Geomate Suparman.pdf

International Journal of GEOMATE, Month,Year, Vol (Issue), pp. 000-000
Geotec., Const. Mat. & Env., ISSN:2186-2990, Japan, DOI: http://dx.doi.org/10.21660/YourID

A NEW ESTIMATION PROCEDURE USING A REVERSIBLE JUMP MCMC ALGORITHM FOR AR MODELS OF EXPONENTIAL WHITE NOISE

*Suparman[1]

[1]Faculty of Teacher Training and Education, University of Ahmad Dahlan, Indonesia

*Corresponding Author, Received: 00 Oct. 2018, Revised: 00 Nov. 2018, Accepted: 00 Dec. 2018

ABSTRACT: White noise using an autoregressive (AR) model is often assumed to be normally distributed. In application however, the white noise usually does not follow a normal distribution. This paper aims to estimate a parameter for an AR model that has an exponential white noise. A Bayesian method is adopted. A prior distribution of the parameter of an AR model is selected and then this prior distribution is combined with a likelihood function of data to get a posterior distribution. Based on this posterior distribution, a Bayesian estimator for the parameter of the AR model is estimated. Because the order of the AR model is considered a parameter, this Bayesian estimator cannot be explicitly calculated. To resolve this problem, a method using a reversible jump Markov Chain Monte Carlo (MCMC) is adopted. The result is an estimation of the parameter for the AR model that can be simultaneously calculated.

Keywords: AR Model, Exponential White Noise, Bayesian, Reversible Jump Markov Chain Monte Carlo

1. INTRODUCTION

An autoregressive (AR) model with normally distributed white noise is a time series model that is often used in many fields. For example, it is used in the field of economics [1]. But there are so many applications where white noise is not normally distributed. An LSE of AR models with heavy-tailed G-GARCH(1,1) noises were studied [1]. A class of non parametric tests on the Pareto tail index of the innovation distribution in the linear autoregressive model is proposed [2]. A study of the autoregressive models with exponential white noise can be found in the literature (see [3-8]). A form of time series models where marginal distributions are in fact exponential distributions is presented in [3]. A Bayesian analysis of threshold AR models with exponential noise is developed in [4]. A robust study of the Bayesian estimation of an AR model with exponential innovation to obtain optimal Bayesian estimator is analyzed [5]. A Bayesian method to estimate the coefficient of the AR(1) models is proposed [6]. Generally, the order of the autoregressive is known and must be estimated by the data.

If the AR model with white noise is fitted to the data, the order and the coefficient of the model will be generally unknown. Let x_t be a time series with $t = 1,2,\cdots,n$ and n be the number of samples. An AR(p) with exponential white noise can be expressed as:

$$x_t = \sum_{i=1}^{p} \phi_i x_{t-i} + z_t \quad (1)$$

where $p < n$ and the $z_t\,(t = 1,\cdots,n)$ are independent and identical exponential random variables with parameter λ, written $z_t \sim \mathrm{Exp}(\lambda)$. For example, Figure 1 shows the graph of the autoregressive model with and $\lambda = 5$.

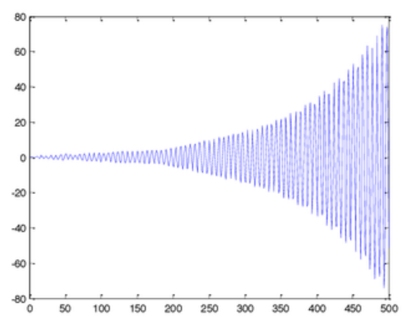

Fig. 1 AR(3) with exponential white noise

Suppose that $\phi^{(p)} = (\phi_1,\cdots,\phi_p)$ is a coefficient vector. Let ψ be the above autoregressive model. Then this parameter ψ can be written as

$$\psi = (p, \phi^{(p)}, \lambda)$$

Suppose that x_t $(t = 1,2,\cdots,n)$ are data. This data is taken from a population having an autoregressive model with exponential white noise. Based on this data, the main problem becomes how to estimate the parameter ψ. This paper aims to provide a procedure to estimate this parameter ψ.

2. METHOD

The parameter ψ is estimated by using a Bayesian method. Unfortunately, the Bayesian estimator cannot be determined analytically because the posterior distribution of parameter ψ has a complicated form. To overcome these problems, a reversible jump MCMC Algorithm [9] is used. An MCMC method is a method producing an ergodic Markov chain with a stationary distribution. This Markov chain can be considered as a random variable whose distribution is the posterior distribution. Furthermore, this Markov chain is then used to estimate the parameter ψ.

3. RESULTS AND DISCUSSION

The parameter ψ is estimated by using a Bayesian method and a likelihood function is determined.

3.1 Likelihood Function

Because the random variable z_t has an exponential distribution with parameter λ for $t=1,2,\cdots,n$, the density function of z_t is

$$f(z_t|\ \lambda)=\lambda\exp-\lambda z_t \tag{2}$$

The variable transformation

$$x_t=\sum_{i=1}^{p}\phi_i x_{t-i}+z_t \tag{3}$$

is used. Then $z_t=x_t-\sum_{i=1}^{p}\phi_i x_{t-i}$ and $\frac{dz_t}{dx_t}=1$.

Let $x=(x_1,x_2,\cdots,x_n)$ be a realization vector of AR(p) with an exponential error. Thus, the density function of x_t is

$$f(x_t|\ \psi)=\lambda\ \exp-\lambda(x_t-\sum_{i=1}^{p}\phi_i x_{t-i}) \tag{4}$$

for $t=1,2,\cdots,n$.

Suppose that $y_0=(x_1,x_2,\cdots,x_p)$ and $y=(x_{p+1},x_{p+2},\cdots,x_n)$. Then the likelihood function of y can be approximated by:

$$L(y|\ \psi)=\lambda^{n-p}\exp-\lambda\sum_{t=p+1}^{n}g(t,p,\phi^{(p)}) \tag{5}$$

where

$$g(t,p,\phi^{(p)})=x_t-\sum_{i=1}^{p}\phi_i x_{t-i} \tag{6}$$

for $t=p+1,2,\cdots,n$, with an initial value y_0. Let S_p be a stationary region and $r^{(p)}=(r_1,r_2,\cdots,r_p)$ be a sample partial autocorrelation vector. By using a transformation

$$F:\phi^{(p)}\in S_p\rightarrow r^{(p)}\in[-1,1]^p \tag{7}$$

then the AR(p) model is stationary if and only if $r^{(p)}\in[-1,1]^p$. Finally, the approximated likelihood function of the y can be written by:

$$L(y|\ \theta)=\lambda^{n-p}\exp-\lambda\sum_{i=p+1}^{n}g(t,p,F^{-1}(r^{(p)})) \tag{8}$$

where $\theta=(p,r^{(p)},\lambda)$ and F^{-1} is a inverse transformation of F.

3.2 Prior Distribution

Before obtaining a posterior distribution, a prior distribution must be selected. The prior distribution is taken as follows. A binomial distribution is chosen for a number of order p ($p=1,2,\cdots,p_{max}$)

$$\pi(p|\varphi)=C_p^{p_{max}}\varphi^p(1-\varphi)^{p_{max}-p} \tag{9}$$

where p_{max} is a maximum of p and μ is an hyper-parameter. A uniform distribution is chosen for a coefficient vector $r^{(p)}$

$$\pi(r^{(p)}|\ p)=U(0,1)^p \tag{10}$$

Also, a uniform distribution is chosen for a parameter λ

$$\pi(\lambda)=U(0,1) \tag{11}$$

Furthermore, a hyper-prior distribution for φ is a uniform distribution.

Let $\pi(\theta,\varphi)$ be a prior distribution for (θ,φ). Because the distribution of θ given φ is $\pi(\theta|\ \varphi)=\frac{\pi(\theta,\varphi)}{\pi(\varphi)}$, the prior distribution for (θ,φ) can be written as follows:

$$\pi(\theta,\varphi)=\pi(\theta|\ \varphi)\pi(\varphi) \tag{12}$$

3.3 Posterior Distribution

Let $\pi(\theta,\varphi|\ y)$ be a posteriori distribution for the parameter and the hyper-parameter (θ,φ). According to the Bayesian Theorem, the posterior distribution for (θ,φ) is given as follows

$$\begin{aligned}\pi(\theta,\varphi|\ y)&\propto f(y|\ \theta)\pi(\theta,\varphi)\\ &\propto f(y|\ \theta)\pi(\theta,\varphi)\ \pi(\theta|\ \varphi)\pi(\varphi)\end{aligned} \tag{13}$$

Unfortunately, the Bayesian estimator cannot be determined analytically because the posterior distribution of parameter θ and hyper-paramater φ has a complicated form. To overcome these problems, reversible jump MCMC Algorithm [9] is used.

3.4 Reversible Jump MCMC

Suppose that $M=(\theta,\varphi)$. An MCMC method for the simulation of a distribution $\pi(\theta,\varphi|\ y)$ produces an ergodic Markov chain $M_1,M_2,\cdots,M_m$ whose stationary distribution is $\pi(\theta,\varphi|\ y)$. This Markov chain $M_1,M_2,\cdots,M_m$ can be considered as a random variable whose distribution is $\pi(\theta,\varphi|\ y)$. Furthermore, the Markov chain $M_1,M_2,\cdots,M_m$ is used to estimate the parameter M. To realize this, the Gibbs sampling algorithm is adopted. It consists of three steps:

- Simulate $\varphi \sim B(p+1, p_{max}-p+1)$
- Simulate $\lambda \sim G(\alpha,\beta)$ with $\alpha = n-p+1$ and $\beta = \sum_{t=p+1}^{n} (x_t - \sum_{i=1}^{p} F^{-1}(r_i) x_{t-i})$
- Simulate $(p, r^{(p)}) \sim \pi(p, r^{(p)}|\ y,\lambda,\varphi)$

Unfortunately, the distribution $\pi(p,r^{(p)}|\ y,\lambda,\varphi)$ is not an explicit form. The exact simulation cannot possibly be done. Since the value p is not known, the MCMC algorithm cannot be used to simulate $\pi(p,r^{(p)}|\ y,\lambda,\varphi)$. Hence the reversible jump MCMC algorithm [9] is adopted.

Let $\omega=(p,r^{(p)})$ be an actual point of the Markov chain. There are 3 types of transformations used, namely: a birth of the order; a death of the order; and a change of the order. Furthermore, let N_p be the probability of the transformation from p to p + 1, let D_p be the probability of transformation from p + 1 to p, and let C_p be the probability of transformation from p to p.

3.4.1 Birth/Death of the Order

A transformation of the birth of the order will change a number of coefficients, from p to the p + 1. Suppose that $\omega=(p,r^{(p)})$ is a current point and $\omega^*=(p+1,r^{(p^*)})$ is an updated point. The birth of the order from $\omega=(p,\phi^{(p)})$ to $\omega^*=(p+1,r^{(p^*)})$ is defined in the following way. Set $p^*=p+1$ and choose a random point $\nu \sim U(-1,1)$. Next, create a new point $\omega^*=(p+1,r^{(p^*)})$ with

$$r_1^* = r_1,\ \ldots,\ r_{p^*-1}^* = r_p,\ r_{p^*}^* = \nu$$

Otherwise, the transformation of the death of the order will change the number of coefficients, from p+1 to p. Suppose that $\omega=(p+1,r^{(p+1)})$ is a current point and $\omega^*=(p+1,r^{(p^*)})$ is an updated point. The death of the order from $\omega=(p+1,\phi^{(p+1)})$ to $\omega^*=(p,r^{(p^*)})$ is defined in the following way. Set $p^*=p$ and create a new point $\omega^*=(p,r^{(p^*)})$ with

$$r_1^* = r_1,\ \ldots,\ r_{p^*}^* = r_p$$

Suppose that a_n and a_d are respectively a probability of acceptance for the birth of the order and death of the order. The probability of acceptance for birth is as follows:

$$a_n(\omega,\omega^*) = \min\left\{1, \frac{\pi(\omega^*|\ \varphi,y)}{\pi(\omega|\ \varphi,y)} \frac{q(\omega_*,\omega)}{q(\omega,\omega^*)}\right\}$$

While the probability of death is as follows:

$$a_d(\omega,\omega^*) = \min\left\{1, \frac{1}{a_n(\omega^*,\omega)}\right\}$$

where

$$\frac{\pi(\omega^*|\ \varphi,y)}{\pi(\omega|\ \varphi,y)} = \frac{D_{k+1}}{N_k} \frac{(\beta^*)^{n-p}}{\beta^{n-p+1}} \frac{1}{n-p} \frac{p+1}{p_{max}-p}$$

and

$$\frac{q(\omega^*,\omega)}{q(\omega,\omega^*)} = 1.$$

3.4.2 Change of the Coefficients

The transformation of the change of order will not change the number of coefficients. This transformation will change the value of coefficients. Suppose that $\omega=(p,r^{(p)})$ is a current point and $\omega^*=(p,r^{(p^*)})$ is an updated point. The change of the coefficients from $\omega=(p,\phi^{(p+1)})$ to $\omega^*=(p,r^{(p^*)})$ is defined in the following way. Set $p^*=p$, choose a random point $i \in \{1,2,\cdots,p\}$, and choose a random point $u \sim U(-1,1)$. Then a new point $\omega^*=(p,r^{(p^*)})$ is created with $r_1^*=r_1$, ...,

$$r_{i-1}^* = r_{i-1},\ r_i^* = u,\ r_{i+1}^* = r_{i+1},\ r_{p^*}^* = r_p$$

Let a_p be a probability of acceptance of a change of coefficient. Then the probability of acceptance for change is as follows:

$$a_p(\omega,\omega^*) = \min\left\{1, \frac{\pi(\omega^*|\ \varphi, y)}{\pi(\omega|\ \varphi, y)} \frac{q(\omega_*,\omega)}{q(\omega,\omega^*)}\right\}$$

where

$$\frac{\pi(\omega^*|\ \varphi, y)}{\pi(\omega|\ \varphi, y)} = \left(\frac{\beta^*}{\beta}\right)^{\alpha}$$

and

$$\frac{q(\omega_*,\omega)}{q(\omega,\omega^*)} = 1.$$

The transformation in statement (7) is used in order to get the stationary conditions for an AR model. Thus the first result of this paper is an AR model that is obtained that is always stationary. The hierarchical Bayesian was adopted to estimate the order of the AR model, the coefficient of the AR model, the variance of the white noise, and its hyper-parameter. The second result of this paper is that both the order of the AR model and the coefficient of the AR model, the variance of the white noise, and the hyper-parameter can be estimated simultaneously.

4. CONCLUSION

The purpose of this paper was to estimate the parameters of an AR model with exponential white noise when the order was unknown. The parameters cannot be estimated by a Markov chain Monte Carlo algorithm, because the order of the AR model is unknown.

The reversible jump Markov chain Monte Carlo algorithm is one of the new methods that can be used to estimate the parameters of AR models when the order of the AR is unknown. The advantage of this method is that both the order of the AR and the coefficient of the AR can be estimated simultaneously.

5. ACKNOWLEDGEMENTS

The author would like to thank the Directorate General of Research and Development Strengthening (Ministry of Research, Technology and Higher Education of Republic of Indonesia) that has given the BSLN grant for presentation of this paper at an international conference in Australia. The author would also like to thank the University of Ahmad Dahlan (UAD) in Indonesia that has supported the grant for this research. Finally, the author gratefully acnowledge the suggestions given by Associate Professor Allan Leslie White, University of Western Sydney in Australia.

6. REFERENCES

[1] Zhang R and Ling S, Asymptotic Inference for AR Models with Heavy-Taliled G-GARCH Noises, Econometric Theory, 2015, 880-890.

[2] Jureckova J, Koul HL, and Picek J, Testing the tail index in autoregressive models, Annals of the Institute of Statistical Mathematics, 2009, 579-598.

[3] Novkovic M, On Exponential Autoregressive Time Series Models, J. Math, 1999, 97-101.

[4] Pereira IMS and Amaral-Turkman MA, Bayesian prediction in threshold autoregressive models with exponential white noise, Test, 2004, 45-64.

[5] Larbi L and Fellag H, Robust Bayesian Analysis of an Autoregressive Model with Exponential Innovations, Afr. Stat., 2016, 955-964.

[6] Ibazizen M and Fellag H, Bayesian Estimation of an AR(1) precess with exponential white noise, A Journal of Theoretical and Applied Statistics, 2003, 365-372.

[7] Suriyakat W, Areepong Y, Sukparungsee S, and Mititelu G. On EWMA Procedure for AR(1) Observations with Exponential White Noise. International Journal od Pure and Applied Mathematics, 2012, 73-83.

[8] Shi Z and Aoyama H, Estimation of the Exponential Autoregressive Time Series Model by Using The Genetic Algorithm, Journal of Sound and Vibration, 1997, 309-321.

[9] Green PJ, Reversible Jump MCMC Computation and Bayesian Model Determination, Biometrika, 1995, 711-732.

Geomate Suparman.pdf

ORIGINALITY REPORT

14%

SIMILARITY INDEX

PRIMARY SOURCES

1 ZHAOYUN SHI, YOSHIYASU TAMURA, TOHRU OZAKI. "Nonlinear time series modelling with the radial basis function-based state-dependent autoregressive model", International Journal of Systems Science, 7/1/1999 — 24 words — 1%
Crossref

2 R.F. Stark. "The declarative semantics of the Prolog selection rule", Proceedings Ninth Annual IEEE Symposium on Logic in Computer Science LICS-94, 1994 — 24 words — 1%
Crossref

3 www.wiete.com.au — 21 words — 1%
Internet

4 www.ism.ac.jp — 18 words — 1%
Internet

5 R. King. "Prior Information in Behavioral Capture-Recapture Methods: Demographic Influences on Drug Injectors' Propensity to Be Listed in Data Sources and Their Drug-related Mortality", American Journal of Epidemiology, 08/24/2005 — 16 words — 1%
Crossref

6 Petcharat, K.; Areepong, Y.; Sukparungsri, S. and Mititelu, G.. "Numerical Approximation to the Performance of CUSUM Charts for EMA (1) Process", World Academy of Science, Engineering & Technology, 2012. — 15 words — 1%
Publications

7 pureapps2.hw.ac.uk — 15 words — 1%
Internet

8	internationalscienceindex.org Internet	13 words — 1%
9	Chih-Yun Yang. "THE STUDY OF REPURCHASE INTENTIONS IN EXPERIENTIAL MARKETING - AN EMPIRICAL STUDY OF THE FRANCHISE RESTAURANT", International Journal of Organizational Innovation/19431813, 20091001 Publications	12 words — 1%
10	Adell, José A., and C. Sangüesa. "Error bounds in divided difference expansions. A probabilistic perspective", Journal of Mathematical Analysis and Applications, 2006. Crossref	12 words — 1%
11	Barnett. "Bayesian Inference", Wiley Series in Probability and Statistics, 07/07/1999 Crossref	12 words — 1%
12	Zhang, Rongmao, and Shiqing Ling. "ASYMPTOTIC INFERENCE FOR AR MODELS WITH HEAVY-TAILED G-GARCH NOISES", Econometric Theory, 2015. Crossref	12 words — 1%
13	Germund Dahlquist, Åke Björck. "4. Interpolation and Approximation", Society for Industrial & Applied Mathematics (SIAM), 2008 Crossref	10 words — < 1%
14	www.uws.edu.au Internet	9 words — < 1%
15	archive.org Internet	9 words — < 1%
16	Rainer Gob. "Control charts based on models derived from differential equations", Quality and Reliability Engineering International, 12/2010 Crossref	9 words — < 1%
17	TU, PHAN HUY, TRAN CAO SON, and CHITTA BARAL.	

"Reasoning and planning with sensing actions, incomplete information, and static causal laws using answer set programming", Theory and Practice of Logic Programming, 2007.
Crossref

8 words — < 1%

18 Fang, Daoyuan, and Ting Zhang. "Discontinuous solutions of the compressible Navier–Stokes equations with degenerate viscosity coefficient and vacuum", Journal of Mathematical Analysis and Applications, 2006.
Crossref

8 words — < 1%

19 stockage.univ-brest.fr
Internet

8 words — < 1%

20 petra.euitio.uniovi.es
Internet

8 words — < 1%

21 Fan, Hong-yi. "Newton–Leibniz integration for ket–bra operators (III)—Application in fermionic quantum statistics", Annals of Physics, 2007.
Crossref

7 words — < 1%

22 Washington, . "Other Applications", Discrete Mathematics and Its Applications, 2008.
Crossref

7 words — < 1%

23 Tao, . "Elements of Rough Set Theory", Rough Multiple Objective Decision Making, 2011.
Crossref

6 words — < 1%

EXCLUDE QUOTES	OFF	EXCLUDE MATCHES	OFF
EXCLUDE BIBLIOGRAPHY	ON		