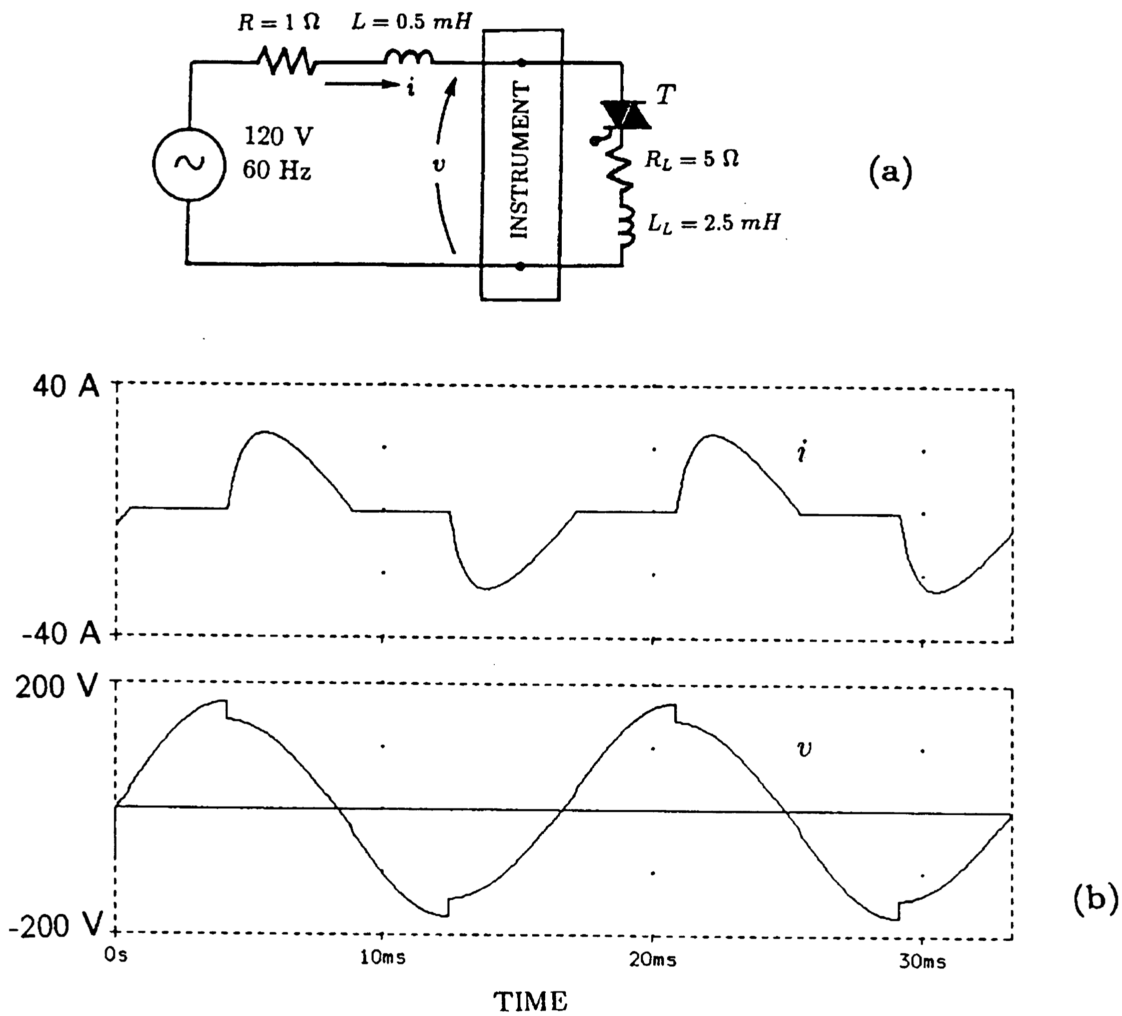


Figure A.1—Single-Phase circuit with thyristorized load
(a) Circuit diagram
(b) Simulated voltage and current waveforms

Table A.1—Percent Harmonic Voltage and Current Phasors.
Base Values: V_1 = 111.09 V; I_1 = 11.17 A

h	1	3	5	7	9	11	13	15
V_h (%)	100.00	5.98	1.74	2.00	1.15	1.14	0.85	0.89
α_h (deg)	23.80	–39.12	173.3	21.39	–120.10	87.24	–60.68	156.40
I_h (%)	100.00	51.20	12.78	12.47	5.38	5.51	3.10	3.04
β_h (deg)	–23.31	115.70	–43.38	161.30	16.11	–141.80	82.66	–79.19

The values of the rms, fundamental, and total harmonic voltage and current are as follows:

$$V = 110.35 \text{ V}$$

$$V_1 = 110.09 \text{ V}$$

$$V_H = 7.55 \text{ V}$$

$$I = 12.75 \text{ A}$$

$$I_1 = 11.17 \text{ A}$$

$$I_H = 6.15 \text{ A}$$

The total harmonic distortions of voltage and current are as follows:

$$THD_V = 0.069$$

$$THD_I = 0.549$$

Computations lead to the results summarized in Table A.2.

Table A.2—Percent powers
Base value: S_1 = 1229.70 VA

$S_e = 114.41$	$S_1 = 100.00$	$S_N = 55.59$ $S_H = 3.80$
$P = 66.35$	$P_1 = 69.61$	$P_H = -3.26$
$N = 93.20$	$Q_1 = 73.07$	$D_I = 55.06$ $D_V = 6.91$ $D_H = 1.96$
$P_F = P/S = 0.580$	$P_{F1} = P_1/S_1 = 0.696$	$S_N/S_1 = 0.556$
	$Q_B = 71.58$	$D_B = 59.69$

The nonlinear load is supplied with a 60 Hz active power $P_1 = 0.6961 \times 1229.7 = 856.04 \text{ W}$, and operates with a power factor $P_F = 0.580$ and a fundamental factor $P_{F1} = \cos\theta_1 = 0.696$. A small part of the 60 Hz active power is converted by the triac into harmonic power (returned to the power system) as follows:

$$P_H = -0.0326 \times 1229.70 = -40.1 \text{ W}$$

The fundamental current lags the fundamental voltage by an angle $\theta_1 = 23.80° + 23.31° = 47.11°$, yielding a 60 Hz reactive power as follows:

$$Q_1 = 0.7307 \times 1229.7 = 898.61 \text{ var}$$

Budeanu's reactive power $Q_B = 880.22 \text{ var}$, is smaller than Q_1.

The degree of distortion can be estimated with the ratio $S_N / S_1 = 0.556$, which is nearly equal to $THD_I = 0.549$.

The overall amount of *harmonic pollution* is quantified with the help of the non-60 Hz apparent power as follows:

$$S_N = 0.556 \times 1229.70 = 683.63 \text{ VA}$$

This value is nearly equal to the current distortion power $D_1 = 677.05 \text{ var}$. The small difference is due to the voltage distortion power $D_V = 84.97 \text{ var}$ and the harmonic apparent power $S_H = 46.73 \text{ VA}$.

The fundamental apparent power S_1 and its components P_1 and Q_1 make up the bulk of the apparent power S. Nevertheless, in this particular example, the nonfundamental apparent power S_N represents a significant amount of the total apparent power.

A.2 Three-Phase balanced nonsinusoidal system

The circuit is shown in Figure A.2. In this example, the third and the ninth harmonic currents are zero-sequence components and cause a large neutral current, which results in additional energy loss in the neutral conductor. Harmonic phasors obtained from this circuit simulation are summarized in Table A.3. Voltage and current components of higher interest have the following rms and total harmonic distortion values:

$$V_a = 279.94 \text{ V}; \quad V_{a1} = 277.25 \text{ V}; \quad V_{aH} = 38.70 \text{ V}; \quad THD_{Va} = 0.139$$

$$V_{ab} = 480.29 \text{ V}; \quad V_{ab1} = 480.20 \text{ V}; \quad V_{abH} = 9.55 \text{ V}; \quad THD_{Vab} = 0.020$$

$$I_a = 129.40 \text{ A}; \quad I_{a1} = 99.58 \text{ A}; \quad I_{aH} = 82.25 \text{ A}; \quad THD_{Ia} = 0.823$$

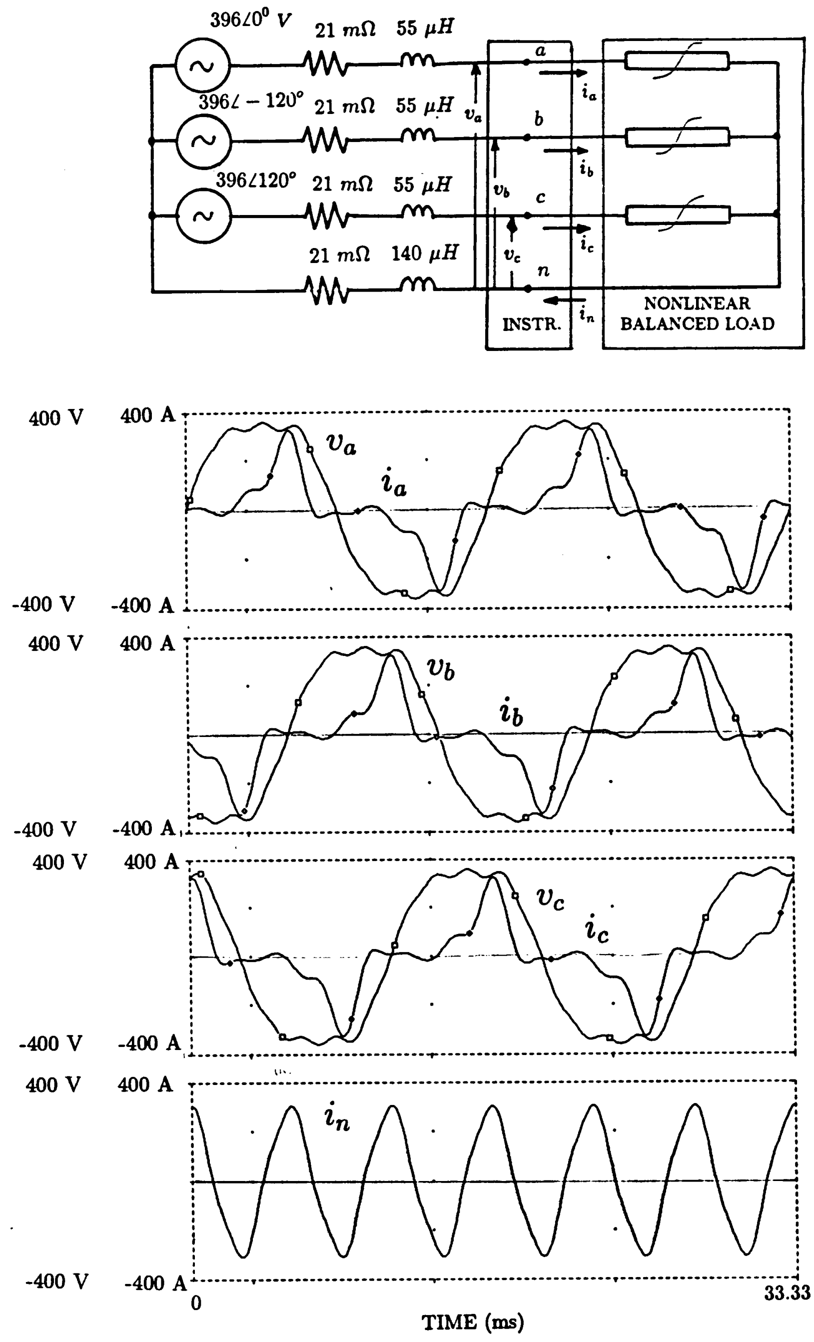
$$I_n = 207.20 \text{ A}; \quad I_{n1} = 0 \text{ A}; \quad I_{nH} = 207.20 \text{ A}$$

The neutral current has no 60 Hz, 300 Hz, or 420 Hz components (i.e., neither positive nor negative sequence components). The line-to-line voltage, however, lacks the 180 Hz and the 540 Hz components (zero-sequence). This situation is reflected in the following apparent power computations:

$$3_{V\ell n} I_a = 3 \times 279.94 \times 129.4 = 108.673 \text{ kVA}$$

$$\sqrt{3} V_{\ell\ell} I_a = \sqrt{3} \times 480.29 \times 129.4 = 107.646 \text{ kVA}$$

A 0.94% difference between these two values is observed.



(a)

(b)

Figure A.2—Three-Phase, four-wire circuit with a nonlinear balanced load
(a) Circuit diagram
(b) Simulated voltage and current waveforms

Table A.3—Percent harmonic voltage and current phasors
Base values: V_1 = 277. 25 V; I_1 = 99.98 A

h	1	3	5	7	9
$V_{\ell nh}$ (%)	100.00	13.26	1.34	1.48	3.48
α_h (deg)	–2.35	7.76	83.54	–163.20	–42.74
I_h (%)	100.00	68.84	34.90	27.84	5.94
β_h (deg)	–22.00	100.00	–175.00	–65.01	47.99
I_{nh} (%)	0	206.52	0	0	17.82
$V_{\ell\ell h}$ (%)	173.20	0	2.31	2.55	0

The normalized equivalent voltages and currents are as follows:

$$V_e = \sqrt{\frac{9V_a^2 + 3V_{ab}^2}{18}} = 1.005; \quad V_{e1} = \sqrt{\frac{9V_{a1}^2 + 3V_{ab1}^2}{18}} = 1.000; \quad V_{eH} = \sqrt{V_e^2 - 3V_{e1}^2} = 0.099$$

$$I_e = \sqrt{\frac{3I_a^2 + I_n^2}{3}} = 1.762; \quad I_{e1} = \sqrt{\frac{3I_{a1}^2 + I_{n1}^2}{3}} = 1.000; \quad I_{eH} = \sqrt{I_e^2 + I_{e1}^2} = 1.451$$

$$THD_{eV} = 0.099; \quad THD_{eI} = 1.451$$

The fundamental apparent power $S_{e1} = 3V_{e1}I_{e1} = 3 \times 277.25 \times 99.98 = 83.158\ \text{kVA}$, is chosen as the base value for the normalized power values given in Table A.4.

Table A.4—Percent powers
Base value: $S_{e1} = 3 \times 277.75 \times 99.98 = 83.158$ kVA

$S_e = 177.13$	$S_{e1} = S_1^+ 100$ $S_{UI} = 0$	$S_{eN} = 146.19$ $S_{eH} = 14.45$
$P = 93.67$	$P_1 = P_1^+ = 94.18$ $P_1^- = 0$	$P_H = -0.513$
$N = 93.20$	$Q_1 = Q_1^+ = 33.62$ $Q_1^- = 0$	$D_{eI} = 145.14$ $D_{eV} = 9.95$ $D_{eH} = 14.45$
$S_A = S_V = 130.68$	$Q_B = 23.36$	$D_B = 88.07$

In a balanced system the effective fundamental apparent power equals the fundamental positive-sequence apparent power, $S_{e1} = S_1^+$. The arithmetic and vector apparent powers are also equal.

$$S_A = S_V = 3V_aI_a = 108.67\ \text{kVA}$$

The normalized values are $S_A = S_V = 130.68\%$. The normalized active power is $P = 93.67\%$ with a normalized fundamental active power $P_1 = 94.18\%$. These active and apparent powers give the following power factors:

$$P_{Fe} = \frac{P}{S_e} = 0.529$$

$$P_{FA} = P_{FV} = \frac{P}{S_A} = 0.717$$

$$P_{F1}^{+} = \cos\theta_1^{+} = \frac{P_1^{+}}{S_1^{+}} = 0.942$$

Significant differences are apparent among these three power factor values. The effective apparent power yields the lowest power factor. This is due to the fact that $S_e > S_A$. The equivalent current I_e covers the thermal effect of the neutral current I_n, hence $I_e > I_a$. The definition of I_e is based on the equivalence of the total line power loss, neutral current included.

$$\Delta P_e = 3rI_e^2 = 3 \times 0.021 \times (1.762 \times 99.98)^2 = 1956 \text{ W}$$

The same result is obtained by using the following expression:

$$\Delta P_e = 3rI_a^2 + r_n I_n^2 = 3 \times 0.021 \times 129.4^2 + 0.021 \times 207.2^2 = 1055 + 901 = 1956 \text{ W}$$

i.e., 1055 W is due to the line currents and 901 W is due to the neutral current.

The vector apparent power S_V definition ignores the fact that, in spite of the balanced load and symmetrical voltages, there is a substantial neutral current. This becomes evident when comparing the following expression:

$$\Delta P_V = r\frac{S_V^2}{V_{ab}^2} = 0.021\left[\frac{107646}{480.29}\right]^2 = 1055 \text{ W}$$

with

$$\Delta P_e = r\frac{S_e^2}{3V_e^2} = 0.021\left[\frac{147298}{\sqrt{3} \times 278.62}\right]^2 = 1956 \text{ W}$$

The harmonic active power $P_H = -0.513\%$ is not a good indicator of the harmonic pollution magnitude. Its value depends not only on the I_{eH} or S_{eN}, but also on the system's Thevenin impedance. A larger r will increase P_H, while S_{eN} will remain practically unchanged. The correct way to evaluate harmonic pollution is with $S_{eN} \approx S_{e1}(THD_{eI})$.

A.3 Three-Phase unbalanced nonsinusoidal system

The previous example was modified by disconnecting phase c, Figure A.3. A capacitor $C = 338\ \mu F$ was connected between terminals b and n to enhance the difference between the current spectra of i_a and i_b.

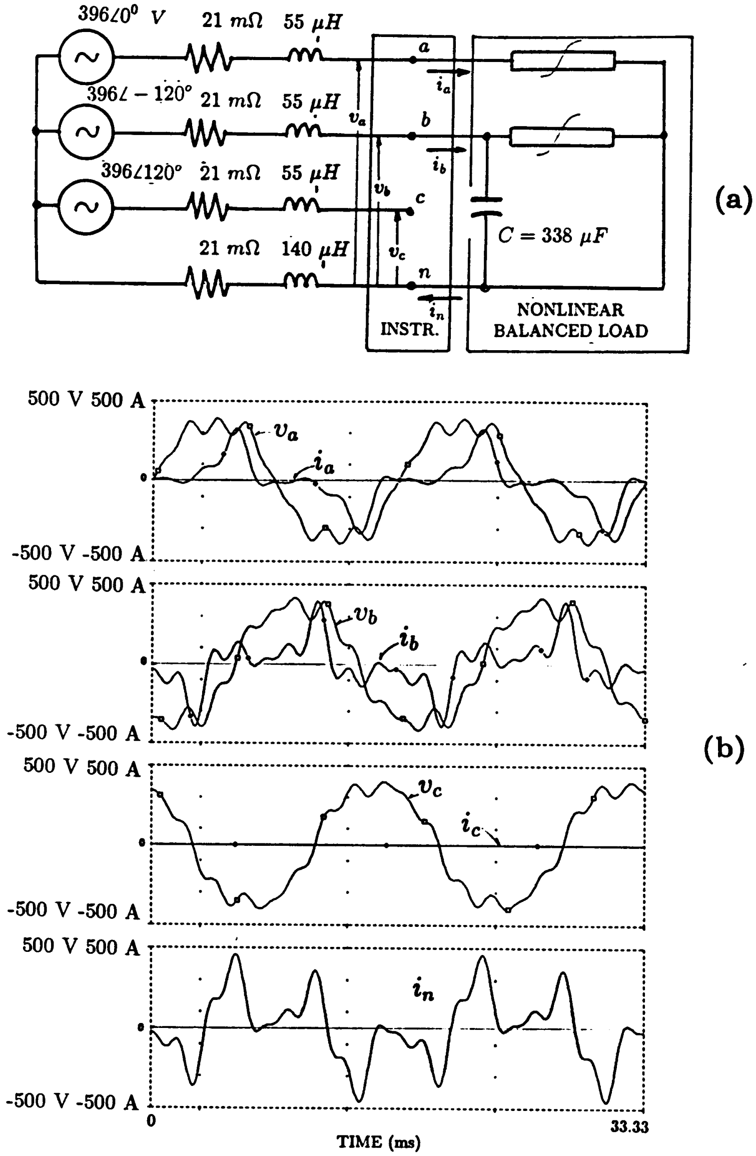


Figure A.3—Three-Phase, four-wire circuit with a nonlinear unbalanced load
(a) Circuit diagram
(b) Simulated voltage and current waveforms

The percent harmonic voltage and current phasors are given in Table A.5.

Table A.5—Percent harmonic voltage and current phasors
Base values: V_{a1} = 271.03 V; I_{a1} = 99.98 A

h	1	3	5	7	9
V_{ah} (%)	100.00	10.28	4.92	7.44	8.64
α_{ah} (deg)	–0.74	6.76	142.30	146.70	–47.40
I_{ah} (%)	100.00	68.84	34.90	27.85	5.93
β_{ah} (deg)	–22.00	100.00	–175.00	–65.00	48.00
V_{bh} (%)	104.49	10.53	5.79	8.58	11.05
α_{bh} (deg)	–121.20	6.28	167.40	125.20	–49.19
I_{bh} (%)	93.49	79.77	42.30	45.81	40.59
β_{bh} (deg)	–120.80	99.49	65.09	–167.90	41.89
V_{ch} (%)	103.73	8.69	4.30	6.58	8.22
α_{ch} (deg)	121.30	9.70	157.70	136.50	–47.35
I_{ch} (%)	0	0	0	0	0
I_{nh} (%)	178.12	21.01	63.28	67.87	65.75
V_{abh} (%)	177.56	0.26	2.48	3.19	2.43
V_{bch} (%)	177.93	1.93	1.65	2.49	2.84
V_{cah} (%)	178.28	1.67	1.36	1.51	0.41

Some of the important computed quantities are as follows:

$$V_a = 274.53\text{ V};\quad V_{a1} = 271.03\text{ V};\quad V_{aH} = 43.70\text{ V}\quad THD_{Va} = 0.161$$

$$V_b = 287.57\text{ V};\quad V_{b1} = 283.20\text{ V};\quad V_{bH} = 49.94\text{ V};\quad THD_{Vb} = 0.176$$

$$V_c = 283.81\text{ V};\quad V_{c1} = 281.13\text{ V};\quad V_{cH} = 38.85\text{ V};\quad THD_{Vc} = 0.138$$

$$V_{ab} = 481.43\text{ V};\quad V_{ab1} = 481.26\text{ V};\quad V_{abH} = 12.79\text{ V};\quad THD_{Vab} = 0.027$$

$$V_{bc} = 482.41\text{ V};\quad V_{bc1} = 482.25\text{ V};\quad V_{bcH} = 12.34\text{ V};\quad THD_{Vbc} = 0.026$$

$$V_{ca} = 483.22\text{ V};\quad V_{ca1} = 483.17\text{ V};\quad V_{caH} = 7.22\text{ V};\quad THD_{Vca} = 0.015$$

$I_a = 129.40 \text{ A}; \quad I_{a1} = 99.98 \text{ A}; \quad I_{aH} = 82.25 \text{ A}; \quad THD_{Ia} = 0.823$

$I_b = 143.64 \text{ A}; \quad I_{b1} = 93.48 \text{ A}; \quad I_{bH} = 109.07 \text{ A}; \quad THD_{Ib} = 1.167$

$I_c = 0 \text{ A}; \quad I_{c1} = 0 \text{ A}; \quad I_{cH} = 0 \text{ A};$

$I_n = 210.66 \text{ A}; \quad I_{n1} = 125.93 \text{ A}; \quad I_{nH} = 168.90 \text{ A}; \quad THD_{In} = 1.341$

The equivalent voltages and currents are as follows:

$$V_e = \sqrt{\frac{3(V_a^2 + V_b^2 + V_c^2) + V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{18}} = 280.25 \text{ V}$$

$$V_{e1} = \sqrt{\frac{3(V_{a1}^2 + V_{b1}^2 + V_{c1}^2) + V_{ab1}^2 + V_{bc1}^2 + V_{ca1}^2}{18}} = 278.45 \text{ V}$$

$$V_{eH} = \sqrt{V_e^2 + V_{e1}^2} = 31.68 \text{ V}$$

$$THD_{Ve} = 0.114$$

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2 + I_n^2}{3}} = 165.08 \text{ A}$$

$$I_{e1} = \sqrt{\frac{I_{a1}^2 + I_{b1}^2 + I_{c1}^2 + I_{n1}^2}{3}} = 107.38 \text{ A}$$

$$I_{eH} = \sqrt{I_e^2 + I_{e1}^2} = 125.41 \text{ A}$$

$$THD_{Ie} = 1.168$$

The following are several key values of computed phasors:

$V_{a1} = 271.03\angle{-0.74°} \text{ V}; \quad V_{b1} = 283.2\angle{-121.2°} \text{ V}; \quad V_{c1} = 281.13\angle{-121.3°} \text{ V}$

$V_1^+ = 288.49\angle{-0.20°} \text{ V}; \quad V_1^- = 0.63\angle{-83.36°} \text{ V}; \quad V_1^0 = 2.98\angle{-143.04°} \text{ V}$

$I_{a1} = 99.98\angle{-22°} \text{ A}; \quad I_{b1} = 93.48\angle{-120.8°} \text{ A}; \quad I_{c1} = 0 \text{ A}$

$I_1^+ = 63.39\angle{-11.76°} \text{ A}; \quad I_1^- = 21.52\angle{-43.15°} \text{ A}; \quad I_1^0 = 42.00\angle{-69.15°} \text{ A}$

Powers normalized to $S_{e1} = 3 \times 278 \times 107.38 = 89.70$ kVA are summarized in Table A.6.

The arithmetic apparent power is as follows:

$$S_A = S_a + S_b + S_c = 35.524 + 41.306 + 0 = 76.83 \text{ kVA}$$

The vector apparent power is as follows:

$$S_V = \sqrt{P^2 + Q_B^2 + D_B^2} = \sqrt{51.333^2 + 2.349^2 + 55.871^2} = 75.91 \text{ kVA}$$

The effective apparent power is as follows:

$$S_e = 3V_eI_e = 3 \times 280.25 \times 165.08 = 138.79 \text{ kVA}$$

The total active power is as follows:

$$P = P_a + P_b + P_c = 24.998 + 26.335 + 0 = 51.33 \text{ kW}$$

The power factors are as follows:

$$P_{FA} = \frac{51.33}{76.83} = 0.668$$

$$P_{FV} = \frac{51.33}{75.91} = 0.676$$

$$P_{Fe} = \frac{51.33}{138.79} = 0.370$$

Table A.6—Percent powers
Base value: $S_{e1} = 3 \times 278 \times 107.38 = 89.70$ kVA

$S_e = 154.73$	$S_{e1} = 100.00$ $S_1^+ = 59.04$ $S_{U1} = 78.88$	$S_{eN} = 118.08$ $S_{eH} = 13.29$
$P = 57.23$	$P_1 = 57.67$ $P_1^+ = 57.84$ $P_1^- = -0.03$ $P_1^0 = -0.14$	$P_H = -0.44$
$N = 143.76$	$Q_1^+ = 11.83$ $Q_1^- = -0.03$ $Q_1^0 = -0.40$	$D_{eI} = 116.77$ $D_{eV} = 11.38$ $D_{eH} = 13.28$
$S_V = 84.62$	$Q_B = 2.62$	$D_B = 62.28$
$S_A = 85.65$		

The 60 Hz positive-sequence powers give the following:

$$P_{F1}^{+} = \cos\theta_1^{+} = \frac{P_1^{+}}{S_1^{+}} = \frac{51.88}{52.96} = 0.980$$

The large differences between the power factor values are due to large differences among apparent powers values.

As in the previous example the total line losses are as follows:

$$\Delta P_e = 3rI_e^2 = 3 \times 0.021 \times 165.08^2 = 1717.8\ \mathrm{W}$$

The neutral conductor power loss, included in ΔP_e, is as follows:

$$\Delta P_n = rI_n^2 = 0.021 \times 210.66^2 = 931.9\ \mathrm{W}$$

The total harmonic distortion of the equivalent current is as follows:

$$THD_{Ie} = \frac{I_{eH}}{I_{e1}} = 1.168$$

and due to the neutral current contribution exceeds phase current distortions.

The ratio $D_{eI}/S_{e1} = 1.117$ indicates a significant nonsinusoidal situation that is quantified by S_{eN} = 100.29 kVA.

This example demonstrates that large differences can occur between Q_B and Q_1^{+}; 2.62% versus 11.83%. This issue is further emphasized in Annex B, which presents actual field studies.

Annex B

(informative)

Practical studies and measurements

B.1 Power survey at a large industrial plant[5]

In 1997, a team of South African engineers (Pretorius, van Wyk, and Swart [B11]) studied and compared a few methods of power resolutions for accuracy, usefulness, and suitability for revenue purpose. The following conclusions were reached by the surveying team:

a) Budeanu's method gives erroneous results.
b) The IEEE Working Group method (presented in this Trial-Use standard), based on the Buchholz-Goodhue definition of effective apparent power S_e and its resolution in S_{e1} and S_{eN} "is encouraged because it is merely an extension of the classical sinusoidal definitions that are already well understood."
c) The IEEE Working Group method "accurately describes the rating of power compensation equipment, that ought to be used in practice and is a good resolution method to indicate the severity of distortion in practical power systems."
d) Other methods studied did yield "mathematically correct results, but are difficult to apply in practice due to measurement problems," the stumbling block being the need for precise measurement of the active power per each harmonic order.

The diagram of the studied system is shown in Figure B.1. This facility is supplied with three-phase, 50 Hz, 11 kV/380 V, from two separate 2MVA, 5.69%, Δ/Y transformers. The major loads consist of 13 pulsed power supplies, ac/dc converters (regulating pulse units) with the following nominal values:

380 V
78.7 A
60.35 kVA
$THD_I \approx 38\%$

This location was selected for its ideal conditions, created by the fact that several passive and active filters were already installed and permission was obtained to connect and disconnect the filters. Some of the results published in Pretorius, van Wyk, and Swart [B11] are listed and discussed in the following:

1. MEASUREMENTS AT THE PRIMARY SIDE OF TRANSFORMER 1

All the filters were disconnected. Voltage and current waveforms and spectra are presented in Figure B.2.

The most important quantities measured are as follows:

$$V_{\ell n} = 6445.1 \text{ V}; \quad I_\ell = 51.36 \text{ A}; \quad THD_V = 0.0016; \quad THD_I = 0.718$$

$$P = 653.78 \text{ kW}; \quad P_5 = -90.7 \text{ W}; \quad P_7 = -50 \text{ W}; \quad P_{11} = -11.3 \text{ W}; \quad P_{13} = -1.7 \text{ W}$$

$$PF_1 = 0.811; \quad PF = 0.658$$

[5]This information is provided with permission from Pretorius, van Wyk, and Swart [B11].

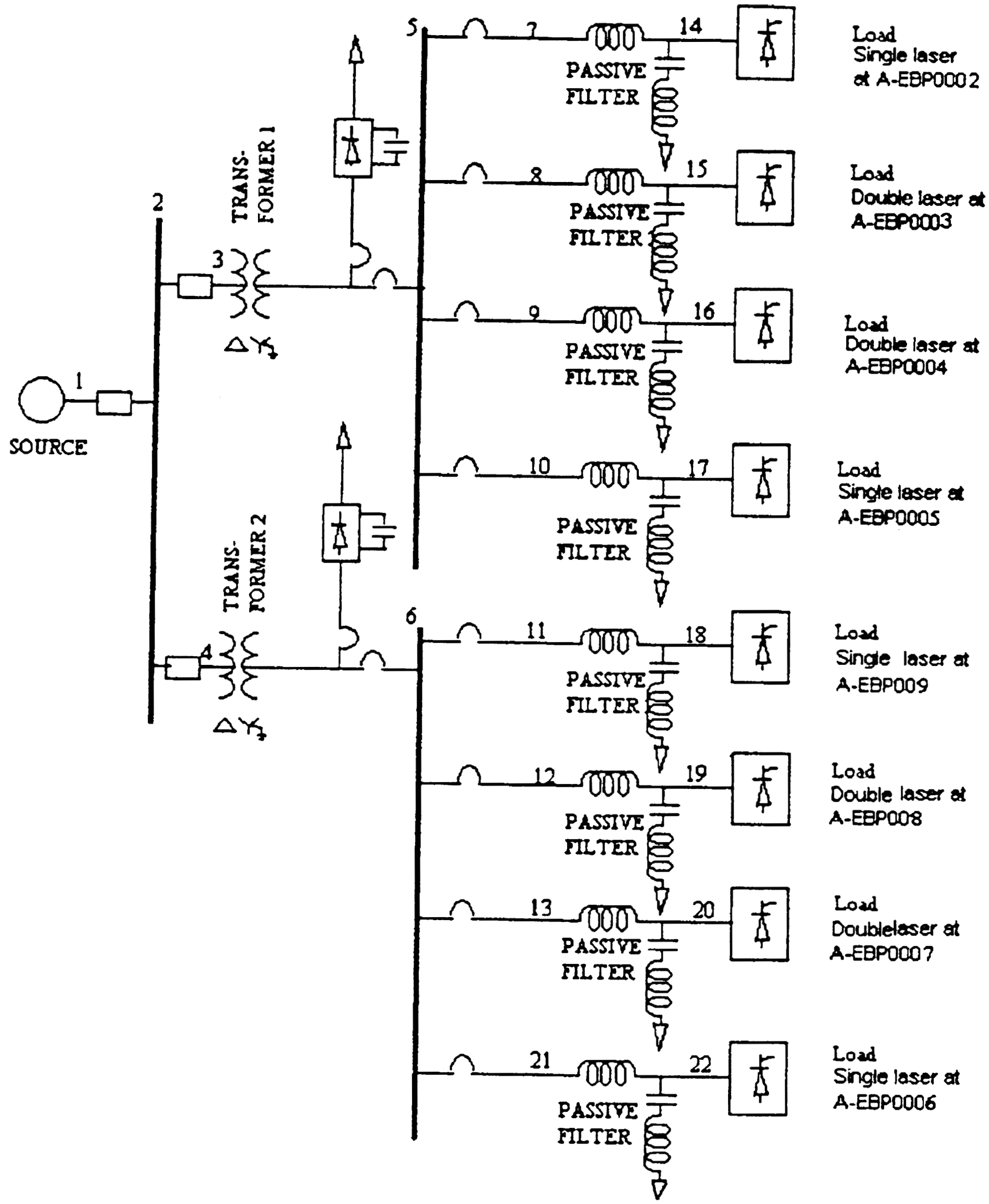


Figure B.1—Schematic of the surveyed industrial plant

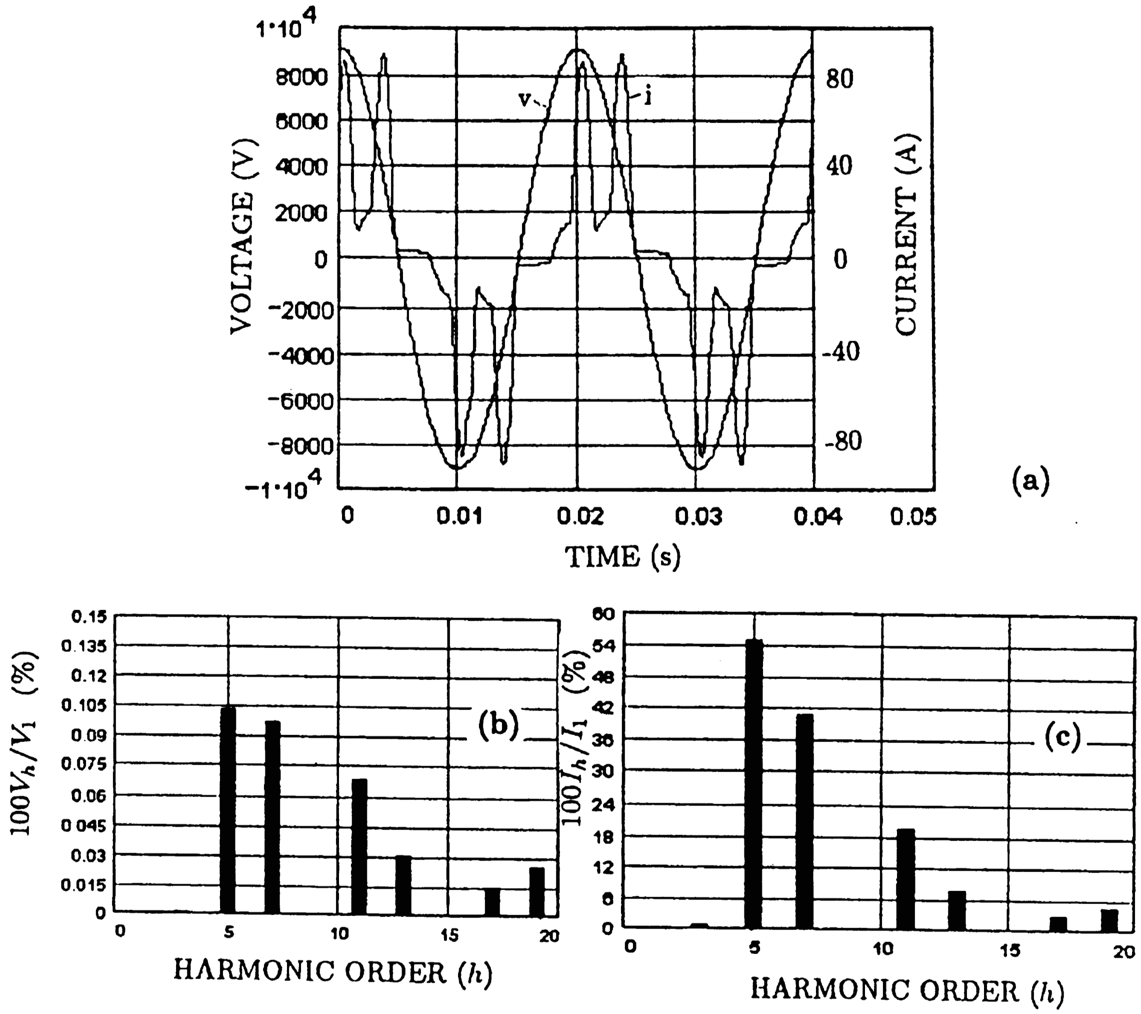


Figure B.2—Measurements at the primary side of Transformer 1
(a) Line-to-Line voltage and line current
(b) Normalized harmonic spectrum
(c) Normalized current spectrum

Values of the measured powers are given in Table B.1.

Table B.1—Powers measured at the primary side of transformer 1

S = 993.08 kVA	S_1 = 806.52 kVA S_{U1} = 0.33 kVA	S_N = 579.42 kVA S_H = 0.96 kVA
P = 653.78 kW	P_1 = 653.93 kW	P_H = –157 W
	Q_1 = 472.07 kvar	D_I = 579.40 kvar D_V = 1.33 kvar D_H = 0.94 kvar
	Q_B = 471.16 kvar	D_B = 580.35 kvar

"The fundamental apparent power (806 kVA) and the fundamental reactive power (472 kvar) indicate the value of the fundamental power factor correction capacitors to overcome the effects of relatively large firing angles used in natural commutated phase-control converters. (This reactive power can be corrected by means of static capacitors used in passive filter configuration.) The nonfundamental apparent power (579 kVA) and the nonfundamental reactive power (not shown) furnishes indications of the required dynamic compensator capacity when used for nonfundamental distortion correction alone." Pretorius, van Wyk, and Swart [B11].

2. MEASUREMENTS AT THE SECONDARY SIDE OF TRANSFORMER 1

Passive harmonic filters were connected at the secondary of each of the two transformers. No dynamic filters were energized. Voltage and current waveforms and spectra are presented in Figure B.3.

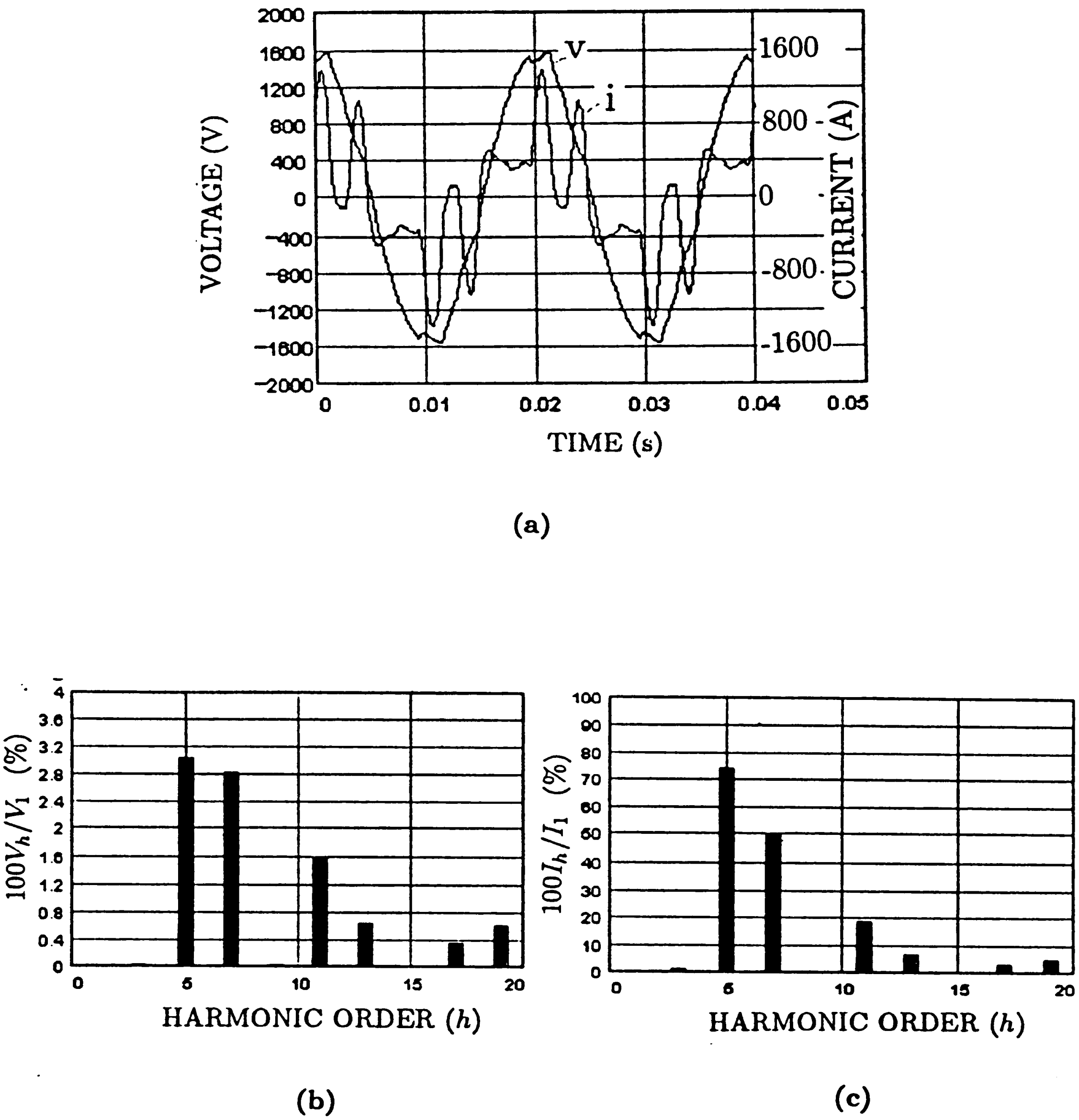


Figure B.3—Measurements at the secondary side of transformer 1
(a) Line-to-Line voltage and line current
(b) Normalized harmonic spectrum
(c) Normalized current spectrum

The most important quantities measured are as follows:

$V_{\ell n} = 242.9$ V ; I = 616.6 A $THD_V = 0.046$; $THD_I = 0.913$

$P = 331.29$ kW; $P_5 = -105.5$ W; $P_7 = -47.6$ W; $P_{11} = -6.3$ W; $P_{13} = -0.7$ W

$P_{F1} = 1.000$; $P_F = 0.737$

Values of the measured powers are given in Table B.2.

Table B.2— Powers measured at the secondary side of Transformer 1

$S = 449.25$ kVA	$S_1 = 331.46$ kVA $S_{U1} = 0.37$ kVA	$S_N = 303.25$ kVA $S_H = 13.91$ kVA
$P = 331.29$ kW	$P_1 = 331.46$ kW	$P_H = -163$ W
—	$Q_1 = 0.94$ kvar	$D_I = 302.53$ kvar $D_V = 15.24$ kvar $D_H = 13.91$ kvar
—	$Q_B = -12.47$ kvar	$D_B = 303.17$ kvar

The existence of filters causes phase shifting of harmonic phasors that produce $Q_B = -12.47$ kvar < 0. Comparing $Q_1 = 0.94$ kvar with $Q_B = -12.47$ kvar, it becomes obvious that Budeanu's Resolution "cannot be used to design or to specify ratings of equipment in any situation and its use in industry must be abandoned in total."

Annex C

(informative)

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