Quadratic Form Optimization with Fuzzy Number Parameters: Multiobjective Approaches

Niswatus S. Al-Mumtazah & Sugiyarto Surono

International Journal of Fuzzy Systems

ISSN 1562-2479
Volume 22
Number 4

Int. J. Fuzzy Syst. (2020) 22:1191-1197
DOI 10.1007/s40815-020-00808-x

Springer

Int. J. Fuzzy Syst. (2020) 22(4):1191–1197
https://doi.org/10.1007/s40815-020-00808-x

Quadratic Form Optimization with Fuzzy Number Parameters: Multiobjective Approaches

Niswatus S. Al-Mumtazah[1] · **Sugiyarto Surono**[1]

Received: 5 September 2019 / Revised: 7 January 2020 / Accepted: 20 January 2020 / Published online: 23 March 2020

Abstract Problems in daily life can be modeled into mathematical forms, one of them is the optimization of the quadratic form where the objective functions are the quadratic function. There are many real problems involving variables that cannot be stated numerically so that fuzzy logic appears. The purpose of this research is to optimize the quadratic form with all its parameters in the form of a fuzzy number by using the method of a triangular fuzzy number. This research resulted in a new completion method by utilizing definitions and arithmetic on the triangular fuzzy numbers. The resulted method is by changing a single fuzzy quadratic program becomes a simple quadratic multiobjective program. By using Sum of Objective Method, the multiobjective problem can be changed into single optimization problem. This single optimization problem is then completed using Karush–Kuhn–Tucker method, resulting in three optimal values which will form optimal values in the form of triangular fuzzy numbers.

Keywords Optimization · Quadratic · Fuzzy program · Triangular fuzzy numbers · Karush–Kuhn–Tucker conditions · Multiobjective optimization problem · Sum of Objective Method

✉ Sugiyarto Surono
sugiyartophd@gmail.com

Niswatus S. Al-Mumtazah
as.is.mumtaz@gmail.com

[1] Department of Mathematics, Faculty of Science and Applied Technology, Ahmad Dahlan University, Yogyakarta, Indonesia

1 Introduction

Problems in daily life can be modeled into a mathematical form or commonly called as mathematical modeling. Mathematic models or programs are problem-solving areas, one of which contains optimization problems. Generally, this best decision making is decided by minimizing or maximizing an objective function with or without constraints. The scope of optimization has attracted much attention, this is due to the advancement of computer technology which develops rapidly, including developing and providing easy-to-use software.

The optimization problems commonly consist of two functions such as objective function to be optimized and constraints function as a condition that becomes a requirement. The objective function can be a linear or non-linear function. Problems of non-linear optimization with objective function in the form of quadratic function are problems of maximizing or minimizing that objective function which depends on the constraint function and variable limitation which is not negative [1].

Many problems in the real world can be formulated as quadratic problems. The examples of that problem can be found in game theory, engineering modeling, design and control, economic problems, location problems and facilities allocation, portfolio, logistics, and so on [2].

In 1965, Lotfi Zadeh introduced the definition of fuzziness to solve problems that contain this uncertainty. Generally, fuzzy logic has shown good potential in modeling non-linear systems, complex, undefined properly or difficult to understand properly. Fuzzy logic is the way to describe the fuzziness mathematically and various applications have been found because they are easy to be implemented, flexible, tolerant toward inaccurate data, and low in implementation cost.

On its development, not only linear programing [3, 4], but also the problems of the quadratic program with parameters in the form of fuzzy numbers have been widely developed [5–9, 10–12]. Ammar [5] has developed an approach method of the quadratic program with fuzzy number parameters and applied it in portfolio optimization. Furthermore, Liu [6, 7] has developed methods to optimize quadratic problems, with its parameters in the form of fuzzy numbers by using a membership function approach. In 2014, Allahviranloo and Moazam [8] developed solutions to solve a quadratic equation where all its parameters, both its coefficient matrix or its variable, are in the form of fuzzy numbers.

The development continues. In 2017, Mirmohseni [9] did a development and related the program. The research that he did focused on the quadratic program with triangular fuzzy numbers where the coefficient of limitation and right-hand side parameters are triangular fuzzy numbers. In 2018, [13] Nezhad did another development where cost coefficients, matrix of quadratic form, vector of right-hand sides, and matrix of constraint coefficients are all triangular fuzzy numbers. The method that is used in this research is almost similar to the method used by Liu [7], both of them are equally produced two simpler optimization problems. The differences are on the two-level mathematic model and the form of end problems. Another research that focuses on fuzzy problem with triangular fuzzy number approach was conducted by Dhanasekar, Hariharan, and Sekar in [11].

The fuzziness of problems in the real world is not only limited to constants or determinant variables, the objective functions or the constraints. This is the background of the author in developing methods to optimize problems with objective functions and its constraints, all over the parameters and its variable in the form of the fuzzy number, so that solution can be applied in any condition. There are several methods that can be used to change fuzzy problem into crisp problem [5–7, 9, 12, 14–17]. The method used in this paper is a triangular fuzzy numbers method because the process is relatively simpler than using the extension principle method or α-cut method. The resulted method is by changing a single fuzzy quadratic program which becomes a simple quadratic multiobjective program. Problem solving of multiobjective problems can be done by using several methods [14–16, 18–20]. By using Sum of Objective Method, the multiobjective problem can be changed into single optimization problem. Then, the final results obtained are finished by using the Karush–Kuhn–Tucker condition.

The remainder of this paper is organized as follows. Section 2 gives a brief review of the literature on optimization of quadratic forms, multiobjective and fuzzy logic. A new completion method for fuzzy quadratic problem is proposed in Sect. 3. The numerical example is described in Sect. 4, while Sect. 5 presents conclusions and suggestions for future research.

2 Preliminaries

2.1 Optimization of Quadratic Forms

Optimization is the core of various problems, and includes decision making in it which can be found both in engineering or economics. A job that is done in making the decision is choosing the best decision from various alternatives. Generally, choosing this best decision is determined by minimizing or maximizing an objective function or without constraints.

Generally, the optimization problems are stated in the following form:

$$\begin{aligned} &\text{Minimize} \quad f(\mathbf{x}) \\ &\text{withconstraint} \quad \mathbf{x} \in \Omega \end{aligned} \tag{1}$$

where function of $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is function of real value that is expected to produce a minimum value. This function is called as objective function. The vector $\mathbf{x}$ is a column vector that contains n of independent variables, such as $\mathbf{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$. Variables $x_1, x_2, \cdots, x_n$ are often called as decision variables, whereas Ω called as set of constraints which is subset of $\mathbb{R}^n$.

The optimization problems above can be seen as a matter of decision making in which to find $\mathbf{x}$ Vector "best" from all of the possible vectors in it. That best vector is expected to be able to produce the smallest value from objectives and referred to as the function vector minimizer f, which is formally stated in the following definition:

Definition 1 [5] Any $\mathbf{x}$ that meets the set of constraints on Eq. (1) is called as a feasible solution. Suppose Q is a set of all feasible solutions of equations (1). $\mathbf{x}^* \in Q$ is said as optimal feasible solution for Eq. (1) if $f(\mathbf{x}^*) \leq f(\mathbf{x}) \quad \forall \mathbf{x} \in Q$.

The optimization problems not only aim to minimize the function f but also in maximizing function f. The problem of maximizing function f also can be stated in the form of (1) because maximizing function f is equivalent to minimizing the function f.

As explained above, optimization problems are problems in optimizing an objective function both minimizing or maximizing. Objective function can be either a linear function or a non-linear function. On non-linear functions, there is one form known as a quadratic form which in its function contains x_j^2 and $x_i x_j (i \neq j)$ and defined as follows:

Definition 2 [21]. A quadratic form $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is a function

$$f(\mathbf{x}) = \mathbf{x}^{\mathrm{T}} Q\mathbf{x} \tag{2}$$

where Q is a real matrix $n \times n$.

Without reducing generality if assumed Q is a symmetric matrix, that is, $Q = Q^{\mathrm{T}}$. If matrix Q_0 is asymmetric, always can be changed becomes matrix symmetric that satisfies the following equation [21],

$$Q = \frac{1}{2}\left(Q_0 + Q_0^{\mathrm{T}}\right). \tag{3}$$

So for an asymmetrical matrix, the objective function will be

$$\mathbf{x}^{\mathrm{T}} Q\mathbf{x} = \mathbf{x}^{\mathrm{T}}\left(\frac{1}{2}Q_0 + \frac{1}{2}Q_0^{\mathrm{T}}\right)\mathbf{x}. \tag{4}$$

The definition of quadratic forms above then can be used to form the problem of quadratic program. Problem of the quadratic program is an optimization problem which consists of objective functions in the form of quadratic and constraint functions which is linear [7]. The general form of the problem of quadratic program is as follows:

$$\begin{aligned} &\text{Min}\, Z = \sum_{j=1}^{n} c_j x_j + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n} q_{ij} x_i x_j \\ &\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1, \ldots, m \\ &x_j \ge 0, \quad j = i, \ldots, n. \end{aligned} \tag{5}$$

In vector–matrix notation, Eq. (5) can be written as follows:

$$\begin{aligned} &\text{Min}\, Z = \mathbf{cx} + \mathbf{x}^{\mathrm{T}}\mathbf{Qx} \\ &\text{s.t}\; \mathbf{Ax} \le \mathbf{b} \\ &\mathbf{x} \ge 0 \end{aligned} \tag{6}$$

where $\mathbf{x} = (x_j; j = 1, \ldots, n)$ is the column vector of the decision variable that will be determined. $\mathbf{c} = (c_j; j = 1, \ldots, n)$ is vector of cost coefficients, $\mathbf{Q}$ is a symmetrical matrix of quadratic form and positive definite, $\mathbf{b} = (x_i; i = 1, \ldots, m)$ is vector of right-hand sides and $\mathbf{A}$ is the matrix of constraint coefficients.

2.2 Optimization of Multiobjective

An optimization problem can have more than one objective function that must be optimized and is called the multiobjective optimization problem.

The general form of multiobjective optimization problems is as follows:

$$\begin{aligned} (P) \quad &\text{Minimize} \quad \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})) \\ &\text{s.t.} \quad g_i(\mathbf{x}) \le 0, \quad i = 1, \ldots, n \\ &\qquad h_j(\mathbf{x}) = 0, \quad j = 1, \ldots, m \\ &\qquad \mathbf{x} \in \mathbf{X}. \end{aligned} \tag{7}$$

where $g_i(x) \le 0$ and $h_j(x) = 0$ are constraint functions, n is the number of constraint functions in the form of inequality, m is the number of equality constraint functions, and k is the number of objective functions.

On optimizing, an objective function can be obtained for one most optimal value, but on multiobjective optimization, it is not easy to obtain the optimal value for all objective functions. Sometimes, the optimal solution is obtained on the first objective function, but less optimal on the other objective functions, or otherwise. This resulted solution that is obtained on the multiobjective optimization is not in the form of one single solution but rather a collection of optimal solutions, in which the choice depends on the decision-maker. In general, the solution chosen depends on the dominance of the solution over other solutions. This solution is called as Pareto optimal solution [22].

Problem solving of multiobjectives can be done by using several methods, one of them is by using the Sum of Objective Method. This method based on the following Theorem:

Theorem 1 [18] *If* $\mathbf{x}^*$ *is an optimal solution of the scalar programing problems* (S) *where*

$$(S) \quad \text{Minimize} \sum_{i=1}^{k} f_i(\mathbf{x}), \quad \mathbf{x} \in P, \tag{8}$$

then $\mathbf{x}^*$ *is a fair solution of the problem* (P).

2.3 Fuzzy Logic

In 1965, Lotfi Zadeh introduces the definition of fuzziness to solve problems, which consist of uncertainty. Lotfi Zadeh wrote about a fuzzy set that is motivated by problems where the analysis method of the traditional system is not suitable for systems where the relationships between its variables cannot be described in a differential equation or another mathematical equation. The use of that traditional analytical methods tends to orient numerically, whereas many world problems involve variables that cannot be stated numerically and need a fuzzy set to describe it.

The fuzzy sets can be introduced as generalizations from one ordinary set where its elements are not only contained members or non-members, but the members have particular membership degrees, as in the following definition:

Definition 3 [9] Suppose R is a real number line, then a fuzzy set P in R is defined as a set of sequential pairs $P = \{(x, \mu_P(x)) | x \in R\}$, where $\mu_P(x)$ called as membership function of fuzzy set.

The degree of membership is value of the membership function $\mu_{\tilde{P}}(x)$ which is a mapping with the following form:

$$\mu_{\tilde{P}}(x) : X \longrightarrow [0, 1] \tag{9}$$

where $\tilde{P}$ is a fuzzy set. The fuzzy set recognizes the following terms [9, 23]:

- Height of fuzzy sets
  The height of the fuzzy set $\tilde{P} \in \tilde{\mathcal{P}}(X)$ is *supremum* (or maximum, if the set is finite) of the membership function $\mu_{\tilde{P}}(x)$:
  $$\text{hgt}(\tilde{P}) = h(\tilde{P}) = \sup_{x \in X} \mu_{\tilde{P}}(x). \tag{10}$$
  If $\text{hgt}(\tilde{P}) = 1$ is called normal; otherwise, it is called subnormal.
- Core of fuzzy set
  The core of the fuzzy set $\tilde{P} \in \tilde{\mathcal{P}}(X)$ is the strict set of all members $x \in X$ that has one membership degree:
  $$\text{core}(\tilde{P}) = C(\tilde{P}) = \{x \in X | \mu_{\tilde{P}}(x) = 1\}. \tag{11}$$
  The value of $\bar{x} = \text{core}(\tilde{p})$ that shows the highest degree of membership function $\mu_{\tilde{P}}(x) = 1$ is called the modal value of fuzzy number $\tilde{p}$. Modal value can also be referred to as peak value, main value or average value, where the last two expressions (main value or average value) are more suitable for symmetrical fuzzy numbers.
- Support fuzzy set
  The support of the fuzzy set $\tilde{P}$ is defined as follows:
  $$\text{supp}(\tilde{P}) = \{x \in R | \mu_{\tilde{P}}(x) > 0\}. \tag{12}$$
- Convex fuzzy set
  Fuzzy set P in R convex if for any $x, y \in R$ and $\lambda \in [0, 1]$ meet
  $$\mu_P(\lambda x + (1 - \lambda)y) \geq \min\{\mu_P(x), \mu_P(y)\}. \tag{13}$$
- Triangle fuzzy number
  As the linear membership function is quite simple, triangular fuzzy numbers or linear fuzzy numbers are one of the frequently used membership functions. The form of triangular fuzzy numbers can be written as follows:
  $$\tilde{p} = \text{tfn}(\bar{x}, l, r) = \langle \bar{x}, l, r \rangle. \tag{14}$$
  The following is the definition of membership functions of triangular fuzzy numbers:
  $$\mu_{\tilde{P}}(x) = \begin{cases} 0, & x \leq \bar{x} - l \\ 1 + \dfrac{x - \bar{x}}{l}, & \bar{x} - l < x < \bar{x} \\ 1 - \dfrac{x - \bar{x}}{r}, & \bar{x} < x < \bar{x} + r \\ 0, & x \geq \bar{x} + r \end{cases} \quad \forall x \in \mathbb{R} \tag{15}$$
- Arithmetic in triangle fuzzy numbers
  Suppose that $\tilde{a} = \langle \bar{x}_a, l_a, r_a \rangle$ and $\tilde{b} = \langle \bar{x}_b, l_b, r_b \rangle$ two triangular fuzzy numbers and $x \in \mathbb{R}$. Addition, subtraction, and multiplication of fuzzy numbers are defined as follows:
  $$\tilde{a} + \tilde{b} = \langle \bar{x}_a + \bar{x}_b, l_a + l_b, r_a + r_b \rangle \tag{16}$$
  $$\tilde{a} - \tilde{b} = \langle \bar{x}_a - \bar{x}_b, l_a - l_b, r_a - r_b \rangle \tag{17}$$
  $$x\tilde{a} = \begin{cases} \langle x\bar{x}_a, xl_a, xr_a \rangle, & x \geq 0 \\ \langle x\bar{x}_a, -xr_a, -xl_a \rangle, & x < 0 \end{cases}. \tag{18}$$

3 Optimization of Fuzzy Numbers Quadratic Program

The fuzzy quadratic programs have a general form which is similar with the general form of crisp number quadratic program in Eq. (5) because basically fuzzy quadratic program is a quadratic program with part or all of its parameters in the form of fuzzy numbers, so the general form of fuzzy quadratic programs can be stated as follows:

$$\begin{aligned} &\text{Min} \quad Z = \sum_{j=1}^{n} \tilde{c}_j x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{q}_{ij} x_i x_j \\ &\text{s.t.} \quad \begin{cases} \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i, & i = 1, 2, \ldots, m \\ x_j \geq 0, & j = 1, 2, \ldots, n \end{cases} \end{aligned} \tag{19}$$

where $\tilde{c}_j, \tilde{q}_{ij}, \tilde{a}_{ij}$ and $\tilde{b}_i$ are fuzzy numbers.

Fuzzy numbers can be expressed in several forms, one of them using triangular fuzzy number, so that the fuzzy quadratic program (19) can be rewritten as follows:

$$\begin{aligned} &\text{Min } Z = \sum_{j=1}^{n} \langle c_j, p_j, t_j \rangle x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \langle q_{ij}, s_{ij}, w_{ij} \rangle x_i x_j \\ &\text{s.t.} \begin{cases} \sum_{j=1}^{n} \langle a_{ij}, l_{ij}, r_{ij} \rangle x_j \leq \langle b_i, u_i, v_i \rangle, & i = 1, 2, \ldots, m \\ x_j \geq 0, & j \in N_n \end{cases} \end{aligned} \tag{20}$$

Broadly speaking, the optimization program consists of two main functions, namely the objective function that will be optimized and the constraint function that limits the objective function.

An optimization problem will produce the most optimal value from the objective function. If the quadratic program that is faced is in the form of fuzzy numbers, then indirectly, the results that will be obtained also will be in the form of fuzzy numbers. The parameters on the quadratic program (20) are triangular fuzzy numbers, then the results of the quadratic program (20) will also be triangular fuzzy numbers.

Consider the parameter $\tilde{c} = \langle c_j, p_j, t_j \rangle$ in (20). Based on the definition of a triangle fuzzy number, in Eq. (15), parameter $\tilde{c}$ consists of three numbers where c_j is a number with a membership value of 1 (core), while $c_j - p_j$ and $c_j + t_j$ have membership value of 0. Because the objective function contains fuzzy triangle number parameters then indirectly the value of the objective function is also in the form of a fuzzy triangle, i.e., $\langle z_m, z_m - z_l, z_m + z_r \rangle$, which by definition, in Eq. (15), z_m is a number with a membership value 1, while $z_m - z_l$ and $z_m + z_r$ have a membership value of 0. By using definitions and applying addition and multiplication to triangular fuzzy numbers, the value of z_m is obtained by adding all the cores of the fuzzy number parameter so that it is obtained as follows:

$$z_m = \sum_{j=1}^{n} c_j x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} x_i x_j. \tag{21}$$

By following this pattern, the objective function of the quadratic fuzzy program (20) can be changed and becomes

$$\text{Min} \begin{cases} z_m = \sum_{j=1}^{n} c_j x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} x_i x_j \\ z_m - z_l = \sum_{j=1}^{n} (c_j - p_j) x_j \\ \quad + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (q_{ij} - s_{ij}) x_i x_j \\ z_m + z_r = \sum_{j=1}^{n} (c_j + t_j) x_j \\ \quad + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (q_{ij} + w_{ij}) x_i x_j \end{cases} \tag{22}$$

while for the constraint function becomes,

$$\text{s.t.} \begin{cases} \sum_{j=1}^{m} a_{ij} x_j \le b_i, & i = 1, 2, \ldots, m \\ \sum_{j=1}^{m} (a_{ij} - l_{ij}) x_j \le (b_i - u_i), & i = 1, 2, \ldots, m \\ \sum_{j=1}^{m} (a_{ij} + r_{ij}) x_j \le (b_i + v_i), & i = 1, 2, \ldots, m. \end{cases} \tag{23}$$

An optimization problem will produce the most optimal value from the objective function. If the quadratic program that is faced is in the form of fuzzy numbers, then indirectly, the results that will be obtained will also be in the form of fuzzy numbers. The parameters on the quadratic program (20) are triangular fuzzy numbers, then the results of the quadratic program (20) will also be triangular fuzzy numbers.

The fuzzy quadratic program (19) produces three new objective functions with the same constraint function, so that it can be formed as a multiobjective quadratic program with the following crisp number parameters.

$$\text{Min} \begin{cases} z_m = \sum_{j=1}^{n} c_j x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} x_i x_j \\ z_m - z_l = \sum_{j=1}^{n} (c_j - p_j) x_j \\ \quad + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (q_{ij} - s_{ij}) x_i x_j \\ z_m + z_r = \sum_{j=1}^{n} (c_j + t_j) x_j \\ \quad + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (q_{ij} + w_{ij}) x_i x_j \end{cases}$$

$$\text{s.t.} \begin{cases} \sum_{j=1}^{m} a_{ij} x_j \le b_i, & i = 1, 2, \ldots, m \\ \sum_{j=1}^{m} (a_{ij} - l_{ij}) x_j \le (b_i - u_i), & i = 1, 2, \ldots, m \\ \sum_{j=1}^{m} (a_{ij} + r_{ij}) x_j \le (b_i + v_i), & i = 1, 2, \ldots, m \\ x_j \ge 0, & j = 1, 2, \ldots, n. \end{cases} \tag{24}$$

Multiobjective problems can be solved using several methods, one of them is Sum of Objective Method [18]. By using this method, the Multiobjective problems (24) can be changed to the following single optimization problem.

$$\begin{aligned} \text{Min } S &= z_m + (z_m - z_l) + (z_m + z_r) \\ &= \sum_{j=1}^{n} c_j x_j + (c_j - p_j) x_j + (c_j + t_j) x_j \\ &\quad + \frac{1}{2} \left[\sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} x_i x_j + (q_{ij} - s_{ij}) x_i x_j \right. \\ &\quad \left. + (q_{ij} + w_{ij}) x_i x_j \right] \end{aligned}$$

$$\text{s.t} \begin{cases} \sum_{j=1}^{m} a_{ij} x_j \le b_i, & i = 1, 2, \ldots, m \\ \sum_{j=1}^{m} (a_{ij} - l_{ij}) x_j \le (b_i - u_i), & i = 1, 2, \ldots, m \\ \sum_{j=1}^{m} (a_{ij} + r_{ij}) x_j \le (b_i + v_i), & i = 1, 2, \ldots, m \\ x_j \ge 0, & j = 1, 2, \ldots, n. \end{cases} \tag{25}$$

The objective function on the optimization problem (25) is in the form of quadratic so that the optimization problem (25) is a simple quadratic program and it can be solved using the existing optimization method.

This method will produce three optimal values such as $z_m, (z_m - z_l)$, and $(z_m + z_r)$, which are the three components forming a triangular fuzzy number of the objective function, $\tilde{z} = \langle z_m, z_l, z_r \rangle$.

This proposed method provides a relatively simpler way to solve the fuzzy quadratic problem by changing this problem into a crisp quadratic program. This change is done by utilizing the definition and arithmetic of triangular fuzzy number and the Sum of Objective Methods. So to obtain optimal value in the initial fuzzy problem, it is enough to solve one crisp quadratic problem of the conversion result.

4 Numerical Example

Consider the quadratic program with all the parameters in the form of the following fuzzy numbers,

$$\begin{aligned}\text{MinZ} &= \langle -5,1,1\rangle x_1 + \langle 1.5,0.5,0.5\rangle x_2 + \frac{1}{2}\left(\langle 6,2,2\rangle x_1^2 \right.\\ &\quad \left. + \langle -4,2,2\rangle x_1x_2 + \langle 4,2,2\rangle x_2^2\right)\\ \text{s.t.} &\begin{cases} x_1 + \langle 1,0.5,0.5\rangle x_2 \le \langle 2,1,1\rangle \\ \langle 2,1,1\rangle x_1 + \langle -1,1,0.5\rangle x_2 \le \langle 4,1,1\rangle \\ x_1, x_2 \ge 0. \end{cases}\end{aligned} \tag{26}$$

The fuzzy quadratic program (26) can be converted into the following multiobjective optimization problem,

$$\text{Min} \begin{cases} z_m = -5x_1 + 1.5x_2 + 3x_1^2 - 2x_1x_2 + 2x_2^2 \\ (z_m - z_l) = -6x_1 + x_2 + 2x_1^2 - 3x_1x_2 + x_2^2 \\ (z_m + z_r) = -4x_1 + 2x_2 + 4x_1^2 - x_1x_2 + 3x_2^2 \end{cases}$$

$$\text{s.t.} \begin{cases} x_1 + x_2 \le 2 \\ x_1 + 0.5x_2 \le 1 \\ x_1 + 1.5x_2 \le 3 \\ 2x_1 - x_2 \le 4 \\ x_1 - 2x_2 \le 3 \\ 3x_1 - 0.5x_2 \le 5 \\ x_1, x_2 \ge 0. \end{cases}$$

This multiobjective optimization problem is then solved by using the Sum of Objective Method. In this method the initial problem, the optimization problem (26), can be formed becomes

$$\text{Min} \quad S = -15x_1 + 4.5x_2 + 9x_1^2 - 6x_1x_2 + 6x_2^2$$

$$\text{s.t.} \begin{cases} x_1 + x_2 \le 2 \\ x_1 + 0.5x_2 \le 1 \\ x_1 + 1.5x_2 \le 3 \\ 2x_1 - x_2 \le 4 \\ x_1 - 2x_2 \le 3 \\ 3x_1 - 0.5x_2 \le 5 \\ x_1, x_2 \ge 0. \end{cases}$$

Furthermore, this simple quadratic program was completed using Kuhn–Tucker's condition. The optimal point obtained is $x_1 = 0.85000$ and $x_2 = 0.0500$, with optimal values $z_m = -2.0875, z_m - z_l = -3.7300$, and $z_m + z_r = -0.4450$ so that the optimal value of objective function obtained is namely $\tilde{z} = \langle z_m, z_l, z_r\rangle = \langle -2.0875, 1.6425, 1.6425\rangle$.

5 Conclusion

In this paper, we introduce a new method to solve a fuzzy quadratic program using a triangular fuzzy number and multiobjective approach. The fuzzy quadratic program is then converted into a multiobjective optimization problem by utilizing definitions and arithmetic on triangular fuzzy numbers. By using Sum of Objective Method, the multiobjective optimization problem can be changed into single optimization problem, and then completed using the Karush–Kuhn–Tucker method. The multiobjective optimization problem will produce three optimal values which will form the optimal value of initial fuzzy problem in the form of triangular fuzzy numbers. Here, an easy and useful way is to solve the fuzzy quadratic problem with triangular fuzzy number.

For future research, several approaches can be used in several sections, such as the form used to express fuzzy numbers, for example, using extension principle, α-cut, trapezoidal fuzzy number, or another form. Another approach that can be used is the method used in solving multiobjective problems such as The Weighted Sum Method, The Goal Programing Method, etc.

Acknowledgements We are grateful for support from Department of Mathematics, Faculty of Science and Applied Technology, Ahmad Dahlan University.

References

1. Peressini, A.L., Sullivan, F.E., Uhl, J.J.: The Mathematics of Nonlinear Programming. Springer, New York (1988)
2. Silva, R.C., Cruz, C., Yamakami, A.: A parametric method to solve quadratic programming probles with fuzzy costs. IFSA-EUSFLAT. 1398–1403, (2009)
3. Javanmard, M., Nehi, H.M.: A solving method for fuzzy linear programming problem with interval type-2 fuzzy number. Int J Fuzzy Syst (2019). https://doi.org/10.1007/s40815-018-0591-3
4. Ghaznavi, M., Soleimani, F., Hoseinpoor, N.: Parametric analysis in fuzzy number linear programming problems. Int J Fuzzy Syst (2016). https://doi.org/10.1007/s40815-015-0123-3
5. Ammar, E., Khalifa, H.A.: Fuzzy portfolio optimization a quadratic programming approach. Chaos Solitons Fractals (2003). https://doi.org/10.1016/S0960-0779(03)00071-7
6. Liu, S,-T.: Solving quadratic programming with fuzzy parameters based on extension principle (2007)

7. Liu, S.-T.: A revisit to quadratic programming with fuzzy parameters. Chaos Solitons Fractal (2009). https://doi.org/10.1016/j.chaos.2008.04.061
8. Allahviranloo, T., Moazam, L.G.: The solution of fully quadratic equation based on optimization theory. Scientific World J (2014). https://doi.org/10.1155/2014/156203
9. Mirmohseni, S.M., Nasseri, S.H.: a quadratic programming with triangular fuzzy number. J Appl Math Phys (2017). https://doi.org/10.4236/jamp.2017.511181
10. Dhanasekar, S., Hariharan, S., Sekar, P.: Fuzzy Hungarian MODI to solve fully fuzzy transportation problems. Int J Fuzzy Syst (2016). https://doi.org/10.1007/s40815-016-0251-4
11. Dourado, A.D.P., Lobato, F.S., Cavalini Jr., A.A., Steffen Jr., V.: Fuzzy reliability-based optimization for engineering system design. Int J Fuzzy Syst (2019). https://doi.org/10.1007/s40815-019-00655-5
12. Ghanbari, R., Moghadam, K.G.: Solving fuzzy quadratic programming problems based on ABS algorithm. Soft Comput (2019). https://doi.org/10.1007/s00500-019-04013-3
13. Nezhad, N.A.T.: A solution approach for solving fully fuzzy quadratic programming problems. J Appl Res Ind Eng (2018). https://doi.org/10.22105/jarie.2018.111797.1028
14. Yang, Y., Zhao, J., Xia, J., Zhuang, G., Zhang, W.: Multiobjective optimization control for uncertain nonlinear stochastic system with state-delay. Int J Fuzzy Syst (2018). https://doi.org/10.1007/s40815-018-0541-0
15. Pourjavad, E., Mayorga, R.V.: Multi-objective fuzzy programming of closed-loop supply chain considering sustained measures. Int J Fuzzy Syst (2018). https://doi.org/10.1007/s40815-018-0551-y
16. Rout, P.K., Nanda, S., Acharya, S.: Multi-objective fuzzy probabilistic quadratic programming problem. Int J Oper Res **34**, 387–408 (2019)
17. Gani, A.N., Saleem, R.A.: Solving fuzzy sequential quadratic programming algorithm for fuzzy non-linear programming. J Phys Sci **23**, 89–96 (2018)
18. Pandian, P.: A simple approach for finding a fair solution to multiobjective programming problems. Bull Math Sci Appl (2012). https://doi.org/10.18052/www.scipress.com/BMSA.2.21
19. Mirzei, N., Mahmoodirad, A., Niroomand, S.: An uncertain multi-objective assembly line balancing problem: a credibility-based fuzzy modeling approach. Int J Fuzzy Syst (2019). https://doi.org/10.1007/s40815-019-00734-7
20. Kecskes, I., Ordy, P.: Multi-scenario multi-objective optimization of a fuzzy motor controller for the Szabad(ka)-II hexapod robot. Acta Polytech Hung **15**, 157–178 (2018)
21. Chong, E.K.P., Zak, S.H.: An Introduction to Optimization, 2nd edn. Wiley-Interscience Publication, Hoboken (2001)
22. Jaimes, A.L., Martinez, S.Z., Coello, C.A.: An Introduction to Multiobjective Optimization Techniques. McGraw-Hill Companies Inc., New York (2009)
23. Hanss, M.: Applied Fuzzy Arithmatic. Springer, New York (2005)

Niswatus S. Al-Mumtazah, was born on 07/03/1991 at Surabaya, East Java, Indonesia. She obtained her Bachelor's degree in Pharmacy from Airlangga University, Surabaya, Indonesia in 2013 and Bachelor's degree in Mathematic from Ahmad Dahlan University, Yogyakarta, Indonesia in 2019. She obtained her Master's degree in Mathematic from Gadjah Mada University, Yogyakarta, Indonesia in 2019. Her research interest includes Fuzzy logic, Mathematical modelling with numerical approach.

Sugiyarto Surono completed his bachelor degree in Mathematics in Universitas Gadjah Mada. He has also completed his master degree in Mathematics and Statistics in the same university. He has completed his doctoral degree in Universiti Teknologi Malaysia, majored in optimization. Currently, he is working as a lecturer in Faculty of Maths and Science of Ahmad Dahlan University while doing several research in mathematics.