# Suparman_Telkomnika.pdf

TELKOMNIKA, Vol.16, No.1, March 2018, pp. xx~xx
ISSN: 1693-6930, accredited A by DIKTI, Decree No: 58/DIKTI/Kep/2013
DOI:10.12928/TELKOMNIKA.v13i2.xxxx 

# Bayesian Segmentation in Signal with Multiplicative Noise Using Reversible Jump MCMC

**Suparman[1], Michel Doisy[2]**
[1]Department of Mathematics Education, Ahmad Dahlan University
Jalan Prof. Dr. Soepomo, SH Yogyakarta, Indonesia, +62 274 563515
[2]Signal and Communications Group, ENSEEIHT
2 Rue Camichel, Toulouse 31071, France
e-mail: suparmancict@yahoo.co.id

***Abstract***

*This paper discusses the important issues in signal segmentation disturbed by multiplicative noise where the number of segments is unknown. A Bayesian approach is proposed to estimate the parameter. In this case the parameter includes the number of segments, the location of the segment, and the amplitude. However, the posterior distribution for the parameter does not have a simple equation so that the Bayes estimator is not easily determined. Reversible Jump MCMC method is adopted to overcome the problem. The Reversible Jump MCMC method creates a Markov chain whose distribution is close to the posterior distribution. The performance of the algorithm is shown by simulation data. The result of this simulation shows that the algorithm works well. As an application, the algorithm is used to segment SAR signal. The advantage of this method is that the number of segments, the position of the segment change, and the amplitude are estimated simultaneously.*

***Keywords***: *Reversible Jump MCMC, Bayesian, Multiplicative Noise, Signal Segmentation*

## 1. Introduction

Signal processing with additive noise has been investigated by several authors, for example Gustafsson *et al.* [1]. But in many applications it is often found that signal with multiplicative noise. Some authors have also discussed signals with multiplicative noise, such as Ullah *et al.* [2], Osoba and Kosko [3], Tian *et al.* [4], and Dong *et al.* [5]. Ullah *et al.* [2] used a variational approach for restoring images with multiplicative noise. Osoba and Kosko [3] used the noisy Expectation-Maximization Algorithm for multiplicative noise injection. Tian *et al.* [4] used an adaptive fractional-order method to eliminate multiplicative noise. Dong *et al.* [5] proposed a method using a sparse analysis model for signal with multiplicative noise. In signal segmentation with multiplicative noise, generally the number of segments is unknown and must be estimated based on the data. This paper discusses the segmentation of signal with multiplicative noise when the number of segments is unknown.

Let N be the many pixels contained in a line from the Synthetic Aperture Radar (SAR) image. The equation of the line can be expressed in the following form (Suparman *et al.* [6], Tourneret *et al.* [7]) :

$$y_t = r_t z_t \qquad t = 1, 2, \cdots, N \tag{1}$$

with $y_t$ is the intensity of the measured SAR image, $r_t$ is SAR intensity, and $z_t$ is a multiplicative noise. In various SAR images, including agricultural images, the properties of $r_t$ and $z_t$ can be defined as follows (Oliver and Quegan [8]) :

(a) SAR intensity $r_t$ is a step function. The equation can be written as :

$$r_t = h_K \qquad n_K < t \le n_{K+1}$$

with $K = 0, 1, \ldots, K_{max}$. Here, $n_K$ is the position of the height change of $K^{th}$ step. (with agreement $n_0 = 0$ and $n_{K+1} = N$) and $h_K$ is the height of $K^{th}$ step, and K is the number of steps.

(b) Multiplicative noise $z_t$ is given in the form of a random variable that follows the gamma

*Received xxxxxx xx, 2017; Revised xxxxxxx, 2017; Accepted xxxxxxx, 2017*

distribution with the mean L and variance 1/L, written $z_t \sim G(L, L)$,

$$f(z_t) = \frac{L^L}{\Gamma(L)} z_t^{L-1} \exp - [L z_t] \qquad t = 1, 2, \cdots, N$$

Here, L is the number of measurements. The value of L is known.
Based on the data $y_t$ (t = 1, 2, ..., N), then we will estimate the value of the parameter

$K$, $n^{(K)} = (n_1, n_2, \ldots, n_{K+1})$ and $h^{(K+1)} = (h_0, h_1, \ldots, h_K)$.

To estimate the value of these parameters, a hierarchical Bayesian approach is used, which will be described in the following sections.

## 2. Research Method

### 2.1. Hiearchical Bayesian

The Bayesian approach [9] is a method for estimating parameter values $\theta = (K, n^{(K)}, h^{(K)})$, which is done based on information obtained from the data $y_n$ (expressed in the probability distribution $f(y|\theta)$) and information on parameter $\theta$ (expressed in the prior distribution $\pi(\theta)$ ).

Due to a multiplicative noise $z_t \sim G(L, L)$, then the probability distribution for $y_t$ can be written as :

$$f(y|\theta) \propto \prod_{i=0}^{K} h_i^{L\tau(n_i, n_{i+1})} \exp - \left[ \frac{L\omega(y, n_i, n_{i+1})}{h_i} \right] \tag{2}$$

with $\tau(a, b) = b - a$, $\omega(y, a, b) = \sum_{n=a+1}^{b} y_n$ , and symbol "$\propto$" means "proportional to".

To use the Bayesian approach, the prior distribution for the parameter $\theta$ should be determined. Prior distribution for parameter $\theta$ is taken the same as in Suparman *et al.* [6]. Suppose $K_{max}$ is the maximum number of steps, then K is assumed to follow a Binomial distribution with parameter $\lambda$. The prior distribution for K can be written as

$$\pi(K | K_{maks}, \lambda) \propto \lambda^K (1-\lambda)^{K_{maks} - K} \quad K = 0, 1, \ldots, K_{max}. \tag{3}$$

For the value of K given, $n^{(K)}$ is assumed to follow the following distribution :

$$\pi(n^{(K)} | K) \propto \prod_{i=0}^{K} (n_{i+1} - n_i - 1) \tag{4}$$

and $h^{(K)}$ follows the inverse gamma distribution with parameters $\alpha$ dan $\beta$. Prior distribution for $h^{(K}$ can be written as

$$\pi(h^{(K+1)} | K, \alpha, \beta) \propto \prod_{i=0}^{K} h_i^{\alpha-1} \exp - [\frac{\beta}{h_i}] \tag{5}$$

The problem that arises is the presence of hyperparameter $\varphi = (\lambda, \alpha, \beta)$ in the above prior distributions. To simplify the problem, in Suparman *et al.* [6] value $\varphi$ is known. In this paper, as in Tourneret *et al.* [7] hyperparameter $\varphi$ is seen as a random variable with a given distribution, here $\lambda$ follows a uniform distribution at interval (0,1) and $\beta$ follows Jeffrey distribution. The value $\alpha$ is taken relatively small.

By using Bayes's theorem, the posterior distribution for $\theta$, written with $\pi(\theta, \varphi | y)$, can be expressed as the product of times of the probability distribution for $y_i$ and prior distribution for $(\theta, \varphi)$ :

$$\pi(\theta, \varphi | y) \propto f(y | \theta) \times \pi(\theta | \varphi) \times \pi(\varphi) \tag{6}$$

Then an estimate of the parameter $\theta$ will be done based on the posterior distribution. For example, the parameter estimator $\theta$ which creates a posterior distribution value $\pi(\theta|y)$ reaches the maximum value. But the shape of the posterior distribution $\pi(\theta|y)$ is very complex, then it is difficult to estimate parameter value $\theta$. To overcome this, we adopt the Markov Chain Monte Carlo (MCMC) method, especially Reversible Jump MCMC method.

### 2.2. Reversible Jump MCMC Method

Suppose $M=(\theta,\varphi)$ is a Markov chain. In general, MCMC method is a sampling method, that is by making a homogeneous Markov chain $M_1, M_2, \cdots, M_m$ which satisfies periodic and irreducible propertiessuch that $M_1, M_2, \cdots, M_m$ can be considered as a random variable following the distribution $\pi(\theta,\varphi|y)$ [10].Therefore, the chain Markov $M_1, M_2, \cdots, M_m$ can be used to estimate parameter M. To realize it was adopted Gibbs algorithm which consists of two stages :

1. Simulate distribution $\pi(\varphi|\theta, y)$
2. Simulate distribution $\pi(\theta|\varphi, y)$

The Gibbs algorithm is used to simulate the distribution $\pi(\varphi|\theta, y)$. Hybrid algorithm is used to simulate the distribution $\pi(\theta|\varphi, y)$. This hybrid algorithm combines the Reversible Jump MCMC algorithm [11] to simulate parameter $\pi(K, n^{(K)}| \varphi, y)$ and algoritma Gibbs to simulate parameter $\pi(h^{(K)}| \varphi, y)$. Reversible Jump MCMC algorithm is an extension of the Metropolis-Hastings algorithm.

#### 2.2.1.Distribution Simulation $\pi(\varphi|\theta, y)$

Conditional distribution of $\varphi$ given $\theta$ and y, the distribution $\pi(\varphi|\theta, y)$ can be expressed as

$$\pi(\varphi|\ \theta, y) \propto \lambda^K (1-\lambda)^{K_{maks}-K} \beta^{\alpha(K+1)} \exp -(\beta \textstyle\sum_{i=0}^{K} \dfrac{1}{h_i})$$

The distribution is the product of distribution $B(K+1, K_{max}-K+1)$ and $G(\alpha(K+1)+1, \sum_{i=0}^{K} 1/h_i)$. So to simulate it we can use Gibbs algorithm.

#### 2.2.2.Distribution Simulation $\pi(\theta|\varphi, y)$

Conditional distribution of $\theta$ given $(\varphi, y)$ can be expressed as

$$\pi(\theta|\varphi, y) \propto C_K^{K_{maks}} \lambda^K (1-\lambda)^{K_{maks}-K} \frac{1}{C_{N-2}^{2K+1}} \left(\frac{\beta^\alpha}{\Gamma(\alpha)}\right)^{K+1} \textstyle\prod_{i=0}^{K} (n_{i+1}-n_i)-1$$
$$\textstyle\prod_{i=0}^{K} h_i^{\Phi(\alpha, L, n_i, n_{i+1})-1} \exp -[\dfrac{\Psi(\beta, L, y, n_i, n_{i+1})}{h_i}]$$

where $\Phi = \alpha + L\tau(a,b)$ and $\Psi(\beta, L, y, a, b) = \beta + L\omega(y, a, b)$.

When it is integrated against $h^{(K)}$, it will be obtained

$$\pi(K, n^{(K)}|\varphi, y) \propto C_K^{K_{maks}} \lambda^K (1-\lambda)^{K_{maks}-K} \frac{1}{C_{N-2}^{2K+1}} \left(\frac{\beta^\alpha}{\Gamma(\alpha)}\right)^{K+1} \textstyle\prod_{i=0}^{K} (n_{i+1}-n_i)-1$$
$$\prod_{i=0}^{K} \frac{\Gamma(\Phi(\alpha, L, n_i, n_{i+1}))}{\Psi(\beta, L, y, n_i, n_{i+1})}$$

On the other hand, we have

$$\pi(h^{(K)} \mid K, n^{(K)}, \varphi, y) \propto \prod_{i=0}^{K} h_i^{\Phi(\alpha, L, n_i, n_{i+1})-1} \exp -[\frac{\Psi(\beta, L, y, n_i, n_{i+1})}{h_i}]$$

$$\otimes_{i=0}^{K} IG(\Phi(\alpha, L, n_i, n_{i+1}, \Psi(\beta, L, y, n_i, n_{i+1}))$$

So we can write the distribution $\pi(\theta|\varphi, y)$ as the product of the distribution $\pi(K, n^{(K)}|\varphi, y)$ and distribution $\pi(h^{(K)} \mid K, n^{(K)}, \varphi, y)$, that is :

$$\pi(\theta|\varphi, y) = \pi(K, n^{(K)}|\varphi, y)\ \pi(h^{(K)} \mid K, n^{(K)}, \varphi, y)$$

Next to simulate the distribution $\pi(\theta|\varphi, y)$, we use a Gibb algorithm consisting of two stages :

- Stage 1 : Simulate distribution $\pi(h^{(K)} \mid K, n^{(K)}, \varphi, y)$
- Stage 2 : Simulate distribution $\pi(K, n^{(K)}|\varphi, y)$

Then to simulate the distribution $\pi(h^{(K)} \mid K, n^{(K)}, \varphi, y)$ we use the Gibbs algorithm. On the other hand, distribution $\pi(K, n^{(K)}|\varphi, y)$ is not explicitly so that the MCMC Reversible Jump algorithm is used to simulate it.

## 3. Results and Analysis

As an illustration, this method is applied to segment simulation multiplicative and real multiplicative signals. As in [12], a simulation study was undertaken to confirm the performance of the Reversible Jump MCMC algorithm whether it works well or not. While case studies are given to provide examples of application of research to solve problems in everyday life.To segment the multiplicative signals of simulation and real multiplicative signals, the Reversible Jump MCMC algorithm is implemented as much as 25 thousand iterations with a 5 thousand burn-in period.

### 3.1. Multiplicative Signal Simulation

Figure 1 is a simulated multiplicative signal created according to the equation (1) above with N = 250 and L = 5. As for value K = 3, vector value $n^{(3)}$ = (75, 125, 200) and vector value $h^{(4)}$ = (1, 7, 3, 5).

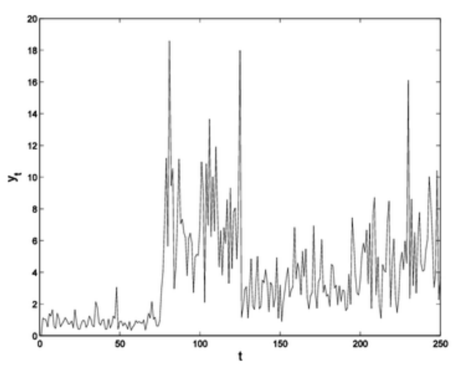


Figure 1. Multiplicative signal simulation

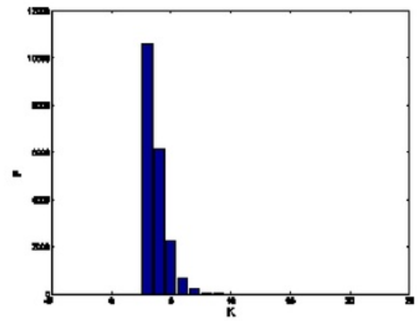


Figure 2. Histogram for K

Furthermore, based on the simulation multiplicative signal in Figure 2, the number of segments K, vector $n^{(K)}$ and vector $h^{(K)}$ is estimated by using Reversible Jump MCMC algorithm. Estimator of K, $n^{(K)}$ and $h^{(K)}$ generated by the algorithm are

$$\hat{K} = 3,\ \mathbf{n}^{(\hat{K})} = (75, 125, 196), \text{ and } h^{(\hat{K})} = (0.9, 7.3, 3.1, 5.1)$$

The histogram for K is given in Figure 2. The signal segmentation generated by the algorithm is presented in Figure 3.

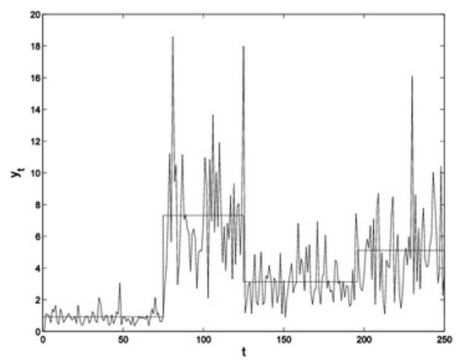


Figure 3. Result of multiplicative segmentation of simulated signal

Figure 4. Images of the Natural Scene Around the Imogiri Tomb, Yogyakarta Indonesia

If we compare the true parameter value of K, $n^{(K)}$, $h^{(K)}$ with the estimated value of parameter K, $n^{(K)}$, $h^{(K)}$ obtained by algorithm, then it appears that the algorithm can work well.

### 3.2. Real Multiplicative Signal

Now, algorithms are used to segment a line on the real image. The image used is 480 x 640 (Figure 4). The image taken using the Nokia 3220 mobile phone is a natural scene around the Imogiri Tomb, Yogyakarta Indonesia.

The 198th column of the real image is presented in Figure 5 below. Then the 198th line will be called a real signal.

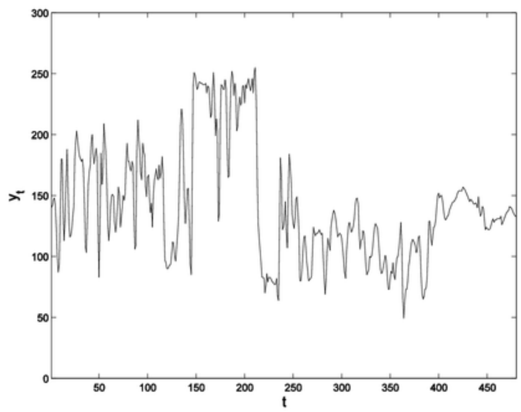


Figure 5. The 198th column

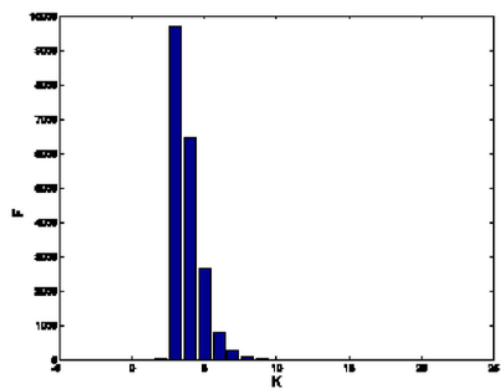


Figure 6. Histogram for K

Once the Reversible Jump MCMC algorithm is implemented in the real signal, we get an estimate for the value K, $n^{(K)}$ dan $h^{(K)}$ as follows :

$$\hat{K} = 3,\ n^{(\hat{K})} = (145, 213, 394) \text{ and } h^{(\hat{K})} = (151, 217, 107, 139).$$

The histogram for K is given in Figure 6. The results of its segment are presented in Figure 7.

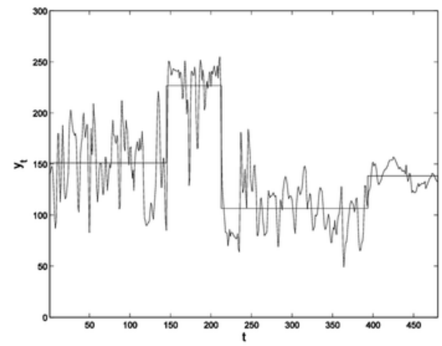

Figure 7. Results of real multiplicative signal segmentation

## 4. Conclusion

The above description was a theoretical study of Reversible Jump MCMC algorithms and their applications for segmenting signal models with multiplicative noise. From the simulation results showed that Reversible Jump MCMC algorithm can segment the signal well.

As an algorithm implementation, real signal was drawn from columns in a natural scene around the Imogiri Tomb,Yogyakarta Indonesia. If the Reversible Jump MCMC algorithm is implemented on each row or column of the image it will generate segmentation of the image.

However, if the image dimension size is large enough, then the method of segmenting the image by segmenting each column or each row in the image will take a longer time. One way to solve this problem is by directly segmenting the image. The development of a Reversible Jump MCMC algorithm to segment images directly will be an interesting research topic.

## References

[1] Gustafsson RT, Hossjer OG, Oberg T. Adaptive Detection of Known Signals in Additive Noise by Means of Kernel Density Estimators, *IEEE Transactions on Information Theory* 1997, Vol 43, 1192-1204.
[2] Ullah A, Chen W, Khan MA. A New Variational Approach for Restoring Images with Multiplicative Noise, *Computers & Mathematics with Applications* 2016, Vol. 71, 2034-2050.
[3] Osoba O, Kosko B. The Noisy Expectation-Maximization Algorithm for Multiplicative Noise Injection, *Fluctuation & Noise Letters* 2016, Vol. 15, 1-23.
[4] Tian D, Du Y, Chen D. An Adaptive Fractional-Order Variation Method for Multiplicative Noise Removal, *Journal of Information Science & Engineering* 2016, Vol. 32, 747-762.
[5] Dong J, Han Z, Zhao Y, Wang W, Prochazka A, Chambers J. Sparse Analysis Model Based Multiplicative Noise Removal with Enchanced Regularization, *Signal Processing* 2017, Vol. 137, 160-176.
[6] Suparman, Doisy M, Tourneret J-Y. *Changepoint Detection using Reversible Jump MCMC Methods. Proceedings of the International Conference on Acoustics, Speech and Signal Processing*, Orlando Florida, 2002; 1569-1572.
[7] Tourneret J-Y, Suparman, Doisy M. Hierarchical Bayesian Segmentation of Signals Corrupted by Multiplicative Noise. *IEEE*2003, Vol. 6, 165-168.
[8] Oliver C, Quegan S. Understanding Synthetic Aperture Radar Images. Raleigh: SciTech. 2004
[9] Robert CP, The Bayesian Choice. From Decision-Theoretic Foundations to Computational Implementation. New York: Springer-Verlag. 2001.
[10] Robert CP, Casella G. Monte Carlo Statistical Methods. New York : Springer-Verlag. 1999.
[11] Green PJ. Reversible Jump Markov Chain Monte Carlo Computation and Bayesian Model Determination, *Biometrika* 1995, Vol. 82, 711-732.
[12] Suparman, Doisy M. Hierarchical Bayesian of ARMA Models Using Simulated Annealing Algorithm, *Telkomnika* 2014, Vol 12, 87-96

# Suparman_Telkomnika.pdf

ORIGINALITY REPORT

1%

SIMILARITY INDEX

PRIMARY SOURCES

| | | |
|---|---|---|
| 1 | journal.uad.ac.id<br>Internet | 16 words — 1% |
| 2 | Utomo, Oei Kurniawan; Santoso, Daniel and Nugroho, Saptadi. "Location and Position Determination Algorithm for Humanoid Soccer Robot", Telkomnika, 2016.<br>Publications | 6 words — < 1% |

| | | | |
|---|---|---|---|
| EXCLUDE QUOTES | OFF | EXCLUDE MATCHES | OFF |
| EXCLUDE BIBLIOGRAPHY | ON | | |