Mathematical Model with Laplace Autoregressive Process for Heart-Rate Signals

By SUPARMAN

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Abstract

Noise that is normally distributed is often added to autoregressive (AR) time series models. Most AR model time series parameter estimation methods are based on normality assumptions. One of the AR time series models that does not verify normality assumptions is the Laplace AR model. If the estimation method based on the normality assumption is used on the Laplace AR time series model, the estimation method will produce a very biased estimate. This study proposes the reversible jump Markov Chain Monte Carlo (MCMC) algorithm to estimate the parameters of the Laplace AR time series models. The parameters of the Laplace AR time series models are model order, model coefficient, and noise variance. The parameter estimation of the Laplace AR time series models is done in the Bayesian framework. The prior distribution for the model order is selected the binomial distribution, the prior distribution for the model coefficient is selected the uniform distribution, the prior distribution for the noise variance is selected the inverse-Gamma distribution. This prior distribution is combined with the likelihood function of the data to get the posterior distribution. Parameter estimation is based on the posterior distribution. The reversible MCMC algorithm allows to estimate the model order, model coefficients, and noise variance simultaneously. The performance of the algorithm is tested by using some synthetic data generated from the simulation. The simulation results show that the reversible jump MCMC algorithm can estimate the Laplace AR models parameters well. The advantage of reversible jump MCMC algorithm is that this algorithm is able to estimate the parameters of the stationary Laplace AR time series models.

***Keywords**: Autoregressive, Bayesian selection, Laplace noise, reversible jump MCMC*

1. Introduction

Autoregressive is a mathematical model that is applied to model data in various fields of life. The autoregressive (AR) model is a flexible model by setting an order and a model parameter. The AR model is adopted to diagnose Parkinson's disease [1]. The AR model is applied to an eye tremor movement [2]. The eye tremor movement is extracted based on the eye position signal. The AR model is used to predict river flow [3]. The AR model is utilized to categorize a gas chromatography output [4]. AR model is implemented to model respiratory rate extraction [5]. The AR model is applied to detect damage [6]. The AR model is adopted to forecast EEG data [7]. The AR model is utilized to detect structural damage [8]. The AR model is implemented to simulate a dynamic light scattering (DLS) signal [9]. An AR model is used to predict a channel in a wireless network [10]. The AR model is applied in an analysis of preearthquake ionospheric anomalies [11]. The AR model is adopted to predict gas temperature that is discharged by a ship's main engine [12]. The AR model can predict a trend of changes in smoke temperatures. The AR model is used to identify a random signal frequency [13]. The AR model is implemented to identify a high accuracy on electroencephalography (EEG) biometrics [14]. The AR model is utilized to detect an atrial fibration [15].

The autoregressive model contains noise. This noise is assumed to have a specific distribution. The autoregressive model is generally considered to have a Gaussian noise, for example [16]. In various applications of autoregressive models, noise present in mathematical models are often found not Gaussian distributed, for example [17], [18], and [19]. The autoregressive model was investigated where the distribution for noise was Pareto [17]. An autoregressive model is examined where noise is exponentially distributed [18]. An autoregressive model was discussed where the noise used was G-GARCH [19]. In the studies above, the autoregressive model order is assumed to be known. The autoregressive model for noise which is exponentially distributed is studied, but the order of the model is unknown [20].

A Laplacian noise is used in a rare signal representation [21]. The use of Laplacian noise in human sensory processing is investigated [22]. A change in body position from the ECG is detected by assuming that changes in body position from the ECG are Laplacian distribution [23]. However, an autoregressive model that has Laplacian noise has not been studied. This study will propose the development of an autoregressive model that has Laplacian noise. The autoregressive model order is assumed to be unknown.

The paper is organized as follows: the first part presents an introduction, the second part describes the method, the third part gives the results and discussion, and the fourth part provides a conclusion with.

2. Method

This study will use the Bayesian approach to estimate an AR model parameter. An autoregressive model order, an autoregressive model coefficient, and a noise variance are treated as a random variable that has a distribution. This distribution is known as a prior distribution. The prior distribution for an order chosen is the Binomial distribution. The prior distribution for a coefficient if given order is a uniform distribution. The prior distribution for the noise variance selected is the inverse exponential distribution. Furthermore, the prior distribution for a parameter is combined with a likelihood function for data to get a posterior distribution for the parameter. Because the order is a parameter, the form of the posterior distribution for a parameter is very complicated. The complexity of the posterior distribution for a parameter that makes the Bayes estimator cannot be determined explicitly. A reversible jump method MCMC [24] was adopted to create a random variable that has a distribution approaching a posterior distribution. The reversible jump MCMC method uses three transformations, namely: an order birth, an order death, and a change in the coefficient. The performance of the reversible jump MCMC method was evaluated using a simulation study. The reversible jump MCMC method was applied to the estimation of a model parameter for heart rate.

3. Result and Discussion

This section describes the likelihood function, prior distribution, posterior distribution, reversible jump MCMC, simulation, and modeling for heart rate data in more detail.

3.1. Likelihood Function

Suppose $x = (x_1, \ldots, x_n)$ is data where n represents the amount of data. The data is said to have a p-order autoregressive model, written with $AR(p)$, if the data satisfies the equation:

$$x_t = z_t - \sum_{i=1}^{p} \phi_i x_{t-i} \tag{1}$$

The variable z_t $(t = 1, \ldots, n)$ is assumed to be a Laplace distribution with the β parameter. The probability function of z_t can be written as

$$g(z_t|\beta) = \frac{1}{2\beta}\, exp - \frac{|z_t|}{\beta} \tag{2}$$

By using variable transformations, the probability function of x_t is

$$g(x_t|\beta) = \frac{1}{2\beta}\, exp - \frac{\left|\sum_{i=1}^{p} \phi_i x_{t-i} + x_t\right|}{\beta} \tag{3}$$

So the probability function of data x is

$$\begin{aligned} f(x|p, \phi^{(p)}, \beta) &= \prod_{t=p+1}^{n} \frac{1}{2\beta}\, exp - \frac{\left|\sum_{i=1}^{p} \phi_i x_{t-i} + x_t\right|}{\beta} \\ &= \left(\frac{1}{2\beta}\right)^{n-p} exp - \frac{1}{\beta}\sum_{t=p+1}^{n}\left|\sum_{i=1}^{p} \phi_i x_{t-i} + x_t\right| \end{aligned} \tag{4}$$

where $\phi^{(p)} = (\phi_1, \ldots, \phi_p)$. Suppose that $r_1, r_2, \cdots, r_p$ states the functions of partial autocorrelation which correspond to the autoregressive model. Let F denote a transformation from $(\phi_1, \ldots, \phi_p) \in S_p$ to $(r_1, r_2, \cdots, r_p) \in (-1,1)^p$ where S_p is the stationary area of the autoregressive model [25]. If the value of p is large, the condition of stationarity will be difficult to determine. Through a reparametrization with the help of transformation F, the condition of stationarity will be easily identified even for large p. By using reparameterization, the likelihood function for data can be written as

$$\begin{aligned} f(x|p, r^{(p)}, \beta) &= \prod_{t=p+1}^{n} \frac{1}{2\beta}\, exp - \frac{\left|\sum_{i=1}^{p} F^{-1}(r_i) x_{t-i} + x_t\right|}{\beta} \\ &= \left(\frac{1}{2\beta}\right)^{n-p} exp - \frac{1}{\beta}\sum_{t=p+1}^{n}\left|\sum_{i=1}^{p} F^{-1}(r_i) x_{t-i} + x_t\right| \end{aligned} \tag{5}$$

where F^{-1} is the inverse transformation of F.

3.2. Prior Distribution

The prior distribution for the order of the AR model chosen is the Binomial distribution with a parameter p_{max} and λ

$$\pi(p|\lambda) = C_p^{p_{max}} \lambda^p (1-\lambda)^{p_{max}-p} \tag{6}$$

While the prior distribution for $r^{(p)}$ is a uniform distribution at intervals $(-1, 1)^p$

$$\pi(r^{(p)}|p) = \frac{1}{2^p} \tag{7}$$

Finally, the prior distribution for β is the inverse Gamma distribution with parameters u and v (8)

$$\pi(\beta|u, v) = \frac{v^u}{\Gamma(u)} \beta^{-(u+1)} exp - \frac{v}{\beta}$$

Here, $u = 1$. In the prior distribution, there are parameters λ and v. The prior distribution for λ selected is a uniform distribution at the interval $(0,1)$. Whereas the prior distribution for v chosen is the prior distribution of Jeffreys $\pi(v) \propto \frac{1}{v}$. Thus, the prior joint distribution and the hyperprior can be written as

$$\begin{aligned} &\pi(p, r^{(p)}, \lambda, \beta, v) \\ &= C_p^{p_{max}} \lambda^p (1-\lambda)^{p_{max}-p} \frac{1}{2^p} \frac{v^u}{\Gamma(u)} \beta^{-(u+1)} exp - \frac{v}{\beta}\frac{1}{v} \end{aligned} \tag{9}$$

3.3. Posterior Distribution

Using the Bayes Theorem, the posterior distribution for $(p, r^{(p)}, \lambda, \beta, v)$ can be expressed by

$$\begin{aligned} &\pi(p, r^{(p)}, \lambda, \beta, v|x) \\ &= \left(\frac{1}{2\beta}\right)^{n-p} exp - \frac{1}{\beta}\sum_{t=p+1}^{n}\left|\sum_{i=1}^{p} F^{-1}(r_i) x_{t-i} + x_t\right| \\ &\quad C_p^{p_{max}} \lambda^p (1-\lambda)^{p_{max}-p} \frac{1}{2^p} \frac{v^u}{\Gamma(u)} \beta^{-(u+1)} exp - \frac{v}{\beta}\frac{1}{v} \\ &= \left(\frac{1}{2}\right)^{n-p} \left(\frac{1}{\beta}\right)^{n-p-1} exp \\ &\qquad - \frac{1}{\beta}\sum_{t=p+1}^{n}\left|\sum_{i=1}^{p} F^{-1}(r_i) x_{t-i} \right. \\ &\qquad \left. + x_t\right| \\ &\quad C_p^{p_{max}} \lambda^p (1-\lambda)^{p_{max}-p} \frac{1}{2^p} \frac{v^{u-1}}{\Gamma(u)} \beta^{-(u+1)} exp - \frac{v}{\beta} \end{aligned} \tag{10}$$

3.4. Reversible Jump MCMC

A simulation for posterior distribution is done in two stages, namely: a conditional distribution simulation for (λ, β, v) if given $(p, r^{(p)})$ and a conditional distribution simulation for $(p, r^{(p)})$ if given (λ, β, v). Because the conditional distribution for (λ, β, v) if given $(p, r^{(p)})$ can be easily recognize the simulation of conditional distribution for (λ, β, v) if given $(p, r^{(p)})$ can be done as follows:

$$\begin{aligned} \beta \sim IG\left(n-p, v + \sum_{t=p+1}^{n}\left|\sum_{i=1}^{p} F^{-1}(r_i) x_{t-i} + x_t\right|\right), \lambda \\ \sim B(p+1, p_{max} - p + 1), v \sim G(u, \frac{1}{\beta}) \end{aligned}$$

However, the conditional distribution for $(p, r^{(p)})$ if given (λ, β, v) has a complex form, then the simulation is the conditional distribution for $(p, r^{(p)})$ if given (λ, β, v) is carried out using a reversible jump MCMC algorithm. This algorithm uses three types of transformations, namely: a change in coefficient, an order birth, and an order death [26].

3.4.1. Change in coefficient

Suppose that $w = (p, r^{(p)})$ is an old Markov chain and $w^* = (p^*, r^{*(p^*)})$ is a new Markov chain. Transformation of coefficient changes does not change the order of the AR model but changes the coefficient of the AR model. So the change from w to w^* is done in two steps. The first step, take $p^* = p$. The second step, select

$i \in \{1, \ldots, p\}$ and define $r_i^* = a$ where $a \sim U(-1,1)$. The ratio between the likelihood function $f(x|w^*)$ and the likelihood function $f(x|w)$ can be expressed as

$$\frac{f(x|w^*)}{f(x|w)} = \frac{exp - \frac{1}{\beta}\sum_{t=p+1}^{n}\left|\sum_{i=1}^{p} F^{-1}(r_i^*)x_{t-i} + x_t\right|}{exp - \frac{1}{\beta}\sum_{t=p+1}^{n}\left|\sum_{i=1}^{p} F^{-1}(r_i)x_{t-i} + x_t\right|} \quad (11)$$

The ratio between the prior distribution for p and the prior distribution for p is $\frac{\pi(p^*)}{\pi(p)} = 1$. The ratio between the conditional prior distribution for r^* if given p^* and the conditional prior distribution for r if given p is $\frac{\pi(r^*|p^*)}{\pi(r|p)} = 1$. The ratio between the distribution of instrument $q(w^*, w)$ and the distribution of instrument $q(w, w^*)$ is

$$\frac{q(w^*, w)}{q(w, w^*)} = \left(\frac{(1+u_i)(1-u_i)}{(1+r_i)(1-r_i)}\right)^{1/2}. \quad (12)$$

So, the probability of acceptance for the change in coefficient $\alpha_c(w, w^*)$ is

$$\alpha_c(w, w^*) = min\left\{1, \frac{f(x|w^*)}{f(x|w)}\frac{q(w^*, w)}{q(w, w^*)}\right\}. \quad (13)$$

3.4.2. Order Birth

Suppose that $w^* = (p^*, r^{*(p^*)})$ is the old Markov chain and $w^* = (p^*, r^{*(p^*)})$ is a new Markov chain. The birth of an order will change both the AR model order and the AR model coefficient. So the change from w to w^* is done in two steps. The first step, take $p^* = p + 1$. The second step, define $w^* = (w, a)$ where $a \sim U(-1,1)$. The ratio between the likelihood function $f(x|w^*)$ and the likelihood function $f(x|w^*)$ can be expressed as

$$\frac{f(x|p^*, q^*, r^*, \rho^*)}{f(x|p, q, r, \rho)} = \frac{exp - \frac{1}{\beta}\sum_{t=p+1}^{n}\left|\sum_{i=1}^{p} F^{-1}(r_i^*)x_{t-i} + x_t\right|}{exp - \frac{1}{\beta}\sum_{t=p+1}^{n}\left|\sum_{i=1}^{p} F^{-1}(r_i)x_{t-i} + x_t\right|}\left(\frac{1}{2\beta}\right) \quad (14)$$

The ratio between the prior distribution for p^* and the prior distribution for p is

$$\frac{\pi(p^*)}{\pi(p)} = \frac{p_{max} - p}{p+1}\frac{\lambda}{1-\lambda} \quad (15)$$

The ratio between the conditional prior distribution for r^* if given p^* and the conditional prior distribution for r if given p is $\frac{\pi(q^*)}{\pi(q)} = 1$. The ratio between the conditional prior distribution for r^* if given p^* and the conditional prior distribution for r if given p is

$$\frac{\pi(r^*|p^*)}{\pi(r|p)} = \frac{1}{2} \quad (16)$$

The ratio between the distribution of instrument $q(w^*, w)$ and the distribution of instrument $q(w, w^*)$ depends on the value of a. If $a < 0$, then the ratio between the distribution of instrument $q(w^*, w)$ and the distribution of instrument $q(w, w^*)$ is

$$\frac{q(w^*, w)}{q(w, w^*)} = \frac{1}{u+1} \quad (17)$$

Whereas if $a > 0$, then the ratio between the distribution of instrument $q(w^*, w)$ and the distribution of instrument $q(w^*, w)$ is

$$\frac{q(w^*, w)}{q(w, w^*)} = \frac{1}{1-u} \quad (18)$$

So, the probability of accepting the birth of the order $\alpha_b(w, w^*)$ is

$$\alpha_b(w, w^*) = min\left\{1, \frac{f(x|p^*, q^*, r^*, \rho^*)}{f(x|p, q, r, \rho)}\frac{\pi(p^*)}{\pi(p)}\frac{\pi(r^*|p^*)}{\pi(r|p)}\frac{q(w^*, w)}{q(w, w^*)}\right\} \quad (19)$$

3.4.3. Order Death

The death of an AR model order is the opposite of the birth of an AR model order. Let $w = (p+1, r^{(p+1)})$ be the old Markov chain and $w^* = (p, r^p)$ the new Markov chain. The death of an order will change both the AR model order and the AR model coefficient. So the change from w to w^* is done in two steps. The first step, take $p^* = p$. The second step, define $w^* = w \setminus \{r_{p+1}\}$. So, the probability of acceptance for the death of order $\alpha_d(w, w^*)$ is

$$\alpha_d(w, w^*) = min\left\{1, \frac{1}{\alpha_b(w, w^*)}\right\}. \quad (20)$$

3.5. Simulations

The performance of the algorithm is tested using a simulation study.

3.5.1. First Simulation

For the first simulation, a total of 250 synthesis time series are made according to equation (1). The AR model parameters are stated in Table 1. Maximum order is $p_{maks} = 10$.

Table 1: Parameters for synthesis time series AR(3)

p	(ϕ_1, ϕ_2, ϕ_3)	β
3	$(0.54, -0.04, -0.75)$	2

The resulting time series is presented in Figure 1.

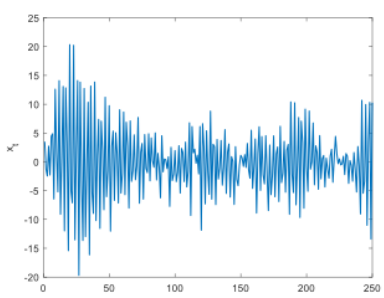

Fig. 1: Synthesis time series AR(3)

Furthermore, the autoregressive model parameters are estimated based on the synthesis time series. The reversible jump MCMC algorithm is used to estimate parameters. The algorithm runs as many as 20000 iterations with 5000 burn-in periods. Histogram for the order is presented in Figure 2.

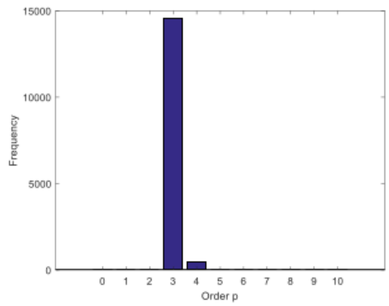

Fig. 2: Histogram for order p

Figure 2 shows that the maximum frequency is reached in order 3. So the estimator for p is p$\hat{p} = 3$. Given $\hat{p} = 3$,, the estimated value of the model coefficient and noise variance are presented in Table 2.

Table 2: Estimated value for an AR(3) parameter

$\hat{p}$	$(\hat{\phi}_1, \hat{\phi}_2, \hat{\phi}_3)$	$\hat{\beta}$
3	$(0.54, -0.035, -0.73)$	2.22

If the estimated value for the AR model parameters in Table 2 is compared with the AR model parameter values in Table 1, then the parameter estimation value approaches the parameter value.

3.5.2. Second Simulation

For the second simulation, a total of 250 synthesis time series are made according to equation (1). The AR model parameters are stated in Table 3. Maximum order is $p_{maks} = 10$.

Table 3: Parameters for synthesis time series AR(5)

p	$(\phi_1, \phi_2, \phi_3, \phi_4, \phi_5)$	β
5	$(0.56, -0.62, 0.03, -0{,}15, 0.44)$	2

The resulting time series is presented in Figure 3.

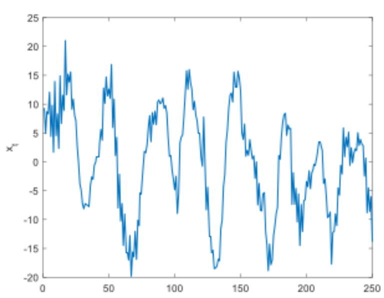

Fig. 3: Synthesis time series AR(5)

Furthermore, the autoregressive model parameters are estimated based on the synthesis time series. The reversible jump MCMC algorithm is used to estimate parameters. The algorithm runs as many as 20000 iterations with 5000 burn-in periods. Histogram for the order is presented in Figure 4.

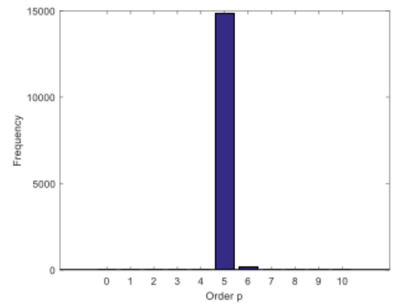

Fig. 4: Histogram for order p

Figure 4 shows that the maximum frequency is reached in order 5. So the estimator for p is $\hat{p} = 5$. Given $\hat{p} = 5$, the estimated value of the model coefficient and noise variance are presented in Table 4.

Table 4: Estimated value for an AR(5) parameter

$\hat{p}$	$(\hat{\phi}_1, \hat{\phi}_2, \hat{\phi}_3, \hat{\phi}_4, \hat{\phi}_5)$	$\hat{\beta}$
5	$(0.55, -0.59, -0.001, -0.1$	2.485

Likewise, if the value of the AR model parameter estimation in Table 4 is compared with the AR model parameter values in Table 3, then the parameter estimation value approaches the parameter value. This simulation study shows that the reversible jump MCMC algorithm can estimate the AR model parameters well.

3.6. Modeling for Heart Rate Data

Furthermore, the AR model is used to model human heart rate data. The number of heart rate per minute is recorded. This data recording is done for 100 minutes. The recording of this data is presented in Figure 5.

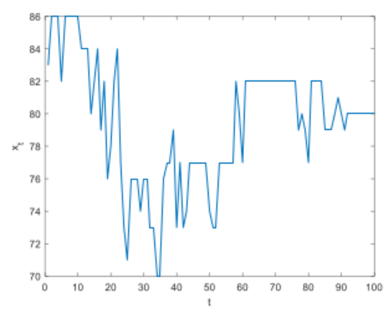

Fig. 5: Heart rate data

This heart rate is modeled with the AR model. The reversible jump MCMC algorithm is implemented to estimate an AR model parameter. The algorithm runs as many as 20000 iterations with a burn-in period of 5000. Figure 6 shows a histogram for the order.

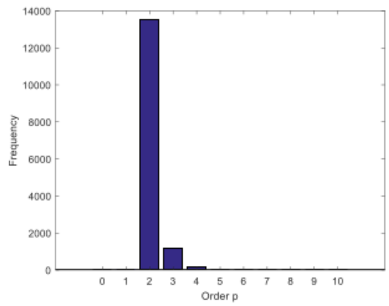

Fig. 6: Histogram for order p

Figure 6 shows that the maximum frequency is reached in order 2. So the estimator for p is $\hat{p} = 2$. Given $\hat{p} = 2$, the estimated value of the model coefficient and noise variance are presented in Table 5.

Table 5: Estimated value for an AR(2) parameter

$\hat{p}$	$(\hat{\phi}_1, \hat{\phi}_2)$	$\hat{\beta}$
3	$(0.2, 0.78)$	4.595

So the mathematical equation for heart rate can be written as

$$x_t = 0.2\, x_{t-1} + 0.78\, x_{t-2} \tag{21}$$

This mathematical equation can be used to predict the heart rate in the 101st minute, namely:

$$\begin{aligned} x_{101} &= 0.2\, x_{100} + 0.78\, x_{99} \\ &= 0.2\,(80) + 0.78\,(80) \\ &= 78.4 \approx 78 \end{aligned} \tag{22}$$

In other words, in the 101st minute, the heart rate will be 78. The equation shows that the heart rate will tend to decrease.

4. Conclusion

This study is the development of an autoregressive model that has Laplace distributed noise. The Bayesian approach is used to estimate an autoregressive model parameter. Because an autoregressive model order is a parameter too, then a Bayes estimator cannot be determined analytically. The Reversible Jump algorithm is used to generate a Markov chain whose distribution approaches a posterior distribution. This Markov chain is used to determine the Bayes estimator. A simulation study shows that the reversible jump MCMC algorithm can estimate model order, autoregressive model coefficients, and noise variance well. The reversible jump MCMC algorithm is implemented to estimate parameters in the heart rate model. This heart rate data can be modeled with the AR model with order 2.

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