Quadratic Form Optimization with Fuzzy Number Parameters: Multiobjective Approaches

By SUGIYARTO

Quadratic Form Optimization with Fuzzy Number Parameters: Multiobjective Approaches

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Abstract: Problems in everyday life can be modelled into mathematical form or commonly referred to as mathematical modelling. One form of mathematical model is optimization, in which the best decision making is made from various alternative types. The problem of optimizing quadratic forms, or commonly referred to as quadratic program problems, is one form of optimization where the objective function is a quadratic function. Many real world problems involve variables that cannot be expressed numerically and require a vague set to describe them, so fuzzy logic emerges which is often used to describe the form of obscurity. The development of quadratic program problems with fuzzy number parameters is then conducted, using the form of triangular fuzzy numbers, which called the fuzzy quadratic programs. The fuzzy quadratic program is then converted into a multiobjective optimization problem by utilizing definitions and arithmetic on triangular fuzzy numbers. By using Sum of Objective Method, the multiobjective problem can be changed into single optimization problem, then be completed using the Karush-Kuhn-Tucker method. The multiobjective optimization problem will produce three optimal values which will form the optimal value in the form of triangular fuzzy numbers.

Keyword: Optimization; Quadratic; Fuzzy Program; Triangular Fuzzy Numbers; Karush-Kuhn-Tucker Conditions; Multiobjective Optimization Problem; Sum of Objective Method.

1. Introduction

Problems in real life can be modeled into mathematical form or commonly referred to as mathematical modeling. Mathematical models or programs are problem solving areas, one of which contains optimization problems. Generally, the selection of the best decision is determined by minimizing or maximizing an objective function with or without constraints. The scope of optimization has attracted a lot of attention, this is because advances in computer technology are developing very rapidly, including the development and provision of easy-to-use software.

Optimization problems generally consist of two functions, namely the objective function to be optimized and the constraint function as the condition that becomes a requirement. The problem of nonlinear optimization with the objective function in the form of a quadratic function is the problem of maximizing or minimizing the objective function which depends on linear constraint functions and non-negative variable boundaries [1]. Many problems in the real world can be formulated as quadratic problems. Examples of these problems can be found in game theory, engineering modeling, design and control, economic problems, location problems and allocation of facilities, portfolios, logistics etc. [2].

In 1965, Lotf Zadeh introduced a definition of fuzziness to solve problems that contained this uncertainty. In general, fuzzy logic has shown good potential in modeling systems that are not linear, complex, not well defined or difficult to understand properly. Fuzzy logic is a way of mathematically describing obscurity and various applications have been found because it is easy to implement, flexible, inaccurate nature of data and low implementation costs.

In its development, the problem of quadratic programs with parameters in the form of fuzzy numbers has been widely developed. Ammar [3] has developed a quadratic program approach method with fuzzy number parameters and applied it to portfolio optimization. Furthermore, Liu ([4], [5]) has developed a method for optimizing the civil problem, with parameters in the form of fuzzy numbers using the membership function approach. In 2014, Allahviranloo and Moazam [6] developed a solution to solve quadratic equations where all the parameters, both the coefficient matrix and the variable, are fuzzy numbers.

Developments continue. In 2017, Mirmohseni [7] carried out a program-related development. This research focuses on quadratic programs with triangular fuzzy numbers where boundary coefficients and right-hand parameters are triangular fuzzy numbers. In 2018, Nezhad [8] carried out a development where cost coefficients, quadratic form matrices, boundary coefficients and right-side parameters were all triangular fuzzy numbers. The method used in this study is almost the same as that used by Liu [5], both of which also produce two simpler optimization problems. The difference lies in the two-level mathematical model and the final form of the problem.

Allegiance to problems in the real world is not only limited to constants or determinant variables, objective functions and constraints. This is the background of the author in developing methods to optimize problems with objective and constraints functions and all parameters and variables in the form of fuzzy numbers, so that the solution can be applied under any conditions. The method used is a triangular fuzzy number method because the process is relatively simpler than using the extension principle method or the α-cut method. Then the final results obtained were completed using Karush-Kuhn-Tucker's condition.

2. Preliminaries

2.1 Optimization of Quadratic Forms

Optimization is the core of various kinds of problems, including decision making that can be found in both technical and economic fields. The work done in decision making is to choose the best decision from various alternatives. Generally, the selection of the best decision is determined by minimizing or maximizing an objective function with or without constraints.

In general, optimization problems are expressed in the following forms:

$$\begin{array}{c}\text{Minimize } f(\mathbf{x}) \\ \text{with constraint } \mathbf{x} \in \Omega\end{array} \qquad (2.1)$$

where function $f: \mathbb{R}^n \to \mathbb{R}$ is a function of real value which is expected to produce a minimum value. This function is called an objective function. Vector $\mathbf{x}$ is a column vector containing n-pieces of independent variables, namely $\mathbf{x} = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$. The variables $x_1, x_2, \dots, x_n$ are often referred to as decision variables. Whereas Ω is referred to as a set of constraints which is a subset of $\mathbb{R}^n$.

The optimization problem above is a problem of decision making in which the search for the "best" $\mathbf{x}$ vector of all possible vectors. The best vector is expected to produce the smallest value of the objective function and is called the minimization vector of function f, which is formally stated in the following definition,

Definition 2.1 [3] *Any* $\mathbf{x}$ *that meets the set of constraints in equation (2.1) is called a feasible solution. Suppose* Q *is a set of all feasible equations (2.1).* $\mathbf{x}^* \in Q$ *is said to be the optimal feasible solution for equation (2.1) if* $f(\mathbf{x}^*) \le f(\mathbf{x})\ \forall \mathbf{x} \in Q$.

Optimization problems not only aim to minimize function f but also maximize function f. The problem of maximizing function f can also be expressed in terms of (2.1) because maximizing f is equivalent to minimizing function f.

As explained above, the optimization problem is a problem in optimizing an objective function both minimizing and maximizing. The objective function can be either a linear function or a non-linear function. In non-linear functions there is one form known as a quadratic form which in its function contains x_j^2 and $x_i x_j (i \neq j)$ and is defined as follows:

Definition 2.2 [9] *A quadratic form* $f: \mathbb{R}^n \to \mathbb{R}$ *is a function*

$$f(\mathbf{x}) = \mathbf{x}^T Q \mathbf{x} \qquad (2.2)$$

where Q *is a real matrix* $n \times n$.

It will not reduce the generality if it is assumed that Q is a symmetric matrix, that is, $Q = Q^T$. If the Q matrix is not symmetrical, it can always be converted into a symmetrical matrix that satisfies the following equation,

$$Q_0 = Q_0^T = \frac{1}{2}(Q + Q^T). \qquad (2.3)$$

So for an asymmetrical matrix, the objective function will be

$$\mathbf{x}^T Q \mathbf{x} = \mathbf{x}^T Q_0 \mathbf{x} = \mathbf{x}^T \left(\frac{1}{2} Q + \frac{1}{2} Q^T\right) \mathbf{x} \qquad (2.4)$$

The definition of the quadratic form above can then be used to form quadratic program problems. A quadratic program problem is an optimization problem consisting of objective functions in the form of quadratic forms and linear constraint functions [5]. The general forms of quadratic program problems are as follows,

$$\begin{aligned} \text{Min} \quad & Z = \sum_{j=1}^{n} c_j x_j + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n} q_{ij} x_i x_j \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i = 1, \dots, m \\ & x_j \geq 0, \quad j = 1, \dots, n \end{aligned} \tag{2.5}$$

In vector-matrix notation, equation (2.5) can be written to be

$$\begin{aligned} \text{Min} \quad & Z = \mathbf{cx} + \mathbf{x}^T \mathbf{Qx} \\ \text{s.t.} \quad & \mathbf{Ax} \leq \mathbf{b} \\ & \mathbf{x} \geq 0 \end{aligned} \tag{2.6}$$

where $\mathbf{x} = (x_j; j = 1, \dots, n)$ is the column vector of the decision variable to be determined. $\mathbf{c} = (c_j\, ; j = 1, \dots, n)$ is a cost coefficients row vector, Q is a symmetrical quadratic matrix and positive definite, $\mathbf{b} = (b_i\, ; i = 1, \dots, m)$ is a right side vector of the constraint function and A is the constraint coefficient matrix.

2.2 Optimization of Multiobjective

An optimization problem can have more than one objective function that must be optimized and is called the multiobjective optimization problem.

The general form of multiobjevtice optimization problems is as follows:

$$\begin{aligned} \text{(P) Min} \quad & \mathbf{f(x)} = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x})) \\ \text{s.t.} \quad & g_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, n \\ & h_j(\mathbf{x}) = 0, \quad j = 1, \dots, m \\ & \mathbf{x} \in \mathbf{X} \end{aligned} \tag{2.7}$$

where $g_i(x) \leq 0$ and $h_j\,(x) = 0$ are constraint functions, n is the number of constraint functions in the form of inequality, m is the number of equality constraint functions and k is the number of objective functions.

In optimizing one objective function, one of the most optimal values can be obtained, but in multi-objective optimization, it is not easy to get an optimal value for all objective functions. This results in the solution obtained in multiobjective optimization not in the form of a single solution but in the form of a collection of optimal solutions, the choice of which depends on the decision maker. In general, the solution depends on the dominance of the solution over other solutions. This solution is called the optimal pareto solution [10].

Solution methods for multiobjective optimization problems have been developed and one of them is the Sum of Objective Method. This method is based on the following theorem,

Theorem 2.3 [11] *If* $\mathbf{x}^*$ *is an optimal solution of the scalar programming problems (S) where*

$$\textit{(S) Minimize} \sum_{i=1}^{k} f_i(\mathbf{x}), \quad \mathbf{x} \in P \tag{2.8}$$

Then $\mathbf{x}^*$ *is a fair solution of the problem* (P).

3. Fuzzy Logic

In 1965, Lotf Zadeh introduced a definition of fuzziness to solve problems that contained uncertainty. Lotf Zadeh writes about a vague set that is motivated from problems where traditional system analysis methods are not suitable for systems where the relationships between variables cannot be described in a differential equation or other mathematical equation. The use of traditional analytical methods tends to be numerically oriented, whereas many real world problems involve variables that cannot be expressed numerically and require a vague set to describe them.

Fuzzy sets can be introduced as generalizations of an ordinary set where the element does not only contain members or non-members, but its members have certain membership degrees, as written in the following definition,

Definition 3.1 [7] *Suppose* R *is a real number line, then a fuzzy* P *set in* R *is defined as a set of sequential pairs* $A = \{(x,$

$\mu_P(x)) | x \in R\}$, *where* $\mu_P(x)$ *is called the fuzzy set membership function*.

The degree of membership is the value of the membership function $\mu_{\tilde{P}}(x)$ which is a mapping with the following form:

$$\mu_{\tilde{P}}(x) : X \to [0,1] \quad (3.1)$$

where $\tilde{P}$ is a fuzzy set. The fuzzy set recognizes the following terms ([7]; [12]):

- **Height of Fuzzy Sets**
  The height of the fuzzy set $\tilde{P} \in \tilde{\mathcal{P}}(X)$ is *supremum* (or maximum, if the set is finite) of the membership function $\mu_{\tilde{P}}(x)$:

$$\text{hgt}(\tilde{P}) = h(\tilde{P}) = \sup_{x \in X} \mu_{\tilde{P}}(x). \quad (3.2)$$

  If $\text{hgt}\mu_{\tilde{P}} = 1$, $\tilde{P}$ is called normal; otherwise, it is called subnormal.
- **Core of Fuzzy Set**
  The essence of the fuzzy set $\tilde{P} \in \tilde{\mathcal{P}}(X)$ is the strict set of all $x \in X$ members who have one membership degree:

$$\text{core}(\tilde{P}) = C(\tilde{P}) = \{x \in X \mid \mu_{\tilde{P}}(x) = 1\}. \quad (3.3)$$

  The value of $\bar{x} = \text{core}(\tilde{p})$ which shows the highest degree of membership function $\mu_{\tilde{P}}(x) = 1$ is called the capital value of fuzzy number $\tilde{p}$. Capital value can also be referred to as peak value, main value or average value, where the last two expressions (main value or average value) are more suitable for symmetrical fuzzy numbers.
- **Support Fuzzy Set**
  The support of the fuzzy set $\tilde{P}$ is defined as follows,

$$supp(\tilde{P}) = \{x \in R \mid \mu_{\tilde{P}}(x) > 0\}. \quad (3.4)$$

- **Convex Fuzzy Set**
  Fuzzy set P in R convex if for any x, $x, y \in R$ and $\lambda \in [0,1]$ meet

$$\mu_P(\lambda x + (1-\lambda)y) \geq \min\{\mu_P(x), \mu_P(y)\}. \quad (3.5)$$

- **Triangle Fuzzy Numbers**
  As the linear membership function is quite simple, triangular fuzzy numbers or linear fuzzy numbers are one of the frequently used membership functions. The form of triangular fuzzy numbers can be written as follows:

$$\tilde{p} = tfn(\bar{x}, l, r) \quad (3.6)$$

  The following is the definition of membership functions of triangular fuzzy numbers:

$$\mu_{\tilde{P}}(x) = \begin{cases} 0, & x \leq \bar{x} - l \\ 1 - \dfrac{x - \bar{x}}{l}, & \bar{x} - l < x < \bar{x} \\ 1 - \dfrac{x - \bar{x}}{r}, & \bar{x} < x < \bar{x} + r \\ 0, & x \geq \bar{x} + l \end{cases} \quad \forall x \in \mathbb{R} \quad (3.7)$$

  $\bar{x}$ is the capital value of fuzzy numbers and $l < \bar{x} < r$.
- **Arithmetic in Triangle Fuzzy Numbers**
  Suppose that $\tilde{a} = \langle \bar{x}_a, l_a, r_a \rangle$ and $\tilde{b} = \langle \bar{x}_b, l_b, r_b \rangle$ two triangular fuzzy numbers and $x \in \mathbb{R}$. Addition, subtraction, and multiplication of fuzzy numbers are defined as:

$$\tilde{a} + \tilde{b} = \langle \bar{x}_a + \bar{x}_b, l_a + l_b, r_a + r_b \rangle \quad (3.9)$$

$$\tilde{a} - \tilde{b} = \langle \bar{x}_a - \bar{x}_b, l_a - l_b, r_a - r_b \rangle \quad (3.10)$$

$$x\tilde{a} = \begin{cases} \langle x\bar{x}_a, xl_a, xr_a \rangle, & x \geq 0 \\ \langle x\bar{x}_a, -xr_a, -xl_a \rangle, & x < 0 \end{cases} \quad (3.11)$$

4. Optimization of Fuzzy Numbers Quadratic Program

Fuzzy quadratic programs have a general form similar to the general form of a strict number quadratic program in equation (2.5) because basically a fuzzy quadratic program is a quadratic program with part or all of its parameters in the form of fuzzy numbers, so the general form of fuzzy quadratic programs can be stated as follows.

$$\begin{aligned} \text{Min} \quad & Z = \sum_{j=1}^{n} \tilde{c}_j x_j + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{q}_{ij} x_i x_j \\ \text{s.t.} \quad & \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i, \quad i = 1, \dots, m \\ & x_j \geq 0, \quad j = 1, \dots, n \end{aligned} \quad (4.1)$$

where $\tilde{c}_j, \tilde{q}_{ij}, \tilde{a}_{ij}$, and $\tilde{b}_i$ are fuzzy numbers.

Fuzzy numbers can be expressed in several forms, one of them using triangular fuzzy number, so that the fuzzy quadratic program (4.1) can be written as follows,

$$\begin{aligned} \text{Min} \quad & Z = \sum_{j=1}^{n} \langle c_j, p_j, t_j \rangle x_j + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \langle q_{ij}, s_{ij}, w_{ij} \rangle x_i x_j \\ \text{s.t.} \quad & \sum_{j=1}^{n} \langle a_{ij}, l_{ij}, r_{ij} \rangle x_j \leq \langle b_i, u_i, v_i \rangle, \quad i = 1, \dots, m \\ & x_j \geq 0, \quad j = 1, \dots, n \end{aligned} \tag{4.2}$$

Broadly speaking, the optimization program consists of two main functions, namely the objective function that will be optimized and the constraint function that limits the objective function. In the quadratic program (4.2), the two functions contain triangular fuzzy numbers, so that by using definitions and applying addition and multiplication to triangular fuzzy numbers, the objective function of the quadratic fuzzy program (4.2) can be changed to

$$\text{Min} \begin{cases} z_m = \sum_{j=1}^{n} c_j x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} x_i x_j \\ z_m - z_l = \sum_{j=1}^{n} (c_j - p_j) x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (q_{ij} - s_{ij}) x_i x_j \\ z_m + z_r = \sum_{j=1}^{n} (c_j + t_j) x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (q_{ij} + w_{ij}) x_i x_j \end{cases} \tag{4.3}$$

while for the constraint function to be,

$$\text{s.t.} \begin{cases} \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i \in N_m \\ \sum_{j=1}^{n} (a_{ij} - l_{ij}) x_j \leq (b_i - u_i), \quad i \in N_m \\ \sum_{j=1}^{n} (a_{ij} + r_{ij}) x_j \leq (b_i + v_i), \quad i \in N_m \end{cases} \tag{4.4}$$

An optimization problem will produce the most optimal value from the objective function. If the quadratic program is in the form of fuzzy numbers, then indirectly, the results to be obtained will also be in the form of fuzzy numbers. Parameters in the quadratic program (4.2) are triangular fuzzy numbers, the results of the quadratic program (4.2) will also be triangular fuzzy numbers.

The fuzzy quadratic program (4.1) produces three new objective functions with the same constraint function, so that it can create a multi-objective quadratic program with the following crisp number parameters.

$$\text{Min} \begin{cases} z_m = \sum_{j=1}^{n} c_j x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} x_i x_j \\ z_m - z_l = \sum_{j=1}^{n} (c_j - p_j) x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (q_{ij} - s_{ij}) x_i x_j \\ z_m + z_r = \sum_{j=1}^{n} (c_j + t_j) x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (q_{ij} + w_{ij}) x_i x_j \end{cases}$$

$$\text{s.t.} \begin{cases} \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i \in N_m \\ \sum_{j=1}^{n} (a_{ij} - l_{ij}) x_j \leq (b_i - u_i), \quad i \in N_m \\ \sum_{j=1}^{n} (a_{ij} + r_{ij}) x_j \leq (b_i + v_i), \quad i \in N_m \end{cases} \tag{4.5}$$

Multi-objective problems can be solved using several methods, one of them is Sum of Objective Method [11]. By using this method, the Multi-objective problems (4.5) can be changed to the following single optimization problem.

$$\begin{aligned} \text{Min } S &= z_m + (z_m - z_l) + (z + m + z_r) \\ &= \sum_{j=1}^{n} c_j x_j + (c_j - p_j) x_j \\ &\quad + (c_j + t_j) x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} x_i x_j \\ &\quad + (q_{ij} - s_{ij}) x_i x_j \\ &\quad + (q_{ij} + w_{ij}) x_i x_j \end{aligned}$$

$$\text{s.t.} \begin{cases} \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i \in N_m \\ \sum_{j=1}^{n} (a_{ij} - l_{ij}) x_j \leq (b_i - u_i), \quad i \in N_m \\ \sum_{j=1}^{n} (a_{ij} + r_{ij}) x_j \leq (b_i + v_i), \quad i \in N_m \end{cases} \tag{4.6}$$

The objective function on the optimization problem (4.6) is quadratic so that the optimization problem (4.6) is a simple quadratic program so it can be solved using the existing optimization method.

This method will produce three optimal values, namely $z_m, (z_m - z_l)$, and $(z_m + z_r)$ which are the three components forming a triangular fuzzy number of the objective function, $\tilde{z} = \langle z_m, z_l, z_r \rangle$.

5. Numerical Example

Consider the quadratic program with all the parameters in the form of the following fuzzy numbers,

$$\text{Min } Z = \langle -5,1,1\rangle x_1 + \langle 1.5,0.5,0.5\rangle x_2 + \frac{1}{2}(\langle 6,2,2\rangle x_1^2 + \langle -4,2,2\rangle x_1 x_2 + \langle 4,2,2\rangle x_2^2) \quad (5.1)$$

$$s.t \begin{cases} x_1 + \langle 1,0.5,0.5\rangle x_2 \le \langle 2,1,1\rangle \\ \langle 2,1,1\rangle x_1 + \langle -1,1,0.5\rangle x_2 \le \langle 4,1,1\rangle \\ x_1, x_2 \ge 0 \end{cases}$$

The fuzzy quadratic program (5.1) can be converted into three simple quadratic programs as follows:

$$\text{Min} \begin{cases} z_m = -5x_1 + 1.5x_2 + 3x_1^2 - 2x_1x_2 + 2x_2^2 \\ z_m - z_l = -6x_1 + x_2 + 2x_1^2 - 3x_1x_2 + x_2^2 \\ z_m + z_r = -4x_1 + 2x_2 + 4x_1^2 - x_1x_2 + 3x_2^2 \end{cases}$$

$$\text{s.t.} \begin{cases} x_1 + x_2 \le 2 \\ x_1 + 0.5x_2 \le 1 \\ x_1 + 1.5x_2 \le 3 \\ 2x_1 - x_2 \le 4 \\ x_1 - 2x_2 \le 3 \\ 3x_1 - 0.5x_2 \le 5 \\ x_1, x_2 \ge 0 \end{cases}$$

This multi-objective optimization problem is the solved using the Sum of Objective Method. In this method the initial problem can be formed into

$$\text{Min } S = -15x_1 + 4.5x_2 + 9x_1^2 - 6x_1x_2 + 6x_2^2$$

$$\text{s.t.} \begin{cases} x_1 + x_2 \le 2 \\ x_1 + 0.5x_2 \le 1 \\ x_1 + 1.5x_2 \le 3 \\ 2x_1 - x_2 \le 4 \\ x_1 - 2x_2 \le 3 \\ 3x_1 - 0.5x_2 \le 5 \\ x_1, x_2 \ge 0 \end{cases}$$

Furthermore, this simple quadratic program was completed using Kuhn-Tucker's condition. The optimal point obtained is $x_1 = 0.85000$ and $x_2 = 0.0500$, with optimal value $z_m = -2.0875, z_m - z_l = -3.7300$ dan $z_m + z_r = -0.4450$ so that optimal triangle number $\tilde{z} = \langle z_m, z_l, z_r \rangle = \langle -2.0875, 1.6425, 1.6425 \rangle$ is produced.

6. Conclusion

In this paper we introduce a new method to solve a fuzzy quadratic program using a triangular fuzzy number approach. The fuzzy quadratic program is then converted into a multiobjective optimization problem by utilizing definitions and arithmetic on triangular fuzzy numbers. By using Sum of Objective Method, the multiobjective optimization problem can be changed into single optimization problem, then be completed using the Karush-Kuhn-Tucker method. The multiobjective optimization problem will produce three optimal values which will form the optimal value in the form of triangular fuzzy numbers.

Acknowledgements

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