Hierarchical Bayesian estimation for stationary autoregressive models using reversible jump MCMC algorithm

By Suparman

Hierarchical Bayesian estimation for stationary autoregressive models using reversible jump MCMC algorithm

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Abstract. The autoregressive model is a mathematical model that is often used to model data in different areas of life. If the autoregressive model is matched against the data then the order and coefficients of the autoregressive model are unknown. This paper aims to estimate the order and coefficients of an autoregressive model based on data. The Bayesian hierarchy approach is used to estimate the order and coefficients of the autoregressive model. In the Bayesian approach, the order and coefficients of the autoregressive model are assumed to have a prior distribution. The prior distribution is combined with the likelihood function to obtain a posterior distribution. Posterior distribution has a complex shape so that the Bayesian estimator is not analytically determined. The reversible jump Markov Chain Monte Carlo (MCMC) algorithm is proposed to obtain Bayesian estimates. The performance of the algorithm is tested by using simulated data. The test results show that the algorithm can estimate the order and coefficients of the autoregressive model very well. Research can be further developed by comparing with other existing methods.

Keywords: autoregressive model, hierarchical Bayesian, reversible jump MCMC

1. Il 22 oduction

The autoregressive model is a time series model that is often used to model data in different areas of life. The autoregressive model (AR) is a flexible model by setting the order and model parageters. Okada [1] used the AR model to dia ose Parkinson's disease. Ramdane-Cherif [2] applies the AR model to the eye tremor movement. The eye tremor movement is extracted from the recorded eye position signal. Kisi [3] uses AR model to predict stream flow. Zhao [4] used the AR model to classify the output from gas chromatography. Lee [5] used the AR model to model the extraction of respiratory rate. Figueiredo [6] uses the AR model to detect damage. Kim [7] uses the AR model to predict EEG data. Jayawardhana [8] uses the AR model to identify structural damage. Zhang [9] used the AR model to simulate dynamic light scattering (DLS) signals. Zhao [10] uses the AR model to predict channels in wireless networks. Dai [11] applied the AR model to the pre-earthquake ionospheric anomaly ana 14s. Yuewen [12] used the AR model to predict the engine's exhaust gas main engine temperature. The AR model can predict the changing trend of smoke temperature. Song [13] uses the AR model to identify the frequency of random signals. Kaewwit [14] uses an AR model to determine the high accuracy of biometric electroencephalography (EEG). Padmavathi [15] used the AR model to detect atrial fibration.

Let $x = (x_1, ..., x_n)$ be n time series data where n denotes many observations. This time series is said to have a p-order AR model, written AR (p), when this time series satisfies the stochastic equation

$$x_{t} = z_{t} + \sum_{i=1}^{p} \phi_{i}^{(p)} x_{t-i}$$
 (1)

as follows: $x_t = z_t + \sum_{i=1}^p \varphi_i^{(p)} x_{t-i}$ for i = 1, ..., n. The random variable z_t is a random error at time t and z_t is assumed to have a normal distribution with mean 0 and variance σ^2 . The vector $\phi^{(p)} = (\phi_1^{(p)}, \dots, \phi_p^{(p)})$ denotes the coefficient vector of model AR (p). The AR model (p) is called stationary if and only if the tribe equation $\varphi(b)=1-\sum_{i=1}^p \varphi_i^{(p)}b^i$ is zero for value b outside the circle with radius equal to one.

If the AR model is matched against the data, generally the order and the AR model coefficients are unknown. Methods to estimate the AR model order have been proposed by several authors, for example: [1] and [16]. Okada [1] uses the Akaike criterion to estimate the AR modeling order.

Khorshidi [16] compares various criteria (FPE, AIC) to estimate the AR modeling order. Likewise, methods for estimating AR model parameters have been proposed by several authors, for example: [16] and [17]. Khorshidi uses the Least-Squares-Forward (LSF) method to estimate the AR model parameters. Chen [17] uses Hubor's M-estimation theory to estimate the AR model. But in the various parameter estimation methods proposed by the researchers, the order model is often assumed to be known.

This paper proposes the order estimation and AR model parameters simultaneously that meet the condition of the stationarity. The AR model of the station is very useful for forecasting. This paper aims to estimate parameter values $(p, \phi^{(p)}, \sigma^2)$ of the AR model simultaneously based on observational data $x = (x_1, ..., x_n)$.

2. Research Method

This research used Bayesian hierarchy approach. Order of AR model, AR model coefficients, and error variance are considered as random variables having a 16 ain distribution. This distribution is known as the prior distribution. Determination of the prior distribution for the parameters $(p, \phi^{(p)}, \sigma^2)$ is done in the following way: The prior distribution of the p-order is chosen by the binomial distribution with the parameter λ . The conditional distribution of the coefficient $\phi^{(p)}$ if known λ is a uniform distribution at the interval of $(-1,1)^p$. The prior distribution of the error variance σ^2 is the distribution of gamma inversions with parameters 1 and $\frac{\beta}{2}$. Hierarchically, the prior distribution of λ is the uniformal distribution at the interval (0,1). The prior distribution of β is Jeffrey's distribution. Then the prior distribution of parameters $(p, \phi^{(p)}, \sigma^2)$ is combined with the probability function of x to obtain the posterior distribution of parameters $(p, \phi^{(p)}, \sigma^2)$. Let $\pi(p, \phi^{(p)}, \sigma^2)$ express the prior distribution for parameters $(p, \phi^{(p)}, \sigma^2)$ and $f(x|p, \phi^{(p)}, \sigma^2)$ can be expressed as follows:

 $\pi(p, \phi^{(p)}, \sigma^2 | \mathbf{x}) \propto f(\mathbf{x}|p, \phi^{(p)}, \sigma^2) \pi(p, \phi^{(p)}, \sigma^2)$

The posterior distribution is proportional to the multiplication of probability and priority distribution functions. Since the order p is not known to result in a posterior distribution having a very complicated form causing the determination of the Bayes estimator cannot be done analytically. Therefore the Bayes estimator is determined using the reversible jump MCMC algorithm [18]. Reversible jump MCMC algorithm allows the transformation from one AR model to another AR model. Transformation is not just from one AR model to another AR model that has the same order, but the transformation from one AR model to another AR model that has a different order. In other words, the transformation is done in a space that has different dimensions. The performance of reversible jump MCMC algorithm was tested using simulated data.

3. Results and Discussion

Let $s = (x_{p+1}, ..., x_n)$ be the realization of the AR(p) model. If the value $s_0 = (x_1, ..., x_p)$ is known, the probable function of s can be written more or less as follows:

ction of s can be written more or less as follows:

$$L\left(s\middle|p,\phi^{(p)},\sigma^2\right) = \left(\frac{1}{2\pi\sigma^2}\right)^{(n-p)/2} \exp\left(-\frac{1}{2\sigma^2}\sum_{t=p+1}^n g^2\left(t,p,\phi^{(p)}\right)\right) \tag{2}$$

where

$$g^{2}(t, p, \phi^{(p)}) = x_{t} - \sum_{i=1}^{p} \phi_{i}^{(p)} x_{t-i}$$

for t = p + 1, ..., n with initial value $x_1 = \cdots = x_p = 0$. Let S_p be the stationarity region. By using transformation

$$F \colon \varphi^{(p)} = \left(\ \varphi_1^{(p)}, \dots, \varphi_p^{(p)} \right) \in S_p \to r^{(p)} = \left(\ r_1 \ , \dots, r_p \ \right) \in (-1,1)^p$$

 $F: \varphi^{(p)} = \left(\varphi_1^{(p)}, \dots, \varphi_p^{(p)} \right) \in S_p \to r^{(p)} = \left(r_1 , \dots, r_p \right) \in (-1,1)^p$ then the 10 del AR $(x_t)_{t \in Z}$ is stationary if and only if $\left(r_1 , \dots, r_p \right) \in (-1,1)^p$ [19]. Further likelihood

function for x can be rewritten as for
$$\frac{1}{23}$$
 vs:
$$L(s|p, \phi^{(p)}, \sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{(n-p)/2} \exp\left(-\frac{1}{2\sigma^2}\sum_{t=p+1}^{n} g^2(t, p, F^{-1}(\phi^{(p)}))\right) \tag{3}$$

3.1. Hierarchy of Bayesian Estimator

The determination of the prior distribution of the parameters mentioned above is as follows:

a) Order p is binomial distributed with parameter λ

$$\pi(p|\lambda) = C_{p_{max}}^{p} \lambda^{p} (1 - \lambda)^{p_{max} - p}$$

- b) For order p is determined, the coefficient vector r^(p) is uniformly distributed at the interval
- Variance σ^2 distributes gamma inversion with parameters $\alpha/2$ and $\beta/2$

$$\pi(\sigma^2|\alpha,\beta) = \frac{(\beta/2)^{\alpha/2}}{\Gamma(\alpha/2)}(\sigma^2)^{-(1+\alpha/2)} exp - \frac{\beta}{2\sigma^2}$$

Here λ is assumed to be uniformly distributed at interval of (-1,1), the value of α is taken equals 2, and the parameter β is assumed to be Jeffrey's distributed. Let $H_1 = (p, r^{(p)}, \sigma^2)$ and $H_2 = (\lambda, \beta)$. Thus the prior distribution for parameters H₁ and H₂ can be presented as follows:

$$\begin{split} \pi(H_1,H_2) &= \pi(p|\lambda)\pi\Big(r^{(p)}\Big|p\Big)\pi(\sigma^2|\alpha,\beta)\pi(\lambda)\pi(\beta) \\ &= C_{p_{max}}^p \lambda^p (1-\lambda)^{p_{max}-p} \Big(\frac{1}{2}\Big)^p \frac{(\beta/2)^{\alpha/2}}{\Gamma(\alpha/2)} (\sigma^2)^{-(1+\alpha/2)} exp - \frac{\beta}{2\sigma^2} \frac{1}{\beta} \end{split}$$

According to Bayes's Theorem, posterior distributions for parameters H1 and H2 can be expressed as $\pi(H_1, H_2|s) \propto L(s|H_1)\pi(H_1, H_2)$

Posterior distribution is a combination of likelihood function and prior distribution that is assumed before the sample is taken. In this case the 15 sterior distribution $\pi(H_1, H_2|s)$ has a very complicated form so that the Bayes estimator cannot be determined by analysis. Therefore the reversible jump MCMC algorithm is proposed to determine Bayes estimators.

3.2. Reversible Jump MCMC Algorithm

Let M=(H₁, H₂). In general, the MCMC method is a sampling method by creating a homogeneous Markov chain M_1, \ldots, M_n that satisfies aperiodic and irreducible properties such that M_1, \ldots, M_n can be considered as a random variable following the distribution $\pi(H_1, H_2|s)$. Thus M_1, \dots, M_n can be used as a means to estimate the parameters of M. To realize it Gibbs Hybrid algorithm is adopted. It consists of two stages: (1) the distribution simulation of $\pi(H_1|H_2,s)$ and (2) the simulation stribution of $\pi(H_2|H_1,s)$. The Gibbs algorithm [20] is used to simulate the distribution $\pi(H_2|H_1,s)$. The reversible jump MCMC algorithm is used to simulate the distribution $\pi(H_1|H_2,s)$

The distribution simulation $\pi(H_2|H_1,s)$ is done in the following way: The conditional distribution H₂ is known to H₁ and s can be expressed as

$$\pi(H_2|H_1,s) \propto \lambda^p (1-\lambda)^{p_{\text{max}}-p} (\beta/2)^{\alpha/2} \exp{-\frac{\beta}{2\sigma^2}\frac{1}{\beta}}$$

Since this distribution is a gamma distribution with parameters $\alpha/2$ and $\frac{1}{2\sigma^2}$, the Gibbs algorithm can be used to simulate the distribution of $\pi(H_2|H_1,s)$.

The distribution simulation $\pi(H_1|H_2,s)$ is done in the following way: The conditional distribution of H_1 if it is known H_2 and s is integral to σ^2 , obtained

$$\pi\left(p, r^{(p)} \middle| H_1, s\right) = \int_{\mathbb{R}^+} \pi(H_1 | H_2, s) d\sigma^2$$

Let $v = \frac{\alpha}{2} + \frac{n - p_{max}}{2}$ and $w = \frac{\beta}{2} + \frac{1}{2} \sum_{t=p_{max}+1}^{n} g^2(t, p, F^{-1}(r^{(p)}))$. Since $\int_{R^+} (\sigma^2)^{-(1+v)} \exp{-\frac{w}{\sigma^2}} d\sigma^2 = \frac{\Gamma(v)}{w^v}$ then

$$\pi \Big(p, r^{(p)} \Big| H_1, s \Big) \propto C_{p_{max}}^p \lambda^p (1-\lambda)^{p_{max}-p} \Big(\frac{1}{2}\Big)^p \frac{(\beta/2)^{\alpha/2}}{\Gamma(\alpha/2)} \frac{1}{\beta} \frac{\Gamma(v)}{w^v}$$

On the other hand, the distribution $\pi(H_1|H_2,s)$ can be expressed as the product of the distribution of $\pi(p,r^{(p)}|H_1,s)$ and the distribution of $\pi(p,r^{(p)}|H_2,s)$, i.e. :

$$(H_1|H_2,s) = \pi(p,r^{(p)}|H_1,s)\pi(\sigma^2|p,r^{(p)},H_2,s)$$

Furthermore, to simulate the distribution of $\pi(H_1|H_2,s)$, a hybrid algorithm is used. It consists of two stages: (a) The distribution simulation $\pi\left(\sigma^2\middle|p,r^{(p)},H_2,s\right)$, (b) The distribution simulation $\pi\left(p,r^{(p)}\middle|H_1,s\right)$. Gibbs algorithm is used to simulate the distribution $\pi\left(\sigma^2\middle|p,r^{(p)},H_2,s\right)$.

The distribution simulation $\pi(p, r^{(p)}|H_1, s)$ is done by using the reversible jump MCMC algorithm. The reversible jump MCMC algorithm uses three types of transformations, namely: birth of the order, death of the order, and change in the coefficient.

3.2.1. Birth / Death of the Order

The birth of the order from the AR(p) model to the AR(p+1) model is done by adding coefficients. Let p be the actual value for the order and $r^{(p)} = (r_1, ..., r_p)$ is the actual value for the AR(p) model coefficient. As in Suparman [21], the random variable u is chosen according to the triangular distribution with mean 0

$$g(u) = \begin{cases} 12 \\ u+1, \\ 1-u, \end{cases} -1 < u < 0 \\ 0 < u < 1 \end{cases}$$

Vector coefficient $\mathbf{r}^{(p)}$ is completed with random variable u, so the proposed new coefficient vector is $\mathbf{r}^{(p+1)} = (r_1, \dots, r_p, u)$. The acceptance/rejection probability corresponds to the birth order is $\alpha_N = \min\{1, r_N\}$ where

$$\alpha_{N} = \min\{1, r_{N}\} \text{ where } \\ r_{N} = \frac{\pi\left(p+1, r^{(p+1)} \middle| H_{2}, s\right)}{\pi\left(p, r^{(p)} \middle| H_{2}, s\right)} \frac{q\left(p+1, r^{(p+1)}; p, r^{(p)}\right)}{q\left(p, r^{(p)}; p+1, r^{(p+1)}\right)} \\ \text{In contrast, the death of the order from the } AR(p+1) \\ \text{model to the } AR(p) \\ \text{model is done by removing}$$

In contrast, the death of the order from the AR(p+1) model to the AR(p) model is done by removing the last coefficient. Let p+1 be the actual value of the order and $r^{(p+1)} = (r_1, \dots, r_p, r_{p+1})$ is the actual value for the AR(p+1) model coefficient. Coefficient r_{p+1} is removed. So the proposed new coefficient vector is $r^{(p)} = (r_1, \dots, r_p)$. The probability of acceptance/rejection corresponding to order death is $\alpha_D = \min\{1, r_N^{-1}\}$.

3.2.2. Change of the Coefficient

The change of coefficient from AR(p) to AR(p) is done by changing each coefficient. Let $r^{(p)} = (r_1, \ldots, r_p)$ is the actual value for the coefficients. For $i=1,\ldots,p$, take the random variable $u_i = sin(r_i+s)$ with s taken according to the uniform distribution at the interval $[-\pi/10,\pi/10]$. So the resulting new coefficient vector is $r^{*(p)} = (r_1^*, \ldots, r_i^* = u_i, \ldots, r_p^*)$. The acceptance/rejection probability corresponding to the coefficient change is $\alpha_C = min\{1, r_C\}$ where

$$r_{C} = \frac{\frac{3}{\pi(p,r^{*(p)}|H_{2},s)} \frac{q(p,r^{*(p)};p,r^{(p)})}{q(p,r^{(p)};p,r^{*(p)})}}{\frac{q(p,r^{(p)};p,r^{*(p)})}{q(p,r^{(p)};p,r^{*(p)})}}$$

3.3. Smulation Study

The reversible jump MCMC algorithm is used to identify the AR order and parameter data simulations. A simulation study is conducted to find out whether the performance of the reversible jump MCMC 8 gorithm worked well or not.

To know the performance of reversible jump MCMC algorithm simulation study is conducted. Figure 1 is an AR simulation data made according to the equation

$$\begin{aligned} x_t &= z_t + \sum\nolimits_{i=1}^p \varphi_i^{(p)} x_{t-i} \\ \text{with } n &= 250, \text{ order } p = 3 \text{ , } \varphi^{(3)} = \left(\varphi_1^{(3)} = -0.36, \; \varphi_2^{(3)} = -0.24, \; \varphi_3^{(3)} = 0.81\right) \text{and } \sigma^2 = 4. \end{aligned}$$

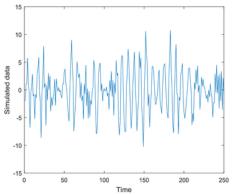


Figure 1. Simulated Data

The reversible jump MCMC algorithm is implemented in this simulation data to estimate the AR model order, AR model coefficients, and error variance. Figure 2 shows the histogram of the AR model order.

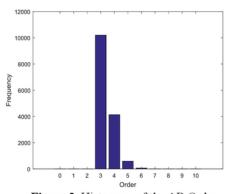


Figure 2. Histogram of the AR Order

Figure 2 shows that the mode of AR order is reached in order 3. This means that the estimator for AR order is p = 3. After it is determined that the most suitable AR model is AR (3) then the estimator for the AR coefficient and corresponding error variance is determined, i.e.:

$$\widehat{\varphi}^{(3)} = \left(\widehat{\varphi}_1^{(3)} = -0.36, \ \widehat{\varphi}_2^{(3)} = -0.26, \ \widehat{\varphi}_3^{(3)} = 0.82\right)$$
 and $\widehat{\sigma}^2 = 3.79$.

Table 1 summarizes the comparison between AR order estimators, AR coefficient estimators, and error variance estimators with AR-order values, AR coefficients, and error variance.

Table 1. Comparison between the value of parameters and the value of estimators

Value of Parameters	Value of Estimators		
p = 3	$\hat{p} = 3$		
$\phi^{(3)} = (-0.36, -0.24, 0.81)$	$\hat{\Phi}^{(3)} = (-0.36, -0.26, 0.82)$		
$\sigma^2 = 4$	$\hat{\sigma}^2 = 3.79$		

Table 1 shows that the reversible jump MCMC algorithm can estimate the AR model order, AR model coefficients, variance error very well.

4. Conclusion

The above description is a review of the theory of the reversible jump MCMC algorithm to estimate the order of AR model, AR coefficient, and error variance. Simulation studies show that the algorithm can estimate AR model parameters very well. The proposed algorithm has the advantage that the resulting estimation is an AR model that meets the condition of the stationarity. Another advantage is that the algorithm can estimate parameters $(p, \phi^{(p)}, \sigma^2)$ simultaneously.

Research can be further developed in comparison with existing estimation methods to determine effectiveness. Research may also be developed on the replacement of assumptions for errors, such as AR models with not normally distributed errors.

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