Artikel di submit tanggal 6 Agustus 2019 dan mendapatkan 2 Editor yaitu Sutarto Hadi dan Minoru Ohtani

Published by:

Journal on Mathematics Education

(P-ISSN: 2087-8885 | E-ISSN: 2407-0610)

Scopus Indexed

#9257 Summary

SUMMARY REVIEW EDITING

Submission

Authors	Heris Hendriana, Rully Charitas Indra Prahmana, Wahyu Hidayat
Title	THE INNOVATION OF LEARNING TRAJECTORY ON MULTIPLICATION OPERATIONS FOR RURAL AREA STUDENTS IN INDONESIA
Original file	9257-21955-1-SM.DOCX 2019-08-06
Supp. files	None
Submitter	Rully Charitas Indra Prahmana
Date submitted	August 6, 2019 - 04:16 AM
Section	Articles
Editor	Sutarto Hadi Minoru Ohtani
Abstract Views	462

Status

Status	Published Vol 10, No 3 (2019)
Initiated	2019-08-28
Last modified	2020-04-11

Submission Metadata

Authors

Name	Heris Hendriana
Affiliation	IKIP Siliwangi, Cimahi
Country	Indonesia
Bio Statement	—
Name	Rully Charitas Indra Prahmana
URL	https://www.scopus.com/authid/detail.uri?authorId=57192302745
Affiliation	(SCOPUS ID: 57192302745), Universitas Ahmad Dahlan, Yogyakarta
Country	Indonesia
Bio Statement	—

Principal contact for editorial correspondence.

Name	Wahyu Hidayat
Affiliation	IKIP Siliwangi, Cimahi
Country	Indonesia
Bio Statement	—

Title and Abstract

Title	THE INNOVATION OF LEARNING TRAJECTORY ON MULTIPLICATION OPERATIONS FOR RURAL AREA STUDENTS IN INDONESIA
Abstract	The rural area's student difficulties in learning the concept of number operation had been documented by several studies, especially for the case of multiplication. The teacher typically introduces the multiplication concepts using the formula without involving the concept itself. Furthermore, this study aims to design learning trajectory on multiplication operations in the Mathematics of GASING (Math GASING) by focusing more on the concept itself than the formula and by starting from the informal to a formal level of teaching. Design research used as the research method to solve this problem consisting of three phases, namely preliminary design, teaching experiment, and retrospective analysis. The research results show that the Math GASING has a real contribution for students to understanding and mastering in the concept of the multiplication operations. This research also explains the strategy and the model discovered by students in learning multiplication that the students used as a basic concept of multiplication. Finally, the students were able to understand the concept of multiplication more easily, and they showed interest in using this learning trajectory.

Indexing

Keywords	multiplication; learning trajectory; design research; rural area
Language	en

Supporting Agencies

Agencies —

References

References

Ahmad, N.S.B. (2010). Multiplication and the reference sum method. Procedia-Social and Behavioral Sciences, 8, 72-78. https://doi.org/10.1016/j.sbspro.2010.12.010.

Akker, J.V.D., Gravemeijer, K., McKenney, S., & Nieveen, N. (2006). Education Design Research. London: Routledge Taylor and Francis Group.

bin Syed Ismail, S.A. (2010). Multiplication with the Vedic method. Procedia-Social and Behavioral Sciences, 8, 129-133. https://doi.org/10.1016/j.sbspro.2010.12.018.

Caron, T.A. (2007). Learning multiplication: The easy way. The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 80(6), 278-282. https://doi.org/10.3200/TCHS.80.6.278-282.

Chang, K. E., Sung, Y.T., Chen, Y.L., & Huang, L.H. (2008). Learning multiplication through computer-assisted learning activities. Computers in Hhuman Bbehavior, 24(6), 2904-2916. https://doi.org/10.1016/j.chb.2008.04.015.
Chung, I. (2004). A comparative assessment of constructivist and traditionalist approaches to establishing mathematical connections in learning multiplication. Education, 125(2), 271-279.
Drews, D., Dudgeon, J., Hansen, A., Lawton, F., & Surtees, L. (Eds.). (2005). Children's Errors in Mathematics: Understanding Common Misconceptions in Primary Schools. SAGE.
Freudenthal, H. (1973). Mathematics as an Educational Task. Dordrecht: Kluwer Academic Publishers.
Gravemeijer, K. (2004). Local instructional theories as means of support for teacher in reform mathematics education. Mathematical Thinking and Learning, 6(2), 105-128. https://doi.org/10.1207/s15327833mtl0602_3.
Ischebeck, A., Zamarian, L., Siedentopf, C., Koppelstätter, F., Benke, T., Felber, S., & Delazer, M. (2006). How specifically do we learn? Imaging the learning of multiplication and subtraction. Neuroimage, 30(4), 1365-1375. https://doi.org/10.1016/j.neuroimage.2005.11.016.
Kilian, L., Cahill, E., Ryan, C., Sutherland, D., & Taccetta, D. (1980). Errors that are common in multiplication. The Arithmetic Teacher, 22-25.Kilian, L., Cahill, E., Ryan, C., Sutherland, D., & Taccetta, D. (1980). Errors that are common in multiplication. The Arithmetic Teacher, 27(5), 22-25.
National Council of Teachers of Mathematics (NCTM). (2000). Principles and Standards for School Mathematics. Reston, VA: National Council of Teachers of Mathematics.
Nuari, L.F., Prahmana, R.C.I., & Fatmawati, I. (2019). Learning of division operation for mental retardations' student through Math GASING. Journal on Mathematics Education, 10(1), 127-142. https://doi.org/10.22342/jme.10.1.6913.127-142.
Prahmana, R.C.I. (2013). Designing division operation learning in the mathematics of GASING. Proceeding in The First South East Asia Design/Development Research (SEA-DR) Conference 2013, 391-398.
Prahmana, R.C.I. (2015). The hypothetical learning trajectory on addition in mathematics GASING. Southeast Asian Mathematics Education Journal, 5(1), 49-61.
Prahmana, R.C.I. (2017). Design Research (Teori dan Implementasinya: Suatu Pengantar) [Design Research (Theory and Its Implementation: An Introduction)]. Depok: Rajawali Pers.
Prahmana, R.C.I., & Suwasti, P. (2014). Local instruction theory on division in mathematics GASING. Journal on Mathematics Education, 5(1), 17-26. https://doi.org/10.22342/jme.5.1.1445.17-26.
Prahmana, R. C. I., Zulkardi, & Hartono, Y. (2012). Learning multiplication using Indonesian traditional game in third grade. Journal on Mathematics Education, 3(2), 115-132. https://doi.org/10.22342/jme.3.2.1931.115-132.
Putri, R.I.I., Dolk, M., & Zulkardi. (2015). Professional development of PMRI teachers for introducing social norms. Journal on Mathematics Education, 6(1), 11-19. https://doi.org/10.22342/jme.6.1.1900.11-19.
Revina, S., Zulkardi, Darmawijoyo, & Galen, F. (2011). Spatial visualization tasks to support students' spatial structuring in learning volume measurement. Journal on Mathematics Education, 2(2), 127-146. http://dx.doi.org/10.22342/jme.2.2.745.127-146.
Reys, R.E., Suydam, M.N., Lindquist, M.M., & Smith, N.L. (1998). Helping Children Learn Mathematics. Boston: Allyn and Bacon.
Shanty, N.O., & Wijaya, S. (2012). Rectangular array model supporting students' spatial structuring in learning multiplication. Journal on Mathematics Education, 3(2), 175-186. http://dx.doi.org/10.22342/jme.3.2.603.175-186.
Surya, Y. (2011). Petunjuk Guru: Dasar-Dasar Pintar Berhitung GASING [Teachers' Guide: Smart Basics of Counting's GASING]. Tangerang: PT. Kandel.
Surya, Y., & Moss, M. (2012). Mathematics education in rural Indonesia. Proceeding in the 12th International Congress on Mathematics Education: Topic Study Group 30. Korea National University of Education, Seoul, South Korea Selatan, 6223-6229.
Tanujaya, B., Prahmana, R. C. I., & Mumu, J. (2017). Mathematics instruction, problems, challenges and opportunities: A case study in Manokwari Regency, Indonesia. World Transactions on Engineering and Technology Education, 15(3), 287-291.
Unlu, M., & Ertekin, E. (2012). Why do pre-service teachers pose multiplication problems instead of division problems in fractions?. Procedia-Social and Behavioral Sciences, 46, 490-494. https://doi.org/10.1016/j.sbspro.2012.05.148.

Journal on Mathematics Education
Doctoral Program on Mathematics Education
Faculty of Teacher Training and Education, Universitas Sriwijaya
Kampus FKIP Bukit Besar
Jl. Srijaya Negara, Bukit Besar
Palembang - 30139
email: jme@unsri.ac.id

p-ISSN: 2087-8885 | e-ISSN: 2407-0610

KEYWORDS

Asian Games Assessment Curriculum Design Research Design research Ethnomathematics Geometry Integers Malaysia Mental models Metacognition Microworlds Modelling PISA PMRI Realistic Mathematics Education Subtraction Yogyakarta culture design research geometry mathematics education

Blind Review Artikel yang di submit pada tanggal 6 Agustus 2019 dengan judul

“The Innovation of Learning Trajectory on Multiplication Operation for Rural Area Student in Indonesia”

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume xx, No. x, January xxxx, pp. x-xx

THE INNOVATION OF LEARNING TRAJECTORY ON MULTIPLICATION OPERATION FOR RURAL AREA STUDENT IN INDONESIA

Abstract

The rural area's student difficulties in learning the concept of number operation had been documented by several studies, especially for the case of multiplication. The teacher always introduces the multiplication concepts using the formula without involving the concept itself. Furthermore, this study aims to design learning trajectory on multiplication operation in the Mathematics of GASING (Math GASING) focused more on the concept itself than the formula and started from the informal to a formal level. Design research used as the research method to solve this problem consisting of three phases' namely preliminary design, teaching experiment, and retrospective analysis. The research results show that the Math GASING has a real contribution for students to understanding and mastering in the concept of the multiplication operation. This research also explains the strategy and model discovered by students in learning multiplication that students use to help their initial understanding of the multiplication concept. Finally, students can understand the concept of multiplication more easily and joyful by using this learning trajectory.

Keywords: multiplication, learning trajectory, design research

Abstrak

Kesulitan siswa di daerah pedesaan dalam mempelajari konsep operasi angka telah didokumentasikan oleh beberapa studi, terutama untuk kasus perkalian. Guru selalu memperkenalkan konsep perkalian menggunakan rumus tanpa melibatkan konsep itu sendiri. Selanjutnya, penelitian ini bertujuan untuk merancang lintasan pembelajaran pada operasi multiplikasi dalam Matematika GASING (Matematika GASING) lebih fokus pada konsep itu sendiri daripada rumus dan mulai dari tingkat informal ke formal. Desain penelitian digunakan sebagai metode penelitian untuk menyelesaikan masalah ini yang terdiri dari tiga fase yaitu desain pendahuluan, eksperimen mengajar, dan analisis retrospektif. Hasil penelitian menunjukkan bahwa GASING Matematika memiliki kontribusi nyata bagi siswa untuk memahami dan menguasai dalam konsep operasi multiplikasi. Penelitian ini juga menjelaskan strategi dan model yang ditemukan oleh siswa dalam mempelajari perkalian yang digunakan siswa untuk membantu pemahaman awal mereka tentang konsep perkalian. Akhirnya, siswa dapat memahami konsep perkalian dengan lebih mudah dan menyenangkan dengan menggunakan lintasan pembelajaran ini.

Kata kunci: perkalian, lintasan belajar, design research

How to Cite: Prahmana, R.C.I. & Zulkardi. (2016). Instructions/Template for Preparing Manuscript for Journal on Mathematics Education. *Journal on Mathematics Education, x* (x), xx-xx.

Learning number operations is important for almost all topics in Mathematics involving numbers (Ahmad, 2010; Freudenthal, 1973; NCTM, 2000; Prahmana, et al. 2012). It's because learning number operations tends to an understanding of symbols, notation, and reference number (other forms to represent) (NCTM, 2000) and plays an important role in determining students' performance in other related Mathematics topics (Ahmad, 2010). Therefore, learning number operation would be one of the prior knowledge that students must have for learn another topics in mathematics.

The concept of number operation, especially in multiplication, is one of students' difficulty to understanding mathematics concept (Ahmad, 2010; bin Syed Ismail, 2010; Drews, et al. 2005; Kilian, et al. 1980; Tanujaya, et al. 2017; Unlu and Ertekin, 2012). Teachers usually teach number operations using symbolic form or something abstract (Unlu and Ertekin, 2012). As the result, students learn number operations more on the process of memorizing than understanding it (bin Syed Ismail, 2010), have several errors which reflected their lack of understanding of various mathematical concepts and also the long multiplication algorithm (Ahmad, 2010), and have the poor understanding of the place value (tens and ones) concept in relation to multiplication (Drews, et al. 2005; Kilian, et al. 1980). The result of the previous research is in line with the preliminary classroom observation results of rural area's student, namely Serui, Ambon, and Sorong Selatan, regarding to learning number operations conducted by researchers in pre-test. Teachers introduced the concept of multiplication using the formula without involving the concept itself (Prahmana and Suwasti, 2014).

Several studies indicated that constructivism approach can improve students' understanding in learn multiplication (Ahmad, 2010; Prahmana, et al. 2012; Chang, et al. 2008; Chung, 2004). The mathematics of GASING (Math GASING) method is one of learning method using constructivism approach (Prahmana and Suwasti, 2014; Prahmana, 2015; Surya, 2011; Surya and Moss, 2012; Shanty and Wijaya, 2012; Prahmana, 2013). This method has been applied to student from rural area in Indonesia which began with the introduction of number and number operations (Prahmana and Suwasti, 2014; Surya and Moss, 2012; Shanty and Wijaya, 2012; Prahmana, 2013). This situation underlies the researcher to try designing learning trajectory on number operation in Math GASING for rural area students derived from Serui, Sorong Selatan, and Ambon, Papua. Hence, the focus of this study was to describe the learning activities on students' performance to do multiplication in Math GASING.

Based on a few things mentioned above, the research question of this study is how the learning trajectory of multiplication in Math GASING is evolved the rural area's students' understanding in multiplication from informal to formal level. Hopefully, the learning trajectory has a role in learning multiplication that makes the learning more easy, joyful, and meaningful for students.

In this study, the literature on Math GASING and number operations were learn to see the typical learning processes used by real situations (concrete) to abstract with the steps that has been in the design.

Number Operation

Integer operations that we know are addition, subtraction, multiplication, and division, where the four operations have any connection with each other (Reys, et al. 1984). The following four relations operation that has a relationship with each other, and students must understand the relationships (Reys, et al. 1984).

Addition and subtraction are inverse operations. There are several ways to teach the concept of integer operations in the learning of mathematics. One of the ways to teach them is Math GASING.

$5 + 8 = 13$ ---------- $13 – 5 = 8$

Multiplication and division are inverse operations

$4 \times 6 = 24$ ---------- $24 : 4 = 6$

Multiplication can be seen as a repeated addition

4×6 ---------- $6 + 6 + 6 + 6$

Division can be seen as a repeated subtraction

$24 : 6$ ---------- $24 - 6 - 6 - 6 - 6$

Mathematics GASING

Surya and Moss (2012) stated that GASING has several basic premises. Firstly, there is no such thing as a child that cannot learn mathematics, only children that have not had the opportunity to learn mathematics in a fun and meaningful way. Secondly, mathematics is based on patterns and these patterns make math understandable. Thirdly, a visual context to mathematical concepts should come before the symbolic notation. Lastly, mathematics is not memorization, but knowing basic facts comes easily with a conceptual and visual understanding. Memorization of basic mathematics facts is easy if it is based on conceptual learning and visual representations.

The learning process make students learning easy, fun, and enjoyable in Math GASING (Shanty and Wijaya, 2012). Easy means the students are introduced to mathematical logic that is easy to learn and to remember. Exciting means the students have motivation which comes from by them to learn mathematics (intrinsic factor). Fun is more in the direction of outside influences such as visual aids and games (extrinsic factor). In the other hand, Prahmana (2013) had been conducted research for division topic in Math GASING, where the learning process begins with the activities share sweets fairly, then move into the process of how each student gets distributed sweets after a fair amount of candy (concrete), ranging from division without remainder to division with remainder, and ends with the completion of division operation in Math GASING (abstract). Math GASING shows how to change a concrete sample into an

abstract symbol so the students will be able to read a mathematical pattern, thus gain the conclusion by themselves.

Math GASING as one of innovations in learning mathematics offers critical point in its learning process. There is a critical point that we must pass that is called GASING's critical point when studying a topic in Math GASING. After reaching this critical point, students will not be difficult anymore to work on the problems in that topic (Surya, 2011). The critical point in learning multiplication is that students must master the multiplication concept of 1×1 to 10×10. The student can learn a variety of multiplication operation problems more easy after pass the critical point.

Hypothetical Learning Trajectory

Hypothetical learning trajectory (HLT) is proposed as a term to identify and describe relevant aspects associated with a mathematics lesson plan, including: A description of the students' mathematical goals, the mathematical activities (including the tasks or problems, that students will work on to achieve the goals), and a hypothetical path that describes the students learning process (Revina, et al. 2011). The HLT in this study had several learning goals expected to be reached by the students during one phase

METHOD

Design research used as the research method of this study. This method is an appropriate way to answer the research questions and achieve the research objectives (Prahmana, et al. 2012; Akker, et al. 2006; Gravemeijer, 2004). Design research has five characteristic. There are interventionist nature, process oriented, reflective component, cyclic character, and theory oriented (Akker, et al. 2006). There are two important aspect related to design research namely Hypothetical Learning Trajectory (HLT) and Local Instruction Theory (LIT). The learning activities as learning paths taken by students in their learning activities must have HLT and LIT.

The HLT consists of three components (Gravemeijer, 2004). First is the purpose of mathematics teaching for students. Second is learning activity and devices or media used in the learning process. Lastly is a conjecture of understanding the process of learning how to learn and strategies students that arise and thrive when learning activities done in class. There are three phases of design research namely preliminary design, teaching experiment, and retrospective analysis seen in Figure 1.

The research data is regarding from multiple sources of data to get a visualization of the

students' mastery of basic concepts of multiplication operations, such as documentation (photo), video recording, and the students' worksheet and observation sheet. Next, the data analysed retrospectively with HLT as a guide. These studies have been completed in 2 days with the subjects are 11 matriculation prospective teachers students at one of institute in Tangerang regarding from Ambon, Serui, Yapen, and South Sorong, Papua, and also a teaching assistant who acted as a model teacher.

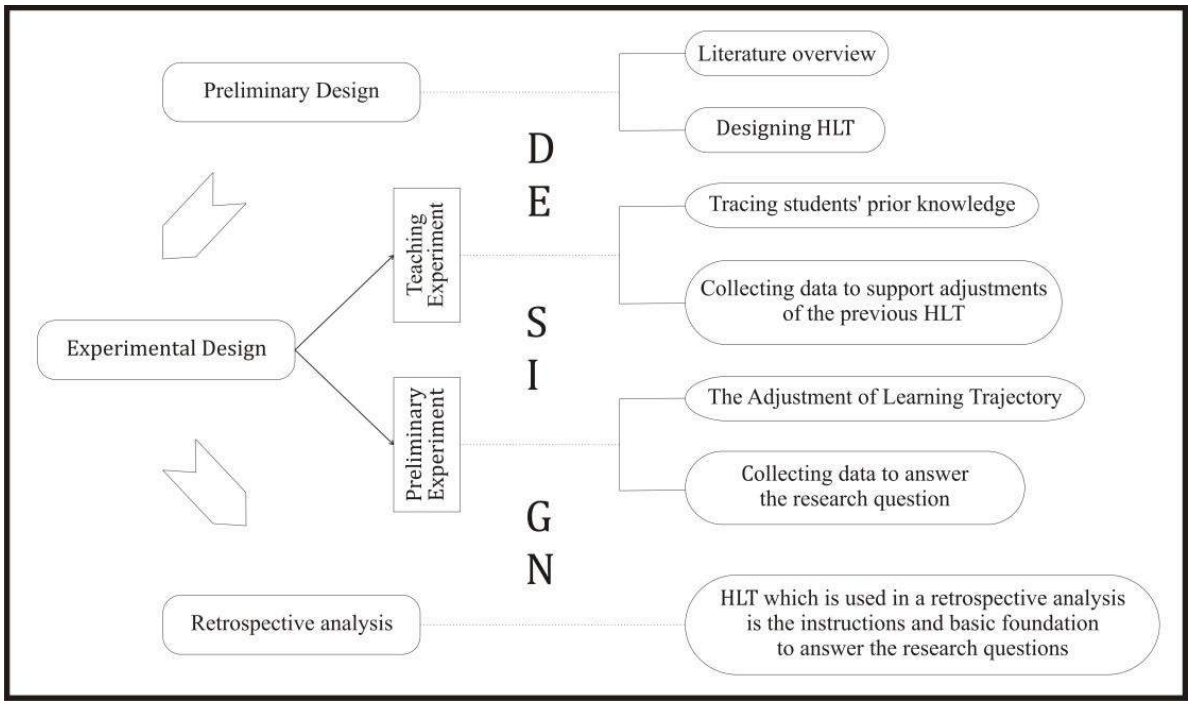


Figure 1. Phase of the design research (Prahmana, et al. 2012).

RESULT AND DISCUSSION

The learning activities start from making same perception of the meaning of boxes containing something in that boxes to introduce the concept of multiplication. Furthermore, students train to memorize the multiplication for 1 to 10 using several methods. Lastly, teacher give evaluation to know the student understanding of multiplication using mental arithmetic activity as one of assessment process in this learning activities and exercise by using student evaluation sheet. As a result, students was able to master the multiplication operation in Math GASING seen from the results of the final evaluation and was pleased to learn Math GASING can be seen from the comments of students who wish to abandon the old way of learning mathematics. The results of this study indicate that learning design of multiplication operation in Math GASING have a very important role as the starting point and improve students' motivation in learning. For more details, researchers will discuss the results of this study, which is divided into three stages that are called preliminary design, teaching experiments, and retrospective analysis.

Preliminary Design

At this stage, researcher is beginning to implement the idea of multiplication operation in Math GASING by reviewing the literature, conducting observations in matriculation class, and designing a sequence of instructional learning for learns multiplication to reach the goals formulated in Table 1 (adapted from Surya (2011)). A set of activities for learning multiplication in Math GASING has been designed based learning trajectory and thinking process of students who hypothesized. The instruction set of activities has been divided into six activities that have been completed in 2 meetings with a variety of fun activities that make students happy in the learning process, and end with the evaluation process.

Table 1. Overview of the learning trajectory of multiplication.

Sequence of activities	Goals	Descriptions
Playing some games using Math GASING learning aids	Understanding the multiplication concept	Students learn multiplication starting from understanding the basic concept of addition using the term of "box", for example 2 × 3 means there are 2 boxes containing 3 things in that box, and so on.
Using some method to memorize this part more easy	Memorizing the multiplication of numbers 1, 10, 9, 2 and 5	Students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students are able to master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on.
Using the patterns of two same numbers multiplication	Memorizing the multiplication of two same numbers, such as $1 \times 1, 2 \times 2, \ldots, 10 \times 10$	Students learn about the same numbers of multiplication, such as 1 × 1, 2 × 2, 3 × 3, ... , 10x10.
Using multiplication characteristics as a commutative operation	Memorizing the multiplication of numbers 3 and 4	Students learn multiplication for 3 and 4 using a commutative operation.
Reducing some part in multiplication that already mastered	Memorizing the multiplication of numbers 8, 7 and 6	Students learn multiplication for 8, 7, and 6. Teacher teaches student by using reduce some part in multiplication that already mastered before.
Evaluation	Determining the student ability in	Teacher evaluates the student about multiplication problem in the formal and informal

learning multiplication form.

Teaching Experiment

In teaching experiment, researcher tests the learning activities have been designed in the preliminary design stage. When the teacher models have started to see students do not get excited, then the teacher models provide educational games that make fun learning activities, because it is becoming one of characteristics in Math GASING learning process. There are five activities in this stage using whiteboard and presentation. First, teacher introduced the concept of multiplication by playing some games using Math GASING learning aids. In that games, students learn the concept of multiplication starting from understanding the basic concept of addition using the term of "box", for example 2 × 3 means there are 2 boxes containing 3 things in that box, and so on. Secondly, students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students are able to master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on. Different with the memorizing process of multiplication order in mathematics in general, students memorize the multiplication start from 1, 10, 9, 2 and 5. Furthermore, students learn about the same numbers of multiplication, such as 1 × 1, 2 × 2, 3 × 3, ... , 10x10. Fourthly, students learn multiplication for 3 and 4 using a commutative operation. Lastly, students learn multiplication for 8, 7, and 6. For this step, teacher teaches student by using reduce some part in multiplication that already mastered before. So, the students can memorize all multiplication concept from one to ten more easily. In the second meeting, teacher evaluates the student about multiplication problem in the formal and informal form. All activities can be shown in Figure 2.

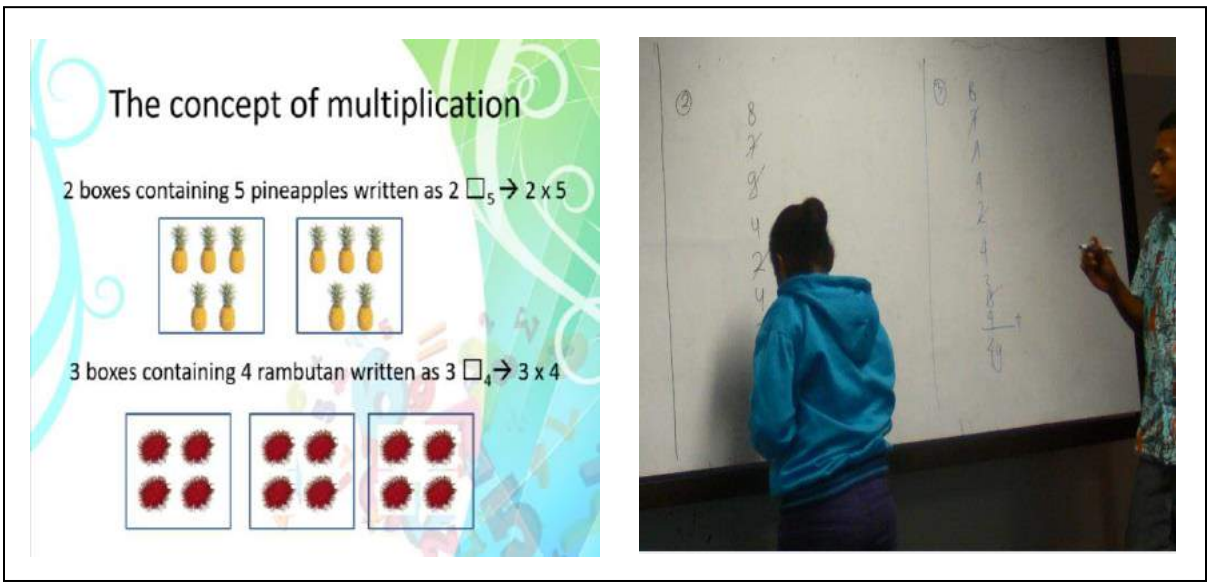


Figure 2. Several activities in teaching experiment phase.

Retrospective Analysis

Multiplication process in Math GASING is different with multiplication process in mathematics in general. As the result, all activities which have been designed can be used to answer the research question above. The activities are as follows:

Learning trajectory which has been modeled in Table 1 are the activities undertaken in this study to guide students mastered multiplication operation. So that, researcher designed an activity using Math GASING aids. The goal is that students are able to understand the concrete form of multiplication using the understanding of boxes and something in there. Student must understand that multiplication in the form of repeated addition. Teacher uses combination learning tools such as presentation and whiteboard to make learning process effective and efficiency (seen in Figure 2).

Furthermore, from these activities, teachers guide students toward the concept of multiplication as the form of repeated addition. Teacher uses several methods to memorize multiplication for one to ten more easily and meaningful. On the other hands, teacher make the order of memorize the multiplication with different order in mathematical in general. First, students memorize the multiplication for 1, 10, 9, 2 and 5. Next, students memorize the multiplication for the same numbers of multiplication, such as 1×1, 2×2, 3×3, ... , 10×10. After that, students memorize the multiplication for 3, 4, 8, 7, and 6. Finally, all students can memorize the multiplication form from one to ten and answer the teacher exercise directly using their mental arithmetic.

Based on all the activities above, it can be seen that the students have gone through the process of activity based on experience using their ability and math GASING learning aids, moving toward a more formal, the understanding of formal level from the critical point, and then reached into the formal level desired as the ultimate goal of this learning activities.

In the design of this study, researcher used the learning steps of multiplication in Math GASING as shown in Table 1. When the activity takes place, the dialogue is very good in the process of introducing the basic concepts of multiplication operations. In the dialogue, it seems that students feel learning multiplication in Math GASING looks so easy and so much fun. As a result, the learning process can guide students in understanding multiplication. It can also be seen from the student evaluation of learning multiplication process given by the teacher to evaluate student understanding (Figure 3). As a result, students seemed to be able to apply multiplication operation process in solving each problem is given in terms of evaluation. Therefore, it can be seen that learning multiplication operation in Math GASING can use to raise students' understanding in integer multiplication operations or in other words, the design of this study can be used as the starting point of learning multiplication.

Figure 3. Student evaluation process using student worksheet.

The retrospective analysis shown that one of the ways making student understanding in learning multiplication is make the learning process can be imaging for students. This results is in line with previous research stated in learning multiplication have several ways to master it (Caron, 2007; Ischebeck, et al. 2006). On the other hands, learning environment also can support the result of the learning process. Finally, all students can solve several problems and exercises regarding in multiplication operation.

CONCLUSION

Researcher can conclude that the learning of multiplication operation in Math GASING have a very important role as the starting point and improve students' motivation in learning multiplication. In addition, the activities that have been designed in such way those students find the concept of multiplication starting from understanding the concept of multiplication to mastering the multiplication concept of 1×1 to 10×10 which is the critical point in learning multiplication in Math GASING. The student can learn a variety of multiplication operation problems more easy after pass the critical point. Lastly, each student can do mental arithmetic for any given multiplication problem and resolve many multiplication questions very quickly and precisely where is both of this are one of assessment forms in Math GASING.

REFERENCES

Ahmad, N. S. B. (2010). Multiplication and the reference sum method. *Procedia-Social and Behavioral Sciences*, *8*, 72-78.

Akker, J.V.D., Gravemeijer, K., McKenney, S. and Nieveen, N. (2006). *Education Design Research*. London: Routledge Taylor and Francis Group.

bin Syed Ismail, S. A. (2010). Multiplication with the Vedic method. *Procedia-Social and Behavioral Sciences*, *8*, 129-133.

Caron, T. A. (2007). Learning multiplication: The easy way. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *80*(6), 278-282.

Chang, K. E., Sung, Y. T., Chen, Y. L. and Huang, L. H. (2008). Learning multiplication through computer-assisted learning activities. *Computers in human behavior*, *24*(6), 2904-2916.

Chung, I. (2004). A comparative assessment of constructivist and traditionalist approaches to establishing mathematical connections in learning multiplication. *Education*, *125*(2), 271-279.

Drews, D., Dudgeon, J., Hansen, A., Lawton, F. and Surtees, L. (Eds.). (2005). *Children's Errors in Mathematics: Understanding Common Misconceptions in Primary Schools*. SAGE.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K. (2004). Local instructional theories as means of support for teacher in reform mathematics education. *Mathematical Thinking and Learning*, *6*(2), 105-128.

Ischebeck, A., Zamarian, L., Siedentopf, C., Koppelstätter, F., Benke, T., Felber, S. and Delazer, M. (2006). How specifically do we learn? Imaging the learning of multiplication and subtraction. *Neuroimage*, *30*(4), 1365-1375.

Kilian, L., Cahill, E., Ryan, C., Sutherland, D., and Taccetta, D. (1980). Errors that are common in multiplication. *The Arithmetic Teacher*, 22-25.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Prahmana, R. C. I. (2013). Designing division operation learning in the mathematics of GASING. *Proceeding in The First South East Asia Design/Development Research (SEA-DR) Conference 2013*, 391-398.

Prahmana, R. C. I. (2015). The hypothetical learning trajectory on addition in mathematics GASING. *Southeast Asian Mathematics Education Journal*, *5*(1), 49-61.

Prahmana, R. C. I. and Suwasti, P. (2014). Local instruction theory on division in mathematics GASING. *Journal on Mathematics Education*, *5*(1), 17-26.

Prahmana, R. C. I., Zulkardi, and Hartono, Y. (2012). Learning multiplication using Indonesian traditional game in third grade. *Journal on Mathematics Education*, *3*(2), 115-132.

Revina, S., Zulkardi, Darmawijoyo and Galen, F. (2011). Spatial visualization tasks to support students' spatial structuring in learning volume measurement. *Journal on Mathematics Education*, *2*(2), 127-146.

Reys, R. E., Suydam, M. N., Lindquist, M. M., and Smith, N. L. (1984). *Helping Children Learn Mathematics*. Boston: Allyn and Bacon.

Shanty, N.O. and Wijaya, S. (2012). Rectangular array model supporting students' spatial structuring in learning multiplication. *Journal on Mathematics Education*, *3*(2), 175-186.

Surya, Y. (2011). *Petunjuk Guru: Dasar-Dasar Pintar Berhitung GASING*. Tangerang: PT. Kandel.

Surya, Y. and Moss, M. (2012). Mathematics education in rural Indonesia. *Proceeding in the 12th International Congress on Mathematics Education: Topic Study Group 30*. Korea National

University of Education, Seoul, Korea Selatan, 6223-6229.

Tanujaya, B., Prahmana, R.C.I. and Mumu, J. (2017). Mathematics instruction, problems, challenges and opportunities: A case study in Manokwari Regency, Indonesia. *World Transactions on Engineering and Technology Education*, *15*(3**)**, 287-291.

Unlu, M. and Ertekin, E. (2012). Why do pre-service teachers pose multiplication problems instead of division problems in fractions?. *Procedia-Social and Behavioral Sciences*, *46*, 490-494.

Keputusan diterima dengan revisi pada pada tanggal 17 Agustus 2019, setelah mendapatkan hasil review dari 3 orang reviewer. Notifikasi ini ditujukan kepada penulis pertama melalui akun corresponding author, dikarenakan terkait informasi untuk merevisi artikel dan biaya publication fee.

Editor
2019-08-17 07:18 AM

Subject: [JME] Editor Decision DELETE

Dear Dr. Heris Hendriana,

Your manuscript entitled "The Innovation of Learning Trajectory on Multiplication Operation for Rural Area Student in Indonesia", which you submitted to Journal on Mathematics Education, has been reviewed. The reviewer comments are included at your account and the bottom of this letter, along with those of the editor who coordinated the review of your paper.

The reviewer(s) would like to see some revisions made to your manuscript before publication. Therefore, I invite you to respond to the reviewer(s)' comments and revise your manuscript.

Furthermore, based on guidelines for Author in Journal on Mathematics Education (JME), https://ejournal.unsri.ac.id/index.php/jme/about/submissions#authorFees. We would like to inform you that the authors or the author's institution are requested to pay a publication fee for each article accepted to continue in the editing process. The publication fee is to support the cost of wide-open access dissemination of research results, to manage the various costs associated with handling and editing of the submitted manuscripts, similarity check using iThenticate, and the Journal management and publication in general.

The publication fee, $200 USD, covers the standard twelve (12) pages manuscript and your paper has 12 pages. Therefore, your publication fee is $200 or Rp. 2.850.000,-. The Publication fee could be paid by bank transfer to:

Bank Account name (please be exact): Dr. Zulkardi, M.Ikom., M.Sc
Address: BNI, Palembang, Indonesia
Bank Account No. : 0216647385
Bank Name: BANK NEGARA INDONESIA - PT (PERSERO)
City: Palembang
Country: Indonesia
Bank SWIFT Code: BNINIDJAPBG

You must send your payment report via email, and we will publish your manuscript on our website. Should you have any questions, please do not hesitate to contact us. For your information, The JME also has been ACCEPTED for inclusion in SCOPUS database since May 30, 2019.
Thank you very much for your cooperation and have a great day.

Sincerely,

Prof. Dr. Minoru Ohtani

Journal on Mathematics Educatioh (p:2087-8885 e:2407-0610)
http://ejournal.unsri.ac.id/index.php/jme

Close

Hasil review dari 3 orang reviewer terlampir pada akun corresponding author.

Hasil review dari 3 reviewer, yang semuanya memberikan catatan pada artikel nya secara langsung, yaitu Reviewer A, B, dan C.

[Paper ID: 9257]

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume xx, No. x, January xxxx, pp. x-xx

THE INNOVATION OF LEARNING TRAJECTORY ON MULTIPLICATION OPERATION FOR RURAL AREA STUDENTS IN INDONESIA

Abstract

The rural area's student difficulties in learning the concept of number operation had been documented by several studies, especially for the case of multiplication. The teacher typically introduces the multiplication concepts using the formula without involving the concept itself. Furthermore, this study aims to design learning trajectory on multiplication operations in the Mathematics of GASING (Math GASING) by focusing more on the concept itself than the formula and by starting from the informal to a formal level of teaching. Design research used as the research method to solve this problem consisting of three phases, namely preliminary design, teaching experiment, and retrospective analysis. The research results show that the Math GASING has a real contribution for students to understanding and mastering in the concept of the multiplication operations. This research also explains the strategy and the model discovered by students in learning multiplication that the students used to help their initial understanding of the multiplication concept. Finally, the students were able to understand the concept of multiplication more easily and they showed interest in using this learning trajectory.

Keywords: multiplication, learning trajectory, design research

Abstrak

Kesulitan siswa di daerah pedesaan dalam mempelajari konsep operasi angka telah didokumentasikan oleh beberapa studi, terutama untuk kasus perkalian. Guru selalu memperkenalkan konsep perkalian menggunakan rumus tanpa melibatkan konsep itu sendiri. Selanjutnya, penelitian ini bertujuan untuk merancang lintasan pembelajaran pada operasi multiplikasi dalam Matematika GASING (Matematika GASING) lebih fokus pada konsep itu sendiri daripada rumus dan mulai dari tingkat informal ke formal. Desain penelitian digunakan sebagai metode penelitian untuk menyelesaikan masalah ini yang terdiri dari tiga fase yaitu desain pendahuluan, eksperimen mengajar, dan analisis retrospektif. Hasil penelitian menunjukkan bahwa GASING Matematika memiliki kontribusi nyata bagi siswa untuk memahami dan menguasai dalam konsep operasi multiplikasi. Penelitian ini juga menjelaskan strategi dan model yang ditemukan oleh siswa dalam mempelajari perkalian yang digunakan siswa untuk membantu pemahaman awal mereka tentang konsep perkalian. Akhirnya, siswa dapat memahami konsep perkalian dengan lebih mudah dan menyenangkan dengan menggunakan lintasan pembelajaran ini.

Kata kunci: perkalian, lintasan belajar, design research

How to Cite: Prahmana, R.C.I. & Zulkardi. (2016). Instructions/Template for Preparing Manuscript for Journal on Mathematics Education. *Journal on Mathematics Education, x* (x), xx-xx.

Learning number operations is important for almost all topics in Mathematics involving numbers (Ahmad, 2010; Freudenthal, 1973; NCTM, 2000; Prahman, et al., 2012). It is because learning number operations involves an understanding of symbols, notation, and reference number (or other forms to represent) (NCTM, 2000), and it also plays an important role in determining students' performance in other related Mathematics topics (Ahmad, 2010). Therefore, learning number operations would be one of the prior knowledge that students must have in order to learn other topics in mathematics.

Commented [SC1]: Should be plural as in Operations

Deleted: always

Deleted: ed

Deleted: ed

Deleted: ’

Deleted: can

Deleted: joyful by

Commented [SC2]: With the changes in the English version of the abstract, please make the necessary edits in the Bahasa part of the abstract.

Deleted: a

Deleted: ’s

Deleted: tends to

Deleted: for

Deleted: an

The concept of number operations, especially in multiplication, is one of the students' difficulties in understanding mathematics concepts (Ahmad, 2010; bin Syed Ismail, 2010; Drews et al., 2005; Kilian et al., 1980; Tanujaya et al., 2017; Unlu & Ertekin, 2012). Teachers usually teach number operations using symbolic form or something abstract (Unlu & Ertekin, 2012). As the result, students learn number operations more on the process of memorizing than understanding it (bin Syed Ismail, 2010), have several errors which reflected their lack of understanding of various mathematical concepts and also the long multiplication algorithm (Ahmad, 2010), and they also have poor understanding of the place value (tens and ones) concept in relation to multiplication (Drews et al., 2005; Kilian et al., 1980). The result of the previous research is in line with the preliminary classroom observation results of the rural area's students, namely Serui, Ambon, and Sorong Selatan, regarding to learning number operations conducted by researchers in pre-test. Teachers introduced the concept of multiplication using the formula without involving the concept itself (Prahmana & Suwasti, 2014).

Several studies indicated that constructivism approach can improve students' understanding in learn multiplication (Ahmad, 2010; Prahmana et al., 2012; Chang et al., 2008; Chung, 2004). The mathematics of GASING (Math GASING) method is one of learning method using constructivism approach (Prahmana & Suwasti, 2014; Prahmana, 2015; Surya, 2011; Surya & Moss, 2012; Shanty & Wijaya, 2012; Prahmana, 2013). This method has been applied to students from rural areas in Indonesia, which began with the introduction of number and number operations (Prahmana & Suwasti, 2014; Surya & Moss, 2012; Shanty & Wijaya, 2012; Prahmana, 2013). This situation underlies the researchers of this present study to try designing learning trajectory on number operations in Math GASING for rural area students derived from Serui, Sorong Selatan, and Ambon, Papua. Hence, the focus of this study is to describe the learning activities on students' performance to do multiplication in Math GASING.

Based on a few things mentioned above, the research question of this study is how the learning trajectory of multiplication in Math GASING is evolved the rural area's students' understanding in multiplication from informal to formal level. Hopefully, the learning trajectory has a role in learning multiplication that makes the learning more easy, joyful, and meaningful for the students.

In this study, the literature on Math GASING and number operations were learn to see the typical learning processes used by real situations (concrete) to abstract with the steps that has been in the design.

Deleted: y
Deleted: to
Deleted: ,
Deleted: ,
Deleted: ,
Deleted: and
Deleted: and
Deleted: the
Deleted: ,
Deleted: ,
Commented [SC3]: Which previous research?
Commented [SC4]: Are these the name of the students or the name of the places in the rural area? The sentence before it indicated those are students' names.
Deleted: and
Deleted: ,
Deleted: ,
Deleted: and
Deleted: and
Deleted: and
Deleted: and
Deleted: and
Deleted: and
Commented [SC5]: For the benefit of International Readers, please explain where in Indonesia these places are located.
Deleted: was

Number Operations

Integer operations that we know are addition, subtraction, multiplication, and division, where the four operations have any connection with each other (Reys, et al., 1984). The following four relations operation that has a relationship with each other, and students must understand the relationships (Reys et al., 1984).

Addition and subtraction are inverse operations. There are several ways to teach the concept of integer operations in the learning of mathematics. One of the ways to teach them is Math GASING.

5 + 8 = 13 ---------- 13 – 5 = 8

Multiplication and division are inverse operations

4 x 6 = 24 ---------- 24 : 4 = 6

Multiplication can be seen as a repeated addition

4 x 6 ---------- 6 + 6 + 6 + 6

Division can be seen as a repeated subtraction

24 : 6 ---------- 24 - 6 - 6 - 6 - 6

Mathematics GASING

Surya and Moss (2012) stated that GASING has several basic premises. Firstly, there is no such thing as a child that cannot learn mathematics, only children that have not had the opportunity to learn mathematics in a fun and meaningful way. Secondly, mathematics is based on patterns and these patterns make math understandable. Thirdly, a visual context to mathematical concepts should come before the symbolic notation. Lastly, mathematics is not memorization, but knowing basic facts comes easily with a conceptual and visual understanding. Memorization of basic mathematics facts is easy if it is based on conceptual learning and visual representations.

The learning process make students' learning easy, fun, and enjoyable in Math GASING (Shanty & Wijaya, 2012). Easy means the students are introduced to mathematical logic that is easy to learn and to remember. Exciting means the students have motivation which comes from by them to learn mathematics (intrinsic factor). Fun is more in the direction of outside influences such as visual aids and games (extrinsic factor). On the other hand, Prahmana (2013) had been conducted research for division topic in Math GASING, where the learning process begins with the activities share sweets fairly, then move into the process of how each student gets distributed sweets after a fair amount of candy (concrete),

Commented [SC6]: Please note that I am only helping in correcting the grammar for some parts of this article. Please make sure to correct the rest for the benefit of the journal's quality.

Deleted: ,

Deleted: ,

Deleted: and

Deleted: In

ranging from division without remainder to division with remainder, and ends with the completion of division operation in Math GASING (abstract). Math GASING shows how to change a concrete sample into an abstract symbol so the students will be able to read a mathematical pattern, thus gain the conclusion by themselves.

Math GASING as one of innovations in learning mathematics offers critical point in its learning process. There is a critical point that we must pass that is called GASING's critical point when studying a topic in Math GASING. After reaching this critical point, students will not be difficult anymore to work on the problems in that topic (Surya, 2011). The critical point in learning multiplication is that students must master the multiplication concept of 1 × 1 to 10 × 10. The student can learn a variety of multiplication operation problems more easy after pass the critical point.

Hypothetical Learning Trajectory

Hypothetical Learning Trajectory (HLT) is proposed as a term to identify and describe relevant aspects associated with a mathematics lesson plan, including: A description of the students' mathematical goals, the mathematical activities (including the tasks or problems, that students will work on to achieve the goals), and a hypothetical path that describes the students learning process (Revina et al., 2011). The HLT in this study had several learning goals expected to be reached by the students during one phase.

Deleted: l

Deleted: t

Deleted: ,

METHOD

Commented [SC7]: Please note that I am only helping in correcting the grammar for some parts of this article. Please make sure to correct the rest for the benefit of the journal's quality.

Design research is used as the research method of this study. This method is an appropriate way to answer the research questions and achieve the research objectives (Prahmana et al., 2012; Akker et al., 2006; Gravemeijer, 2004). Design research has five characteristics. These are interventionist nature, process oriented, reflective component, cyclic character, and theory oriented (Akker et al., 2006). There are two important aspect related to design research namely Hypothetical Learning Trajectory (HLT) and Local Instruction Theory (LIT). The learning activities as learning paths taken by students in their learning activities must have HLT and LIT.

Deleted: ,

Deleted: ,

Deleted: re

Deleted: ,

The HLT consists of three components (Gravemeijer, 2004). First is the purpose of mathematics teaching for the students. Second is learning activity and devices or media used in the learning process. Lastly is a conjecture of understanding the process of learning how to learn and strategies students that arise and thrive when learning activities done in class. There are three phases of design research namely preliminary design, teaching experiment, and

retrospective analysis seen in Figure 1.

The research data is from multiple sources of data in order to get a visualization of the students' mastery of basic concepts of multiplication operations, such as documentation (photo), video recording, and the students' worksheet and observation sheet. Next, the data analysed retrospectively with HLT as a guide. This present study was conducted and completed in 2 days with the subjects of 11 matriculation prospective teachers students at one of institute in Tangerang from Ambon, Serui, Yapen, and South Sorong, Papua, and also a teaching assistant who acted as a model teacher.

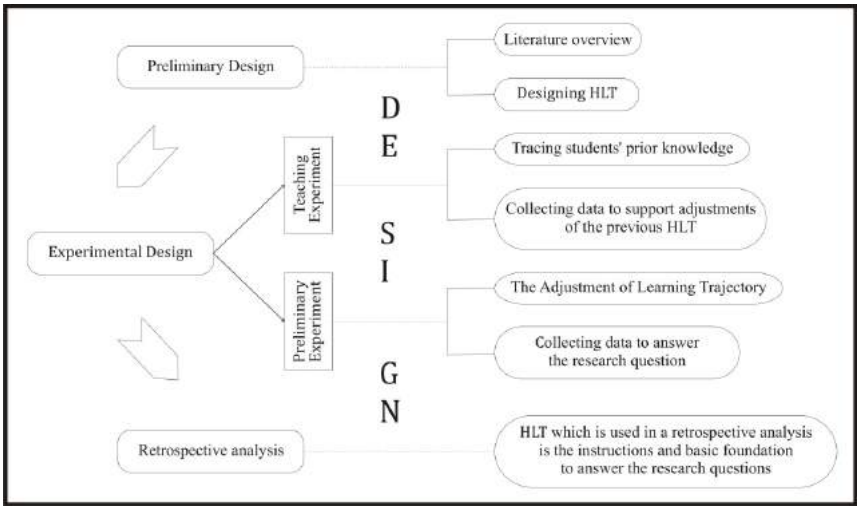


Figure 1. Phase of the design research (Prahmana et al., 2012).

RESULTS AND DISCUSSIONS

The learning activities start from making the same perceptions of the meaning of boxes containing something in that boxes to introduce the concept of multiplication. Furthermore, the students were trained to memorize the multiplication for 1 to 10 using several methods. Lastly, the teacher gave the evaluation to investigate the students' understanding of multiplication using mental arithmetic activity as one of assessment process in this learning activities and exercise by using the student evaluation sheet. As a result, the students were able to master the multiplication operation in Math GASING as shown from the results of the final evaluation and importantly, the learning of Math GASING could be detected from the comments from the students who wished to abandon the old way of learning mathematics. The results of this study indicated that learning design of multiplication operation in Math GASING have a very important role as the starting point and improvement in the

Deleted: regarding

Deleted: These studies have been

Deleted: are

Deleted: regarding

Deleted: ,

Commented [SC8]: Please note that I am only helping in correcting the grammar for some parts of this article. Please make sure to correct the rest for the benefit of the journal's quality.

Commented [SC9]: What boxes? You will need to elaborate here first because this term only appears later in Table 1, and in later parts of the results section.

Deleted:

Deleted: ive

Deleted: know

Deleted: as

Deleted: seen

Deleted: was pleased to learn

Deleted: can be seen

Deleted: of

students' motivation in learning. For more details, the researchers will discuss the results of this study, which is divided into three stages that are called preliminary design, teaching experiments, and retrospective analysis.

Preliminary Design

At this stage, the researchers began to implement the idea of multiplication operation in Math GASING by reviewing the literature, conducting observations in matriculation class, and designing a sequence of instructional learning for the learning of multiplication to reach the goals formulated in Table 1 (adapted from Surya (2011)). A set of activities for learning multiplication in Math GASING has been designed based learning trajectory and thinking process of students who hypothesized. The instruction set of activities has been divided into six activities that have been completed in two meetings with a variety of fun activities that made the students interested and engaged in the learning process, and end with the evaluation process.

Table 1. Overview of the learning trajectory of multiplication (adapted from Surya (2011)).

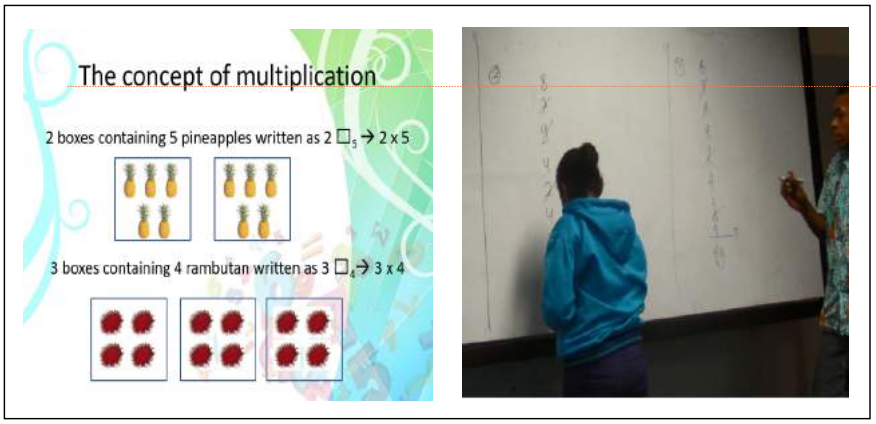
Sequence of activities	Goals	Descriptions
Playing some games using Math GASING learning aids	Understanding the multiplication concept	Students learn multiplication starting from understanding the basic concept of addition using the term of "box", for example 2 × 3 means there are 2 boxes containing 3 things in that box, and so on.
Using some method to memorize this part more easy	Memorizing the multiplication of numbers 1, 10, 9, 2 and 5	Students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students are able to master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on.
Using the patterns of two same numbers multiplication	Memorizing the multiplication of two same numbers, such as 1 × 1,2 × 2,…,10 × 10	Students learn about the same numbers of multiplication, such as 1 × 1, 2 × 2, 3 × 3, ... , 10x10.
Using multiplication characteristics as a commutative operation	Memorizing the multiplication of numbers 3 and 4	Students learn multiplication for 3 and 4 using a commutative operation.
Reducing some part in multiplication that	Memorizing the multiplication of	Students learn multiplication for 8, 7, and 6. Teacher teaches student by using reduce some

Deleted: is
Deleted: inning
Deleted: learns
Deleted: 2
Deleted: k
Deleted: happy in
Formatted: Left
Formatted: Left
Formatted: Left
Formatted: Left
Formatted: Left

already mastered	numbers 8, 7 and 6	part in multiplication that already mastered before.
Evaluation	Determining the student ability in learning multiplication	Teacher evaluates the student about multiplication problem in the formal and informal form.

Teaching Experiment

In teaching experiment, researchers test the learning activities that had been designed in the preliminary design stage. When the teacher models have started to see students do not get excited, then the teacher models provide educational games that make fun learning activities, because it is becoming one of characteristics in Math GASING learning process. There are five activities in this stage using whiteboard and presentation. First, teacher introduced the concept of multiplication by playing some games using Math GASING learning aids. In that games, students learn the concept of multiplication starting from understanding the basic concept of addition using the term of "box", for example 2 × 3 means there are 2 boxes containing 3 things in that box, and so on. Secondly, students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students are able to master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on. Different with the memorizing process of multiplication order in mathematics in general, students memorize the multiplication start from 1, 10, 9, 2 and 5. Furthermore, students learn about the same numbers of multiplication, such as 1 × 1, 2 × 2, 3 × 3, ... , 10x10. Fourthly, students learn multiplication for 3 and 4 using a commutative operation. Lastly, students learn multiplication for 8, 7, and 6. For this step, teacher teaches student by using reduce some part in multiplication that already mastered before. So, the students can memorize all multiplication concept from one to ten more easily. In the second meeting, teacher evaluates the student about multiplication problem in the formal and informal form. All activities can be shown in Figure 2.



Formatted: Left, Indent: First line: 0 cm
Formatted: Left
Deleted: researcher tests
Deleted: ve
Commented [SC10]: Please cover the man's face

Figure 2. Several activities in teaching experiment phase.

Retrospective Analysis

Multiplication process in Math GASING is different with multiplication process in mathematics in general. As a result, all activities, which have been designed, can be used to answer the research question above. The activities are given as follows.

Learning trajectory that has been modeled in Table 1 are the activities undertaken in this study to guide students in mastering the multiplication operations. So that, researcher designed an activity using Math GASING aids. The goal is that students are able to understand the concrete form of multiplication using the understanding of boxes and something in there. Student must understand that multiplication in the form of repeated addition. The teacher used combination learning tools such as presentation and whiteboard to make learning process effective and efficiency (seen in Figure 2).

Furthermore, from these activities, teachers guided students toward the concept of multiplication as the form of repeated addition. Teacher uses several methods to memorize multiplication for one to ten more easily and meaningful. On the other hands, teacher make the order of memorize the multiplication with different order in mathematical in general. First, students memorize the multiplication for 1, 10, 9, 2 and 5. Next, students memorize the multiplication for the same numbers of multiplication, such as 1×1, 2×2, 3×3, ... , 10×10. After that, students memorize the multiplication for 3, 4, 8, 7, and 6. Finally, all students can memorize the multiplication form from one to ten and answer the teacher exercise directly using their mental arithmetic.

Based on all the activities above, it can be seen that the students have gone through the process of activity based on experience using their ability and math GASING learning aids, moving toward a more formal, the understanding of formal level from the critical point, and then reached into the formal level desired as the ultimate goal of this learning activities.

In the design of this study, the researchers used the learning steps of multiplication in Math GASING as shown in Table 1. When the activity takes place, the dialogue is very good in the process of introducing the basic concepts of multiplication operations. In the dialogue, it seems that students feel learning multiplication in Math GASING looks so easy and so much fun. As a result, the learning process can guide students in understanding multiplication. It can also be seen from the student evaluation of learning multiplication process given by the teacher to evaluate student understanding (Figure 3). As a result, students seemed to be able to apply multiplication operation process in solving each problem is given in terms of evaluation. Therefore, it can be seen that learning multiplication operation in Math GASING can use to raise students' understanding in integer multiplication operations or in other words, the design of this study can be used as the starting point of learning multiplication.

Commented [SC11]: Please note that I am only helping in correcting the grammar for some parts of this article. Please make sure to correct the rest for the benefit of the journal's quality.

Deleted: the

Deleted: activities which have been designed

Deleted: :

Deleted: trajectory which has been modeled in Table 1

Deleted: ed

Deleted: T

Deleted: s

Commented [SC12]: It just shows a picture doing some work. I think this is irrelevant and needs to be remove. What reviewers/researchers/future readers are interested to see are the results of the students' evaluation (not them that showed that they are answering the evaluation form).

Figure 3. Student evaluation process using student worksheet.

The retrospective analysis shown that one of the ways making student understanding in learning multiplication is make the learning process can be imaging for students. This results is in line with previous research stated in learning multiplication have several ways to master it (Caron, 2007; Ischebeck, et al. 2006). On the other hand, learning environment also can support the result of the learning process. Finally, all students can solve several problems and exercises regarding in multiplication operation.

CONCLUSION

The researchers can conclude that the learning of multiplication operation in Math GASING have a very important role as the starting point and improve students' motivation in learning multiplication. In addition, the activities that have been designed in such way those students find the concept of multiplication starting from understanding the concept of multiplication to mastering the multiplication concept of 1×1 to 10×10 which is the critical point in learning multiplication in Math GASING. The student can learn a variety of multiplication operation problems more easily after passing the critical point. Lastly, each student can do mental arithmetic for any given multiplication problem and resolve many multiplication questions very quickly and precisely where is both of this are one of assessment forms in Math GASING.

REFERENCES

Ahmad, N. S. B. (2010). Multiplication and the reference sum method. *Procedia-Social and Behavioral Sciences*, *8*, 72-78.

Commented [SC13]: Please remove. Refer to comments above.

Commented [SC14]: Will need to include this result to strengthen the paper further.

Deleted: s

Commented [SC15]: This is a sweeping statement without any concrete evidences.

Commented [SC16]: Please note that I am only helping in correcting the grammar for some parts of this article. Please make sure to correct the rest for the benefit of the journal's quality.

Deleted: R

Deleted: y

Commented [SC17]: Not convincing because it is not supported by evidences as mentioned above.

Akker, J.V.D., Gravemeijer, K., McKenney, S., & Nieveen, N. (2006). *Education Design Research*. London: Routledge Taylor and Francis Group.

bin Syed Ismail, S. A. (2010). Multiplication with the Vedic method. *Procedia-Social and Behavioral Sciences*, *8*, 129-133.

Caron, T. A. (2007). Learning multiplication: The easy way. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *80*(6), 278-282.

Chang, K. E., Sung, Y. T., Chen, Y. L., & Huang, L. H. (2008). Learning multiplication through computer-assisted learning activities. *Computers in Human Behavior*, *24*(6), 2904-2916.

Chung, I. (2004). A comparative assessment of constructivist and traditionalist approaches to establishing mathematical connections in learning multiplication. *Education*, *125*(2), 271-279.

Drews, D., Dudgeon, J., Hansen, A., Lawton, F., & Surtees, L. (Eds.). (2005). *Children's Errors in Mathematics: Understanding Common Misconceptions in Primary Schools*. SAGE.

Freudenthal, H. (1973). *Mathematics as an Educational Task.* Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K. (2004). Local instructional theories as means of support for teacher in reform mathematics education. *Mathematical Thinking and Learning*, *6*(2), 105-128.

Ischebeck, A., Zamarian, L., Siedentopf, C., Koppelstätter, F., Benke, T., Felber, S., & Delazer, M. (2006). How specifically do we learn? Imaging the learning of multiplication and subtraction. *Neuroimage*, *30*(4), 1365-1375.

Kilian, L., Cahill, E., Ryan, C., Sutherland, D., & Taccetta, D. (1980). Errors that are common in multiplication. *The Arithmetic Teacher*, 22-25.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Prahmana, R. C. I. (2013). Designing division operation learning in the mathematics of GASING. *Proceeding in The First South East Asia Design/Development Research (SEA-DR) Conference 2013*, 391-398.

Prahmana, R. C. I. (2015). The hypothetical learning trajectory on addition in mathematics GASING. *Southeast Asian Mathematics Education Journal*, *5*(1), 49-61.

Prahmana, R. C. I., & Suwasti, P. (2014). Local instruction theory on division in mathematics GASING. *Journal on Mathematics Education*, *5*(1), 17-26.

Prahmana, R. C. I., Zulkardi, & Hartono, Y. (2012). Learning multiplication using Indonesian traditional game in third grade. *Journal on Mathematics Education*, *3*(2), 115-132.

Revina, S., Zulkardi, Darmawijoyo, & Galen, F. (2011). Spatial visualization tasks to support students' spatial structuring in learning volume measurement. *Journal on Mathematics Education*, *2*(2), 127-146.

Deleted: and

Deleted: and

Deleted: *h*

Deleted: *b*

Deleted: and

Deleted: and

Deleted: and

Deleted: and

Deleted: and

Deleted: and

Reys, R. E., Suydam, M. N., Lindquist, M. M., & Smith, N. L. (1984). *Helping Children Learn Mathematics*. Boston: Allyn and Bacon.

Shanty, N. O. , & Wijaya, S. (2012). Rectangular array model supporting students' spatial structuring in learning multiplication. *Journal on Mathematics Education*, *3*(2), 175-186.

Surya, Y. (2011). *Petunjuk Guru: Dasar-Dasar Pintar Berhitung GASING*. Tangerang: PT. Kandel.

Surya, Y., & Moss, M. (2012). Mathematics education in rural Indonesia. *Proceeding in the 12th International Congress on Mathematics Education: Topic Study Group 30*. Korea National University of Education, Seoul, South Korea, 6223-6229.

Tanujaya, B., Prahmana, R. C. I., & Mumu, J. (2017). Mathematics instruction, problems, challenges and opportunities: A case study in Manokwari Regency, Indonesia. *World Transactions on Engineering and Technology Education*, *15*(3), 287-291.

Unlu, M., & Ertekin, E. (2012). Why do pre-service teachers pose multiplication problems instead of division problems in fractions? *Procedia-Social and Behavioral Sciences*, *46*, 490-494.

Deleted: and

Deleted: and

Deleted: and

Deleted: Selatan

Deleted: and

Deleted: and

Deleted: .

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume xx, No. x, January xxxx, pp. x-xx

THE INNOVATION OF LEARNING TRAJECTORY ON MULTIPLICATION OPERATION FOR RURAL AREA STUDENT IN INDONESIA

Abstract

The rural area's student difficulties in learning the concept of number operation had been documented by several studies, especially for the case of multiplication. The teacher always introduces the multiplication concepts using the formula without involving the concept itself. Furthermore, this study aims to design learning trajectory on multiplication operation in the Mathematics of GASING (Math GASING) focused more on the concept itself than the formula and started from the informal to a formal level. Design research used as the research method to solve this problem consisting of three phases' namely preliminary design, teaching experiment, and retrospective analysis. The research results show that the Math GASING has a real contribution for students to understanding and mastering in the concept of the multiplication operation. This research also explains the strategy and model discovered by students in learning multiplication that students use to help their initial understanding of the multiplication concept. Finally, students can understand the concept of multiplication more easily and joyful by using this learning trajectory.

Keywords: multiplication, learning trajectory, design research

Abstrak

Kesulitan siswa di daerah pedesaan dalam mempelajari konsep operasi angka telah didokumentasikan oleh beberapa studi, terutama untuk kasus perkalian. Guru selalu memperkenalkan konsep perkalian menggunakan rumus tanpa melibatkan konsep itu sendiri. Selanjutnya, penelitian ini bertujuan untuk merancang lintasan pembelajaran pada operasi multiplikasi dalam Matematika GASING (Matematika GASING) lebih fokus pada konsep itu sendiri daripada rumus dan mulai dari tingkat informal ke formal. Desain penelitian digunakan sebagai metode penelitian untuk menyelesaikan masalah ini yang terdiri dari tiga fase yaitu desain pendahuluan, eksperimen mengajar, dan analisis retrospektif. Hasil penelitian menunjukkan bahwa GASING Matematika memiliki kontribusi nyata bagi siswa untuk memahami dan menguasai dalam konsep operasi multiplikasi. Penelitian ini juga menjelaskan strategi dan model yang ditemukan oleh siswa dalam mempelajari perkalian yang digunakan siswa untuk membantu pemahaman awal mereka tentang konsep perkalian. Akhirnya, siswa dapat memahami konsep perkalian dengan lebih mudah dan menyenangkan dengan menggunakan lintasan pembelajaran ini.

Kata kunci: perkalian, lintasan belajar, design research

How to Cite: Prahmana, R.C.I. & Zulkardi. (2016). Instructions/Template for Preparing Manuscript for Journal on Mathematics Education. *Journal on Mathematics Education, x* (x), xx-xx.

Learning number operations is important for almost all topics in Mathematics involving numbers (Ahmad, 2010; Freudenthal, 1973; NCTM, 2000; Prahmana, et al. 2012). It's because learning number operations tends to an understanding of symbols, notation, and reference number (other forms to represent) (NCTM, 2000) and plays an important role in determining students' performance in other related Mathematics topics (Ahmad, 2010). Therefore, learning number operation would be one of the prior knowledge that students must have for learn another topics in mathematics.

Commented [AWK1]: It is not wise to say 'always'. In fact, some teachers do not teach in that way. It should be 'often'

Commented [AWK2]: design or develop?? HLT needs to be developed, not only designed

Commented [AWK3]: Use 'it is', rather than it is

The concept of number operation, especially in multiplication, is one of students' difficulty to understanding mathematics concept (Ahmad, 2010; bin Syed Ismail, 2010; Drews, et al. 2005; Kilian, et al. 1980; Tanujaya, et al. 2017; Unlu and Ertekin, 2012). Teachers usually teach number operations using symbolic form or something abstract (Unlu and Ertekin, 2012). As the result, students learn number operations more on the process of memorizing than understanding it (bin Syed Ismail, 2010), have several errors which reflected their lack of understanding of various mathematical concepts and also the long multiplication algorithm (Ahmad, 2010), and have the poor understanding of the place value (tens and ones) concept in relation to multiplication (Drews, et al. 2005; Kilian, et al. 1980). The result of the previous research is in line with the preliminary classroom observation results of rural area's student, namely Serui, Ambon, and Sorong Selatan, regarding to learning number operations conducted by researchers in pre-test. Teachers introduced the concept of multiplication using the formula without involving the concept itself (Prahmana and Suwasti, 2014).

Several studies indicated that constructivism approach can improve students' understanding in learn multiplication (Ahmad, 2010; Prahmana, et al. 2012; Chang, et al. 2008; Chung, 2004). The mathematics of GASING (Math GASING) method is one of learning method using constructivism approach (Prahmana and Suwasti, 2014; Prahmana, 2015; Surya, 2011; Surya and Moss, 2012; Shanty and Wijaya, 2012; Prahmana, 2013). This method has been applied to student from rural area in Indonesia which began with the introduction of number and number operations (Prahmana and Suwasti, 2014; Surya and Moss, 2012; Shanty and Wijaya, 2012; Prahmana, 2013). This situation underlies the researcher to try designing learning trajectory on number operation in Math GASING for rural area students derived from Serui, Sorong Selatan, and Ambon, Papua. Hence, the focus of this study was to describe the learning activities on students' performance to do multiplication in Math GASING.

Based on a few things mentioned above, the research question of this study is how the learning trajectory of multiplication in Math GASING is evolved the rural area's students' understanding in multiplication from informal to formal level. Hopefully, the learning trajectory has a role in learning multiplication that makes the learning more easy, joyful, and meaningful for students.

In this study, the literature on Math GASING and number operations were learned to see the typical learning processes used by real situations (concrete) to abstract with the steps that has been in the design.

Number Operation

Commented [AWK4]: Previous studies about students' learning experiences especially in learning numbers in rural areas

Commented [AWK5]: It is better if the summary or the reviews about findings about some mathematics GASIS methods implemented in Indonesia. It may be related to what learning experiences, students' learning difficulties, and other interesting point of findings, and of course, what can be learned from those findings.

Commented [AWK6]: The author should describe the background or situations of this area. In other words, why this area is categorized as rural area? What makes it unique?

Commented [AWK7]: The authors should provide some reviews about learning trajectory about multiplication, then tell what did the authors propose regarding this learning trajectory

Commented [AWK8]: Instead of describing what **Number Operation** is, the authors should better describe the summary of reviews on **Learning Number Operation,** which implies some literature discussing about how to learn number operation based on experts' expectation or previous findings

Integer operations that we know are addition, subtraction, multiplication, and division, where the four operations have any connection with each other (Reys, et al. 1984). The following four relations operation that has a relationship with each other, and students must understand the relationships (Reys, et al. 1984).

Addition and subtraction are inverse operations. There are several ways to teach the concept of integer operations in the learning of mathematics. One of the ways to teach them is Math GASING.

5 + 8 = 13 ---------- 13 – 5 = 8

Multiplication and division are inverse operations

4 x 6 = 24 ---------- 24 : 4 = 6

Multiplication can be seen as a repeated addition

4 x 6 ---------- 6 + 6 + 6 + 6

Division can be seen as a repeated subtraction

24 : 6 ---------- 24 - 6 - 6 - 6 - 6

Mathematics GASING

Surya and Moss (2012) stated that GASING has several basic premises. Firstly, there is no such thing as a child that cannot learn mathematics, only children that have not had the opportunity to learn mathematics in a fun and meaningful way. Secondly, mathematics is based on patterns and these patterns make math understandable. Thirdly, a visual context to mathematical concepts should come before the symbolic notation. Lastly, mathematics is not memorization, but knowing basic facts comes easily with a conceptual and visual understanding. Memorization of basic mathematics facts is easy if it is based on conceptual learning and visual representations.

The learning process make students learning easy, fun, and enjoyable in Math GASING (Shanty and Wijaya, 2012). Easy means the students are introduced to mathematical logic that is easy to learn and to remember. Exciting means the students have motivation which comes from by them to learn mathematics (intrinsic factor). Fun is more in the direction of outside influences such as visual aids and games (extrinsic factor). In the other hand, Prahmana (2013) had been conducted research for division topic in Math GASING, where the learning process begins with the activities share sweets fairly, then move into the process of how each student gets distributed sweets after a fair amount of candy (concrete), ranging from division without remainder to division with remainder, and ends with the completion of division operation in Math GASING (abstract). Math GASING shows how to change a concrete sample into an

abstract symbol so the students will be able to read a mathematical pattern, thus gain the conclusion by themselves.

Math GASING as one of innovations in learning mathematics offers critical point in its learning process. There is a critical point that we must pass that is called GASING's critical point when studying a topic in Math GASING. After reaching this critical point, students will not be difficult anymore to work on the problems in that topic (Surya, 2011). The critical point in learning multiplication is that students must master the multiplication concept of 1 × 1 to 10 × 10. The student can learn a variety of multiplication operation problems more easy after pass the critical point.

Commented [AWK9]: The authors need elaborate more about how MATH GASING would be implemented in number operation learning

Hypothetical Learning Trajectory

Hypothetical learning trajectory (HLT) is proposed as a term to identify and describe relevant aspects associated with a mathematics lesson plan, including: A description of the students' mathematical goals, the mathematical activities (including the tasks or problems, that students will work on to achieve the goals), and a hypothetical path that describes the students learning process (Revina, et al. 2011). The HLT in this study had several learning goals expected to be reached by the students during one phase

Commented [AWK10]: The authors need to cite some more references about the conceptualization of HLT, primarily from primary (original) resources.

METHOD

Commented [AWK11]: Use past tense for all of the descriptions or this section

Design research used as the research method of this study. This method is an appropriate way to answer the research questions and achieve the research objectives (Prahmana, et al. 2012; Akker, et al. 2006; Gravemeijer, 2004). Design research has five characteristics. There are interventionist nature, process oriented, reflective component, cyclic character, and theory oriented (Akker, et al. 2006). There are two important aspect related to design research namely Hypothetical Learning Trajectory (HLT) and Local Instruction Theory (LIT). The learning activities as learning paths taken by students in their learning activities must have HLT and LIT.

The HLT consists of three components (Gravemeijer, 2004). First is the purpose of mathematics teaching for students. Second is learning activity and devices or media used in the learning process. Lastly is a conjecture of understanding the process of learning how to learn and strategies students that arise and thrive when learning activities done in class. There are three phases of design research namely preliminary design, teaching experiment, and retrospective analysis seen in Figure 1.

The research data is regarding from multiple sources of data to get a visualization of the

students' mastery of basic concepts of multiplication operations, such as documentation (photo), video recording, and the students' worksheet and observation sheet. Next, the data analysed retrospectively with HLT as a guide. These studies have been completed in 2 days with the subjects are 11 matriculation prospective teachers students at one of institute in Tangerang regarding from Ambon, Serui, Yapen, and South Sorong, Papua, and also a teaching assistant who acted as a model teacher.

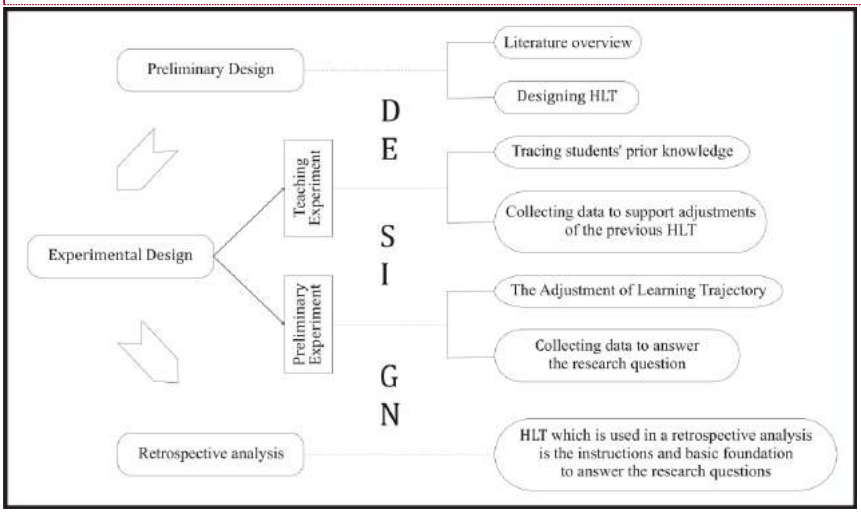


Figure 1. Phase of the design research (Prahmana, et al. 2012).

Commented [AWK12]: The authors should add some information about figure 1. In what stage does the authors modify the stages?

Commented [AWK13]: In which part of this stages, the authors clarify the validation and the reliability of the instrument and the data obtained?

RESULT AND DISCUSSION

The learning activities start from making same perception of the meaning of boxes containing something in that boxes to introduce the concept of multiplication. Furthermore, students train to memorize the multiplication for 1 to 10 using several methods. Lastly, teacher give evaluation to know the student understanding of multiplication using mental arithmetic activity as one of assessment process in this learning activities and exercise by using student evaluation sheet. As a result, students was able to master the multiplication operation in Math GASING seen from the results of the final evaluation and was pleased to learn Math GASING can be seen from the comments of students who wish to abandon the old way of learning mathematics. The results of this study indicate that learning design of multiplication operation in Math GASING have a very important role as the starting point and improve students' motivation in learning. For more details, researchers will discuss the results of this study, which is divided into three stages that are called preliminary design, teaching experiments, and retrospective analysis.

Preliminary Design

At this stage, researcher is beginning to implement the idea of multiplication operation in Math GASING by reviewing the literature, conducting observations in matriculation class, and designing a sequence of instructional learning for learns multiplication to reach the goals formulated in Table 1 (adapted from Surya (2011)). A set of activities for learning multiplication in Math GASING has been designed based learning trajectory and thinking process of students who hypothesized. The instruction set of activities has been divided into six activities that have been completed in two meetings with a variety of fun activities that make students happy in the learning process, and end with the evaluation process.

Deleted: 2

Table 1. Overview of the learning trajectory of multiplication.

Sequence of activities	Goals	Descriptions
Playing some games using Math GASING learning aids	Understanding the multiplication concept	Students learn multiplication starting from understanding the basic concept of addition using the term of "box", for example 2 × 3 means there are 2 boxes containing 3 things in that box, and so on.
Using some method to memorize this part more easily	Memorizing the multiplication of numbers 1, 10, 9, 2 and 5	Students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students are able to master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on.
Using the patterns of two same numbers multiplication	Memorizing the multiplication of two same numbers, such as $1 \times 1, 2 \times 2, \ldots, 10 \times 10$	Students learn about the same numbers of multiplication, such as 1 × 1, 2 × 2, 3 × 3, ... , 10x10.
Using multiplication characteristics as a commutative operation	Memorizing the multiplication of numbers 3 and 4	Students learn multiplication for 3 and 4 using a commutative operation.
Reducing some part in multiplication that already mastered	Memorizing the multiplication of numbers 8, 7 and 6	Students learn multiplication for 8, 7, and 6. Teacher teaches student by using reduce some part in multiplication that already mastered before.
Evaluation	Determining the	Teacher evaluates the student about

Deleted: y

Commented [AWK14]: The demand of asking students to 'memorize' is too narrow. It does not measure a deeper understanding on the topic they were studying. Furthermore, by only memorizing the multiplication of numbers does not guarantee the students' understanding on the concept of multiplication of integers, does it?
I suggest the authors to use another word indicating more-demanded cognitive skills, such as analyze, compare, hypothesyze, etc.

student ability in multiplication problem in the formal and informal learning multiplication form.

Commented [AWK15]: Change the sentence, into the form of Students learn....., instead of Teacher teaches/evaluates....

Teaching Experiment

Commented [AWK16]: Write using Past Tense

In teaching experiment, researcher tests the learning activities have been designed in the preliminary design stage. When the teacher models have started to see students do not get excited, then the teacher models provide educational games that make fun learning activities, because it is becoming one of characteristics in Math GASING learning process. There are five activities in this stage using whiteboard and presentation. First, teacher introduced the concept of multiplication by playing some games using Math GASING learning aids. In that games, students learn the concept of multiplication starting from understanding the basic concept of addition using the term of "box", for example 2 × 3 means there are 2 boxes containing 3 things in that box, and so on. Secondly, students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students are able to master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on. Different with the memorizing process of multiplication order in mathematics in general, students memorize the multiplication start from 1, 10, 9, 2 and 5. Furthermore, students learn about the same numbers of multiplication, such as 1 × 1, 2 × 2, 3 × 3, ... , 10x10. Fourthly, students learn multiplication for 3 and 4 using a commutative operation. Lastly, students learn multiplication for 8, 7, and 6. For this step, teacher teaches student by using reduce some part in multiplication that already mastered before. So, the students can memorize all multiplication concept from one to ten more easily. In the second meeting, teacher evaluates the student about multiplication problem in the formal and informal form. All activities can be shown in Figure 2.

Commented [AWK17]: How did these two forms were performed by the students?

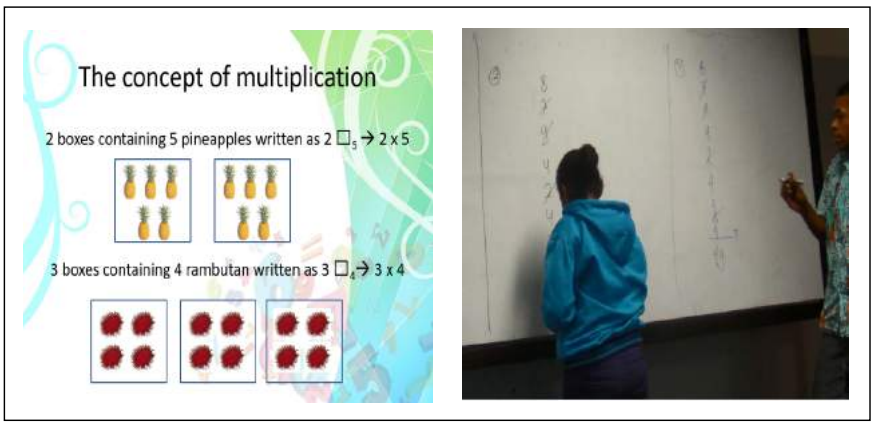


Figure 2. Several activities in teaching experiment phase.

Retrospective Analysis

Commented [AWK18]: The authors do not provide any analysis of the progression of learning trajectory within the HLT they designed. It should declare the local instructional theory of this study based on the HLT

Multiplication process in Math GASING is different with multiplication process in mathematics in general. As the result, all activities which have been designed can be used to answer the research question above. The activities are as follows:

Learning trajectory which has been modeled in Table 1 are the activities undertaken in this study to guide students mastered multiplication operation. So that, researcher designed an activity using Math GASING aids. The goal is that students are able to understand the concrete form of multiplication using the understanding of boxes and something in there. Student must understand that multiplication in the form of repeated addition. Teacher uses combination learning tools such as presentation and whiteboard to make learning process effective and efficiency (seen in Figure 2).

Furthermore, from these activities, teachers guide students toward the concept of multiplication as the form of repeated addition. Teacher uses several methods to memorize multiplication for one to ten more easily and meaningful. On the other hands, teacher make the order of memorize the multiplication with different order in mathematical in general. First, students memorize the multiplication for 1, 10, 9, 2 and 5. Next, students memorize the multiplication for the same numbers of multiplication, such as 1×1, 2×2, 3×3, ... , 10×10. After that, students memorize the multiplication for 3, 4, 8, 7, and 6. Finally, all students can memorize the multiplication form from one to ten and answer the teacher exercise directly using their mental arithmetic.

Based on all the activities above, it can be seen that the students have gone through the process of activity based on experience using their ability and math GASING learning aids, moving toward a more formal, the understanding of formal level from the critical point, and then reached into the formal level desired as the ultimate goal of this learning activities.

In the design of this study, researcher used the learning steps of multiplication in Math GASING as shown in Table 1. When the activity takes place, the dialogue is very good in the process of introducing the basic concepts of multiplication operations. In the dialogue, it seems that students feel learning multiplication in Math GASING looks so easy and so much fun. As a result, the learning process can guide students in understanding multiplication. It can also be seen from the student evaluation of learning multiplication process given by the teacher to evaluate student understanding (Figure 3). As a result, students seemed to be able to apply multiplication operation process in solving each problem is given in terms of evaluation. Therefore, it can be seen that learning multiplication operation in Math GASING can use to raise students' understanding in integer multiplication operations or in other words, the design of this study can be used as the starting point of learning multiplication.

Commented [AWK19]: This is not operational. How to indicate that the dialogue is good?

Figure 3. Student evaluation process using student worksheet.

The retrospective analysis shown that one of the ways making student understanding in learning multiplication is make the learning process can be imaging for students. This results is in line with previous research stated in learning multiplication have several ways to master it (Caron, 2007; Ischebeck, et al. 2006). On the other hands, learning environment also can support the result of the learning process. Finally, all students can solve several problems and exercises regarding in multiplication operation.

CONCLUSION

It is concluded that the learning of multiplication operation in Math GASING have a very important role as the starting point and improve students' motivation in learning multiplication. In addition, the activities that have been designed in such way those students find the concept of multiplication starting from understanding the concept of multiplication to mastering the multiplication concept of 1×1 to 10×10 which is the critical point in learning multiplication in Math GASING. The student can learn a variety of multiplication operation problems more easy after pass the critical point. Lastly, each student can do mental arithmetic for any given multiplication problem and resolve many multiplication questions very quickly and precisely where is both of this are one of assessment forms in Math GASING.

REFERENCES

Ahmad, N. S. B. (2010). Multiplication and the reference sum method. *Procedia-Social and Behavioral Sciences*, *8*, 72-78.

Akker, J.V.D., Gravemeijer, K., McKenney, S. and Nieveen, N. (2006). *Education Design Research*. London: Routledge Taylor and Francis Group.

Commented [AWK20]: What does this figure tell the readers?

Commented [AWK21]: In this stage, the authors should also examine the HLT they designed and the literature review about the HLT related to multiplication.

Deleted: Researcher can

Commented [AWK22]: As design research, the authors should describe the conclusion by clarifying the development of the HLT the designed. Furthermore, they also need to state explicitly the LIT (local instructional theory) produced in this research.

bin Syed Ismail, S. A. (2010). Multiplication with the Vedic method. *Procedia-Social and Behavioral Sciences*, *8*, 129-133.

Caron, T. A. (2007). Learning multiplication: The easy way. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *80*(6), 278-282.

Chang, K. E., Sung, Y. T., Chen, Y. L. and Huang, L. H. (2008). Learning multiplication through computer-assisted learning activities. *Computers in human behavior*, *24*(6), 2904-2916.

Chung, I. (2004). A comparative assessment of constructivist and traditionalist approaches to establishing mathematical connections in learning multiplication. *Education*, *125*(2), 271-279.

Drews, D., Dudgeon, J., Hansen, A., Lawton, F. and Surtees, L. (Eds.). (2005). *Children's Errors in Mathematics: Understanding Common Misconceptions in Primary Schools*. SAGE.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K. (2004). Local instructional theories as means of support for teacher in reform mathematics education. *Mathematical Thinking and Learning*, *6*(2), 105-128.

Ischebeck, A., Zamarian, L., Siedentopf, C., Koppelstätter, F., Benke, T., Felber, S. and Delazer, M. (2006). How specifically do we learn? Imaging the learning of multiplication and subtraction. *Neuroimage*, *30*(4), 1365-1375.

Kilian, L., Cahill, E., Ryan, C., Sutherland, D., and Taccetta, D. (1980). Errors that are common in multiplication. *The Arithmetic Teacher*, 22-25.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Prahmana, R. C. I. (2013). Designing division operation learning in the mathematics of GASING. *Proceeding in The First South East Asia Design/Development Research (SEA-DR) Conference 2013*, 391-398.

Prahmana, R. C. I. (2015). The hypothetical learning trajectory on addition in mathematics GASING. *Southeast Asian Mathematics Education Journal*, *5*(1), 49-61.

Prahmana, R. C. I. and Suwasti, P. (2014). Local instruction theory on division in mathematics GASING. *Journal on Mathematics Education*, *5*(1), 17-26.

Prahmana, R. C. I., Zulkardi, and Hartono, Y. (2012). Learning multiplication using Indonesian traditional game in third grade. *Journal on Mathematics Education*, *3*(2), 115-132.

Revina, S., Zulkardi, Darmawijoyo and Galen, F. (2011). Spatial visualization tasks to support students' spatial structuring in learning volume measurement. *Journal on Mathematics Education*, *2*(2), 127-146.

Reys, R. E., Suydam, M. N., Lindquist, M. M., and Smith, N. L. (1984). *Helping Children Learn Mathematics*. Boston: Allyn and Bacon.

Shanty, N.O. and Wijaya, S. (2012). Rectangular array model supporting students' spatial structuring in learning multiplication. *Journal on Mathematics Education*, *3*(2), 175-186.

Surya, Y. (2011). *Petunjuk Guru: Dasar-Dasar Pintar Berhitung GASING*. Tangerang: PT. Kandel.

Surya, Y. and Moss, M. (2012). Mathematics education in rural Indonesia. *Proceeding in the 12th International Congress on Mathematics Education: Topic Study Group 30*. Korea National

University of Education, Seoul, Korea Selatan, 6223-6229.

Tanujaya, B., Prahmana, R.C.I. and Mumu, J. (2017). Mathematics instruction, problems, challenges and opportunities: A case study in Manokwari Regency, Indonesia. *World Transactions on Engineering and Technology Education*, *15*(3**)**, 287-291.

Unlu, M. and Ertekin, E. (2012). Why do pre-service teachers pose multiplication problems instead of division problems in fractions?. *Procedia-Social and Behavioral Sciences*, *46*, 490-494.

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume xx, No. x, January xxxx, pp. x-xx

THE INNOVATION OF LEARNING TRAJECTORY ON MULTIPLICATION OPERATION FOR RURAL AREA STUDENT IN INDONESIA

Abstract

The rural area's student difficulties in learning the concept of number operation had been documented by several studies, especially for the case of multiplication. The teacher always introduces the multiplication concepts using the formula without involving the concept itself. Furthermore, this study aims to design learning trajectory on multiplication operation in the Mathematics of GASING (Math GASING) focused more on the concept itself than the formula and started from the informal to a formal level. Design research used as the research method to solve this problem consisting of three phases' namely preliminary design, teaching experiment, and retrospective analysis. The research results show that the Math GASING has a real contribution for students to understanding and mastering in the concept of the multiplication operation. This research also explains the strategy and model discovered by students in learning multiplication that students use to help their initial understanding of the multiplication concept. Finally, students can understand the concept of multiplication more easily and joyful by using this learning trajectory.

Keywords: multiplication, learning trajectory, design research

Abstrak

Kesulitan siswa di daerah pedesaan dalam mempelajari konsep operasi angka telah didokumentasikan oleh beberapa studi, terutama untuk kasus perkalian. Guru selalu memperkenalkan konsep perkalian menggunakan rumus tanpa melibatkan konsep itu sendiri. Selanjutnya, penelitian ini bertujuan untuk merancang lintasan pembelajaran pada operasi multiplikasi dalam Matematika GASING (Matematika GASING) lebih fokus pada konsep itu sendiri daripada rumus dan mulai dari tingkat informal ke formal. Desain penelitian digunakan sebagai metode penelitian untuk menyelesaikan masalah ini yang terdiri dari tiga fase yaitu desain pendahuluan, eksperimen mengajar, dan analisis retrospektif. Hasil penelitian menunjukkan bahwa GASING Matematika memiliki kontribusi nyata bagi siswa untuk memahami dan menguasai dalam konsep operasi multiplikasi. Penelitian ini juga menjelaskan strategi dan model yang ditemukan oleh siswa dalam mempelajari perkalian yang digunakan siswa untuk membantu pemahaman awal mereka tentang konsep perkalian. Akhirnya, siswa dapat memahami konsep perkalian dengan lebih mudah dan menyenangkan dengan menggunakan lintasan pembelajaran ini.

Kata kunci: perkalian, lintasan belajar, design research

How to Cite: Prahmana, R.C.I. & Zulkardi. (2016). Instructions/Template for Preparing Manuscript for Journal on Mathematics Education. *Journal on Mathematics Education, x* (x), xx-xx.

Learning number operations is important for almost all topics in Mathematics involving numbers (Ahmad, 2010; Freudenthal, 1973; NCTM, 2000; Prahmana, et al. 2012). It's because learning number operations tends to an understanding of symbols, notation, and reference number (other forms to represent) (NCTM, 2000) and plays an important role in determining students' performance in other related Mathematics topics (Ahmad, 2010). Therefore, learning number operation would be one of the prior knowledge that students must have for learn another topics in mathematics.

Commented [WU1]: This part is not clear

Commented [WU2]: Formula is also part of concept. The sentence needs to restructure

Commented [WU3]: The result should be connected to learning trajectory design...for example how the hlt looks like

Commented [WU4]: English needs to be revised

The concept of number operation, especially in multiplication, is one of students' difficulty to understanding mathematics concept (Ahmad, 2010; bin Syed Ismail, 2010; Drews, et al. 2005; Kilian, et al. 1980; Tanujaya, et al. 2017; Unlu and Ertekin, 2012). Teachers usually teach number operations using symbolic form or something abstract (Unlu and Ertekin, 2012). As the result, students learn number operations more on the process of memorizing than understanding it (bin Syed Ismail, 2010), have several errors which reflected their lack of understanding of various mathematical concepts and also the long multiplication algorithm (Ahmad, 2010), and have the poor understanding of the place value (tens and ones) concept in relation to multiplication (Drews, et al. 2005; Kilian, et al. 1980). The result of the previous research is in line with the preliminary classroom observation results of rural area's student, namely Serui, Ambon, and Sorong Selatan, regarding to learning number operations conducted by researchers in pre-test. Teachers introduced the concept of multiplication using the formula without involving the concept itself (Prahmana and Suwasti, 2014).

Several studies indicated that constructivism approach can improve students' understanding in learn multiplication (Ahmad, 2010; Prahmana, et al. 2012; Chang, et al. 2008; Chung, 2004). The mathematics of GASING (Math GASING) method is one of learning method using constructivism approach (Prahmana and Suwasti, 2014; Prahmana, 2015; Surya, 2011; Surya and Moss, 2012; Shanty and Wijaya, 2012; Prahmana, 2013). This method has been applied to student from rural area in Indonesia which began with the introduction of number and number operations (Prahmana and Suwasti, 2014; Surya and Moss, 2012; Shanty and Wijaya, 2012; Prahmana, 2013). This situation underlies the researcher to try designing learning trajectory on number operation in Math GASING for rural area students derived from Serui, Sorong Selatan, and Ambon, Papua. Hence, the focus of this study was to describe the learning activities on students' performance to do multiplication in Math GASING.

Based on a few things mentioned above, the research question of this study is how the learning trajectory of multiplication in Math GASING is evolved the rural area's students' understanding in multiplication from informal to formal level. Hopefully, the learning trajectory has a role in learning multiplication that makes the learning more easy, joyful, and meaningful for students.

In this study, the literature on Math GASING and number operations were learn to see the typical learning processes used by real situations (concrete) to abstract with the steps that has been in the design.

Commented [WU5]: Grammatical error

Number Operation

Integer operations that we know are addition, subtraction, multiplication, and division, where the four operations have any connection with each other (Reys, et al. 1984). The following four relations operation that has a relationship with each other, and students must understand the relationships (Reys, et al. 1984).

Addition and subtraction are inverse operations. There are several ways to teach the concept of integer operations in the learning of mathematics. One of the ways to teach them is Math GASING.

5 + 8 = 13 ---------- 13 – 5 = 8

Multiplication and division are inverse operations

4 x 6 = 24 ---------- 24 : 4 = 6

Multiplication can be seen as a repeated addition

4 x 6 ---------- 6 + 6 + 6 + 6

Division can be seen as a repeated subtraction

24 : 6 ---------- 24 - 6 - 6 - 6 - 6

Mathematics GASING

Commented [WU6]: HLT and math gasing is better to be elaborated so that it will be more clear how the hlt with math gasing looks like

Surya and Moss (2012) stated that GASING has several basic premises. Firstly, there is no such thing as a child that cannot learn mathematics, only children that have not had the opportunity to learn mathematics in a fun and meaningful way. Secondly, mathematics is based on patterns and these patterns make math understandable. Thirdly, a visual context to mathematical concepts should come before the symbolic notation. Lastly, mathematics is not memorization, but knowing basic facts comes easily with a conceptual and visual understanding. Memorization of basic mathematics facts is easy if it is based on conceptual learning and visual representations.

The learning process make students learning easy, fun, and enjoyable in Math GASING (Shanty and Wijaya, 2012). Easy means the students are introduced to mathematical logic that is easy to learn and to remember. Exciting means the students have motivation which comes from by them to learn mathematics (intrinsic factor). Fun is more in the direction of outside influences such as visual aids and games (extrinsic factor). In the other hand, Prahmana (2013) had been conducted research for division topic in Math GASING, where the learning process begins with the activities share sweets fairly, then move into the process of how each student gets distributed sweets after a fair amount of candy (concrete), ranging from division without remainder to division with remainder, and ends with the completion of division operation in Math GASING (abstract). Math GASING shows how to change a concrete sample into an

Commented [WU7]: How is the math gasing in this study compared to Shanty and Wijaya

abstract symbol so the students will be able to read a mathematical pattern, thus gain the conclusion by themselves.

Math GASING as one of innovations in learning mathematics offers critical point in its learning process. There is a critical point that we must pass that is called GASING's critical point when studying a topic in Math GASING. After reaching this critical point, students will not be difficult anymore to work on the problems in that topic (Surya, 2011). The critical point in learning multiplication is that students must master the multiplication concept of 1×1 to 10×10. The student can learn a variety of multiplication operation problems more easy after pass the critical point.

Hypothetical Learning Trajectory

Hypothetical learning trajectory (HLT) is proposed as a term to identify and describe relevant aspects associated with a mathematics lesson plan, including: A description of the students' mathematical goals, the mathematical activities (including the tasks or problems, that students will work on to achieve the goals), and a hypothetical path that describes the students learning process (Revina, et al. 2011). The HLT in this study had several learning goals expected to be reached by the students during one phase

METHOD

Design research used as the research method of this study. This method is an appropriate way to answer the research questions and achieve the research objectives (Prahmana, et al. 2012; Akker, et al. 2006; Gravemeijer, 2004). Design research has five characteristic. There are interventionist nature, process oriented, reflective component, cyclic character, and theory oriented (Akker, et al. 2006). There are two important aspect related to design research namely Hypothetical Learning Trajectory (HLT) and Local Instruction Theory (LIT). The learning activities as learning paths taken by students in their learning activities must have HLT and LIT.

The HLT consists of three components (Gravemeijer, 2004). First is the purpose of mathematics teaching for students. Second is learning activity and devices or media used in the learning process. Lastly is a conjecture of understanding the process of learning how to learn and strategies students that arise and thrive when learning activities done in class. There are three phases of design research namely preliminary design, teaching experiment, and retrospective analysis seen in Figure 1.

The research data is regarding from multiple sources of data to get a visualization of the

students' mastery of basic concepts of multiplication operations, such as documentation (photo), video recording, and the students' worksheet and observation sheet. Next, the data analysed retrospectively with HLT as a guide. These studies have been completed in 2 days with the subjects are 11 matriculation prospective teachers students at one of institute in Tangerang regarding from Ambon, Serui, Yapen, and South Sorong, Papua, and also a teaching assistant who acted as a model teacher.

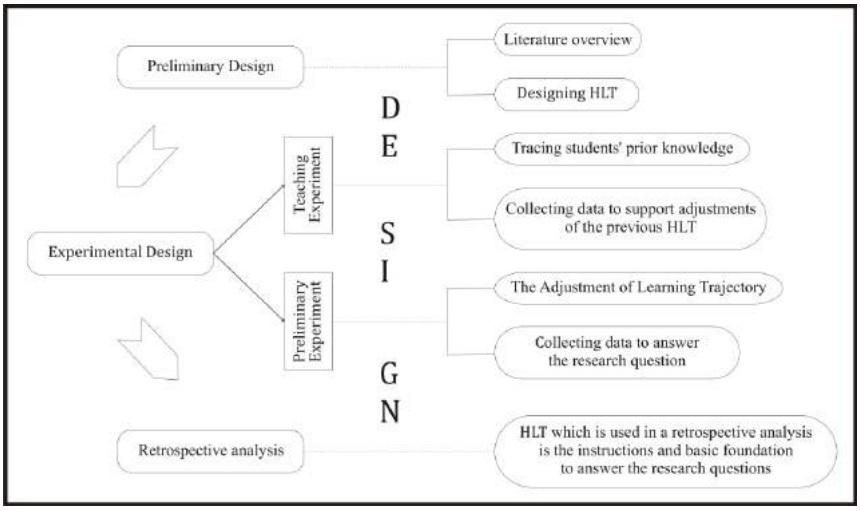


Figure 1. Phase of the design research (Prahmana, et al. 2012).

RESULT AND DISCUSSION

The learning activities start from making same perception of the meaning of boxes containing something in that boxes to introduce the concept of multiplication. Furthermore, students train to memorize the multiplication for 1 to 10 using several methods. Lastly, teacher give evaluation to know the student understanding of multiplication using mental arithmetic activity as one of assessment process in this learning activities and exercise by using student evaluation sheet. As a result, students was able to master the multiplication operation in Math GASING seen from the results of the final evaluation and was pleased to learn Math GASING can be seen from the comments of students who wish to abandon the old way of learning mathematics. The results of this study indicate that learning design of multiplication operation in Math GASING have a very important role as the starting point and improve students' motivation in learning. For more details, researchers will discuss the results of this study, which is divided into three stages that are called preliminary design, teaching experiments, and retrospective analysis.

Commented [WU8]: Revise the english

Preliminary Design

At this stage, researcher is beginning to implement the idea of multiplication operation in Math GASING by reviewing the literature, conducting observations in matriculation class, and designing a sequence of instructional learning for learns multiplication to reach the goals formulated in Table 1 (adapted from Surya (2011)). A set of activities for learning multiplication in Math GASING has been designed based learning trajectory and thinking process of students who hypothesized. The instruction set of activities has been divided into six activities that have been completed in 2 meetings with a variety of fun activities that make students happy in the learning process, and end with the evaluation process.

Table 1. Overview of the learning trajectory of multiplication.

Sequence of activities	Goals	Descriptions
Playing some games using Math GASING learning aids	Understanding the multiplication concept	Students learn multiplication starting from understanding the basic concept of addition using the term of "box", for example 2 × 3 means there are 2 boxes containing 3 things in that box, and so on.
Using some method to memorize this part more easy	Memorizing the multiplication of numbers 1, 10, 9, 2 and 5	Students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students are able to master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on.
Using the patterns of two same numbers multiplication	Memorizing the multiplication of two same numbers, such as 1 × 1,2 × 2,…,10 × 10	Students learn about the same numbers of multiplication, such as 1 × 1, 2 × 2, 3 × 3, ... , 10x10.
Using multiplication characteristics as a commutative operation	Memorizing the multiplication of numbers 3 and 4	Students learn multiplication for 3 and 4 using a commutative operation.
Reducing some part in multiplication that already mastered	Memorizing the multiplication of numbers 8, 7 and 6	Students learn multiplication for 8, 7, and 6. Teacher teaches student by using reduce some part in multiplication that already mastered before.
Evaluation	Determining the student ability in	Teacher evaluates the student about multiplication problem in the formal and informal

Commented [WU9]: Add the hypothetical path of students learning process (this can include alternative learning path

learning multiplication form.

Teaching Experiment

In teaching experiment, researcher tests the learning activities have been designed in the preliminary design stage. When the teacher models have started to see students do not get excited, then the teacher models provide educational games that make fun learning activities, because it is becoming one of characteristics in Math GASING learning process. There are five activities in this stage using whiteboard and presentation. First, teacher introduced the concept of multiplication by playing some games using Math GASING learning aids. In that games, students learn the concept of multiplication starting from understanding the basic concept of addition using the term of "box", for example 2 × 3 means there are 2 boxes containing 3 things in that box, and so on. Secondly, students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students are able to master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on. Different with the memorizing process of multiplication order in mathematics in general, students memorize the multiplication start from 1, 10, 9, 2 and 5. Furthermore, students learn about the same numbers of multiplication, such as 1 × 1, 2 × 2, 3 × 3, ... , 10x10. Fourthly, students learn multiplication for 3 and 4 using a commutative operation. Lastly, students learn multiplication for 8, 7, and 6. For this step, teacher teaches student by using reduce some part in multiplication that already mastered before. So, the students can memorize all multiplication concept from one to ten more easily. In the second meeting, teacher evaluates the student about multiplication problem in the formal and informal form. All activities can be shown in Figure 2.

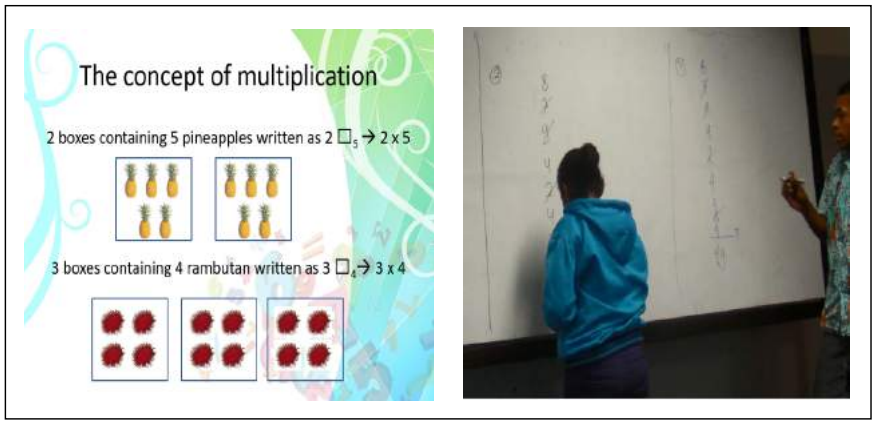


Figure 2. Several activities in teaching experiment phase.

Retrospective Analysis

Multiplication process in Math GASING is different with multiplication process in mathematics in general. As the result, all activities which have been designed can be used to answer the research question above. The activities are as follows:

Learning trajectory which has been modeled in Table 1 are the activities undertaken in this study to guide students mastered multiplication operation. So that, researcher designed an activity using Math GASING aids. The goal is that students are able to understand the concrete form of multiplication using the understanding of boxes and something in there. Student must understand that multiplication in the form of repeated addition. Teacher uses combination learning tools such as presentation and whiteboard to make learning process effective and efficiency (seen in Figure 2).

Furthermore, from these activities, teachers guide students toward the concept of multiplication as the form of repeated addition. Teacher uses several methods to memorize multiplication for one to ten more easily and meaningful. On the other hands, teacher make the order of memorize the multiplication with different order in mathematical in general. First, students memorize the multiplication for 1, 10, 9, 2 and 5. Next, students memorize the multiplication for the same numbers of multiplication, such as 1×1, 2×2, 3×3, ... , 10×10. After that, students memorize the multiplication for 3, 4, 8, 7, and 6. Finally, all students can memorize the multiplication form from one to ten and answer the teacher exercise directly using their mental arithmetic.

Based on all the activities above, it can be seen that the students have gone through the process of activity based on experience using their ability and math GASING learning aids, moving toward a more formal, the understanding of formal level from the critical point, and then reached into the formal level desired as the ultimate goal of this learning activities.

In the design of this study, researcher used the learning steps of multiplication in Math GASING as shown in Table 1. When the activity takes place, the dialogue is very good in the process of introducing the basic concepts of multiplication operations. In the dialogue, it seems that students feel learning multiplication in Math GASING looks so easy and so much fun. As a result, the learning process can guide students in understanding multiplication. It can also be seen from the student evaluation of learning multiplication process given by the teacher to evaluate student understanding (Figure 3). As a result, students seemed to be able to apply multiplication operation process in solving each problem is given in terms of evaluation. Therefore, it can be seen that learning multiplication operation in Math GASING can use to raise students' understanding in integer multiplication operations or in other words, the design of this study can be used as the starting point of learning multiplication.

Figure 3. Student evaluation process using student worksheet.

The retrospective analysis shown that one of the ways making student understanding in learning multiplication is make the learning process can be imaging for students. This results is in line with previous research stated in learning multiplication have several ways to master it (Caron, 2007; Ischebeck, et al. 2006). On the other hands, learning environment also can support the result of the learning process. Finally, all students can solve several problems and exercises regarding in multiplication operation.

CONCLUSION

Researcher can conclude that the learning of multiplication operation in Math GASING have a very important role as the starting point and improve students' motivation in learning multiplication. In addition, the activities that have been designed in such way those students find the concept of multiplication starting from understanding the concept of multiplication to mastering the multiplication concept of 1×1 to 10×10 which is the critical point in learning multiplication in Math GASING. The student can learn a variety of multiplication operation problems more easy after pass the critical point. Lastly, each student can do mental arithmetic for any given multiplication problem and resolve many multiplication questions very quickly and precisely where is both of this are one of assessment forms in Math GASING.

REFERENCES

Ahmad, N. S. B. (2010). Multiplication and the reference sum method. *Procedia-Social and Behavioral Sciences*, *8*, 72-78.

Akker, J.V.D., Gravemeijer, K., McKenney, S. and Nieveen, N. (2006). *Education Design Research*. London: Routledge Taylor and Francis Group.

Commented [WU10]: What is the contribution of this figure to this study?. The analysis of students' evaluation as well as the worksheet used within the learning process will be more appropriate

Commented [WU11]: Describe about the study and do not use researcher as subject

bin Syed Ismail, S. A. (2010). Multiplication with the Vedic method. *Procedia-Social and Behavioral Sciences*, *8*, 129-133.

Caron, T. A. (2007). Learning multiplication: The easy way. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *80*(6), 278-282.

Chang, K. E., Sung, Y. T., Chen, Y. L. and Huang, L. H. (2008). Learning multiplication through computer-assisted learning activities. *Computers in human behavior*, *24*(6), 2904-2916.

Chung, I. (2004). A comparative assessment of constructivist and traditionalist approaches to establishing mathematical connections in learning multiplication. *Education*, *125*(2), 271-279.

Drews, D., Dudgeon, J., Hansen, A., Lawton, F. and Surtees, L. (Eds.). (2005). *Children's Errors in Mathematics: Understanding Common Misconceptions in Primary Schools*. SAGE.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K. (2004). Local instructional theories as means of support for teacher in reform mathematics education. *Mathematical Thinking and Learning*, *6*(2), 105-128.

Ischebeck, A., Zamarian, L., Siedentopf, C., Koppelstätter, F., Benke, T., Felber, S. and Delazer, M. (2006). How specifically do we learn? Imaging the learning of multiplication and subtraction. *Neuroimage*, *30*(4), 1365-1375.

Kilian, L., Cahill, E., Ryan, C., Sutherland, D., and Taccetta, D. (1980). Errors that are common in multiplication. *The Arithmetic Teacher*, 22-25.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Prahmana, R. C. I. (2013). Designing division operation learning in the mathematics of GASING. *Proceeding in The First South East Asia Design/Development Research (SEA-DR) Conference 2013*, 391-398.

Prahmana, R. C. I. (2015). The hypothetical learning trajectory on addition in mathematics GASING. *Southeast Asian Mathematics Education Journal*, *5*(1), 49-61.

Prahmana, R. C. I. and Suwasti, P. (2014). Local instruction theory on division in mathematics GASING. *Journal on Mathematics Education*, *5*(1), 17-26.

Prahmana, R. C. I., Zulkardi, and Hartono, Y. (2012). Learning multiplication using Indonesian traditional game in third grade. *Journal on Mathematics Education*, *3*(2), 115-132.

Revina, S., Zulkardi, Darmawijoyo and Galen, F. (2011). Spatial visualization tasks to support students' spatial structuring in learning volume measurement. *Journal on Mathematics Education*, *2*(2), 127-146.

Reys, R. E., Suydam, M. N., Lindquist, M. M., and Smith, N. L. (1984). *Helping Children Learn Mathematics*. Boston: Allyn and Bacon.

Shanty, N.O. and Wijaya, S. (2012). Rectangular array model supporting students' spatial structuring in learning multiplication. *Journal on Mathematics Education*, *3*(2), 175-186.

Surya, Y. (2011). *Petunjuk Guru: Dasar-Dasar Pintar Berhitung GASING*. Tangerang: PT. Kandel.

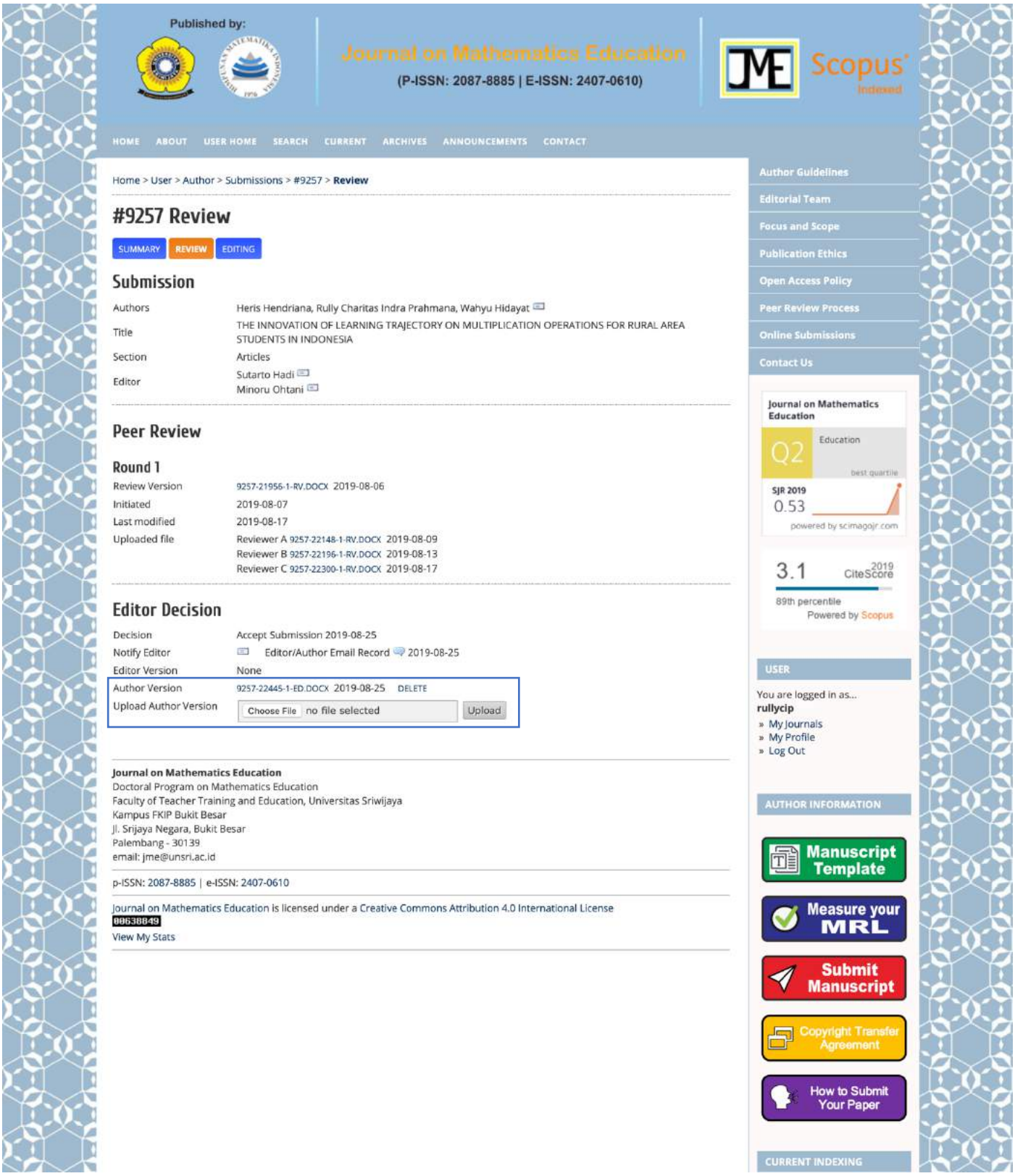
Surya, Y. and Moss, M. (2012). Mathematics education in rural Indonesia. *Proceeding in the 12th International Congress on Mathematics Education: Topic Study Group 30*. Korea National

University of Education, Seoul, Korea Selatan, 6223-6229.

Tanujaya, B., Prahmana, R.C.I. and Mumu, J. (2017). Mathematics instruction, problems, challenges and opportunities: A case study in Manokwari Regency, Indonesia. *World Transactions on Engineering and Technology Education*, *15*(3**)**, 287-291.

Unlu, M. and Ertekin, E. (2012). Why do pre-service teachers pose multiplication problems instead of division problems in fractions?. *Procedia-Social and Behavioral Sciences*, *46*, 490-494.

Hasil revisi di kirim pada tanggal 25 Agustus 2019 dengan perubahan signifikan pada konten isi dan sedikit pada judul.



Paper hasil revisi dengan perubahan sedikit pada judul seperti berikut,

“The Innovation of Learning Trajectory on Multiplication Operations for Rural Area Students in Indonesia”

[Paper ID: 9257]

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume xx, No. x, January xxxx, pp. x-xx

THE INNOVATION OF LEARNING TRAJECTORY ON MULTIPLICATION OPERATIONS FOR RURAL AREA STUDENTS IN INDONESIA

Abstract

The rural area's student difficulties in learning the concept of number operation had been documented by several studies, especially for the case of multiplication. The teacher typically introduces the multiplication concepts using the formula without involving the concept itself. Furthermore, this study aims to design learning trajectory on multiplication operations in the Mathematics of GASING (Math GASING) by focusing more on the concept itself than the formula and by starting from the informal to a formal level of teaching. Design research used as the research method to solve this problem consisting of three phases namely preliminary design, teaching experiment, and retrospective analysis. The research results show that the Math GASING has a real contribution for students to understanding and mastering in the concept of the multiplication operations. This research also explains the strategy and the model discovered by students in learning multiplication that the students used to help their initial understanding of the multiplication concept. Finally, the students were able to understand the concept of multiplication more easily and they showed interest in using this learning trajectory.

Keywords: multiplication, learning trajectory, design research

Abstrak

Kesulitan siswa di daerah pedesaan dalam mempelajari konsep operasi angka telah didokumentasikan oleh beberapa studi, terutama untuk kasus perkalian. Guru selalu memperkenalkan konsep perkalian menggunakan rumus tanpa melibatkan konsep itu sendiri. Selanjutnya, penelitian ini bertujuan untuk merancang lintasan pembelajaran pada operasi multiplikasi dalam Matematika GASING (Matematika GASING) lebih fokus pada konsep itu sendiri daripada rumus dan mulai dari tingkat informal ke formal. Desain penelitian digunakan sebagai metode penelitian untuk menyelesaikan masalah ini yang terdiri dari tiga fase yaitu desain pendahuluan, eksperimen mengajar, dan analisis retrospektif. Hasil penelitian menunjukkan bahwa GASING Matematika memiliki kontribusi nyata bagi siswa untuk memahami dan menguasai dalam konsep operasi multiplikasi. Penelitian ini juga menjelaskan strategi dan model yang ditemukan oleh siswa dalam mempelajari perkalian yang digunakan siswa untuk membantu pemahaman awal mereka tentang konsep perkalian. Akhirnya, siswa dapat memahami konsep perkalian dengan lebih mudah dan menyenangkan dengan menggunakan lintasan pembelajaran ini.

Kata kunci: perkalian, lintasan belajar, design research

How to Cite: . (2016). Instructions/Template for Preparing Manuscript for Journal on Mathematics Education. *Journal on Mathematics Education, x* (x), xx-xx.

Learning number operations is important for almost all topics in Mathematics involving numbers (Ahmad, 2010; Freudenthal, 1973; NCTM, 2000; Prahmana, et al., 2012). It is because learning number operations involves an understanding of symbols, notation, and reference number (or other forms to represent) (NCTM, 2000), and it also plays an important role in determining students' performance in other related Mathematics topics (Ahmad, 2010). Therefore, learning number operations would be one of the prior knowledges that

students must have in order to learn other topics in mathematics.

The concept of number operations, especially in multiplication, is one of the students' difficulties in understanding mathematics concepts (Ahmad, 2010; bin Syed Ismail, 2010; Drews et al., 2005; Kilian et al., 1980; Tanujaya et al., 2017; Unlu & Ertekin, 2012). Teachers usually teach number operations using symbolic form or something abstract (Unlu & Ertekin, 2012). As the result, students learn number operations more on the process of memorizing than understanding it (bin Syed Ismail, 2010), have several errors which reflected their lack of understanding of various mathematical concepts and also the long multiplication algorithm (Ahmad, 2010), and they also have poor understanding of the place value (tens and ones) concept in relation to multiplication (Drews et al., 2005; Kilian et al., 1980). The result of the previous research is in line with the preliminary classroom observation results of the rural area's students came from, namely Serui, Ambon, and Sorong Selatan, regarding to learning number operations conducted by researchers in pre-test. Teachers introduced the concept of multiplication using the formula without involving the concept itself (Prahmana & Suwasti, 2014).

Several studies indicated that constructivism approach can improve students' understanding in learn multiplication (Ahmad, 2010; Prahmana et al., 2012; Chang et al., 2008; Chung, 2004). The mathematics of GASING (Math GASING) method is one of learning method using constructivism approach (Prahmana & Suwasti, 2014; Prahmana, 2015; Surya, 2011; Surya & Moss, 2012; Shanty & Wijaya, 2012; Prahmana, 2013). This method has been applied to students from rural areas in Indonesia, which began with the introduction of number and number operations (Prahmana & Suwasti, 2014; Surya & Moss, 2012; Shanty & Wijaya, 2012; Prahmana, 2013). This situation underlies the researchers of this present study to try designing learning trajectory on number operations in Math GASING for rural area students derived from Serui, Sorong Selatan, and Ambon, Papua, Indonesia. Hence, the focus of this study is to describe the learning activities on students' performance to do multiplication in Math GASING.

Based on a few things mentioned above, the research question of this study is how the learning trajectory of multiplication in Math GASING is evolved the rural area's students' understanding in multiplication from informal to formal level. Hopefully, the learning trajectory has a role in learning multiplication that makes the learning more easy, joyful, and meaningful for the students.

In this study, the literature on Math GASING and number operations were learn to see the typical learning processes used by real situations (concrete) to abstract with the steps that

has been in the design.

Number Operations

Integer operations that we know are addition, subtraction, multiplication, and division, where the four operations have any connection with each other (Reys et al., 1984). The following four relations operation that has a relationship with each other, and students must understand the relationships (Reys et al., 1984).

Addition and subtraction are inverse operations. There are several ways to teach the concept of integer operations in the learning of mathematics. One of the ways to teach them is Math GASING.

5 + 8 = 13 ---------- 13 – 5 = 8

Multiplication and division are inverse operations

4 x 6 = 24 ---------- 24 : 4 = 6

Multiplication can be seen as a repeated addition

4 x 6 ---------- 6 + 6 + 6 + 6

Division can be seen as a repeated subtraction

24 : 6 ---------- 24 - 6 - 6 - 6 - 6

Mathematics GASING

Surya and Moss (2012) stated that GASING has several basic premises. Firstly, there is no such thing as a child that cannot learn mathematics, only children that have not had the opportunity to learn mathematics in a fun and meaningful way. Secondly, mathematics is based on patterns and these patterns make math understandable. Thirdly, a visual context to mathematical concepts should come before the symbolic notation. Lastly, mathematics is not memorization, but knowing basic facts comes easily with a conceptual and visual understanding. Memorization of basic mathematics facts is easy if it is based on conceptual learning and visual representations.

The learning process make students' learning easy, fun, and enjoyable in Math GASING (Shanty & Wijaya, 2012). Easy means the students are introduced to mathematical logic that is easy to learn and to remember. Exciting means the students have motivation which comes from by them to learn mathematics (intrinsic factor). Fun is more in the direction of outside influences such as visual aids and games (extrinsic factor). On the other hand, Prahmana (2013) had been conducted research for division topic in Math GASING, where the learning process begins with the activities share sweets fairly, then move into the

process of how each student gets distributed sweets after a fair amount of candy (concrete), ranging from division without remainder to division with remainder, and ends with the completion of division operation in Math GASING (abstract). Math GASING shows how to change a concrete sample into an abstract symbol so the students will be able to read a mathematical pattern, thus gain the conclusion by themselves.

Math GASING as one of innovations in learning mathematics offers critical point in its learning process. There is a critical point that we must pass that is called GASING's critical point when studying a topic in Math GASING. After reaching this critical point, students will not be difficult anymore to work on the problems in that topic (Surya, 2011). The critical point in learning multiplication is that students must master the multiplication concept of 1×1 to 10×10. The student can learn a variety of multiplication operation problems more easy after pass the critical point.

Hypothetical Learning Trajectory

Hypothetical Learning Trajectory (HLT) is proposed as a term to identify and describe relevant aspects associated with a mathematics lesson plan, including: A description of the students' mathematical goals, the mathematical activities (including the tasks or problems, that students will work on to achieve the goals), and a hypothetical path that describes the students learning process (Revina et al., 2011). The HLT in this study had several learning goals expected to be reached by the students during one phase.

METHOD

Design research is used as the research method of this study. This method is an appropriate way to answer the research questions and achieve the research objectives (Prahmana et al., 2012; Akker et al., 2006; Gravemeijer, 2004). Design research has five characteristics. These are interventionist nature, process oriented, reflective component, cyclic character, and theory oriented (Akker et al., 2006). There are two important aspect related to design research namely Hypothetical Learning Trajectory (HLT) and Local Instruction Theory (LIT). The learning activities as learning paths taken by students in their learning activities must have HLT and LIT.

The HLT consists of three components (Gravemeijer, 2004). First is the purpose of mathematics teaching for the students. Second is learning activity and devices or media used in the learning process. Lastly is a conjecture of understanding the process of learning how to learn and strategies students that arise and thrive when learning activities done in class. There

are three phases of design research namely preliminary design, teaching experiment, and retrospective analysis seen in Figure 1.

The research data is from multiple sources of data in order to get a visualization of the students' mastery of basic concepts of multiplication operations, such as documentation (photo), video recording, and the students' worksheet and observation sheet. Next, the data analysed retrospectively with HLT as a guide. This present study was conducted and completed in 2 days with the subjects of 11 matriculation prospective teachers students at one of institute in Tangerang from Ambon, Serui, Yapen, and South Sorong, Papua, and also a teaching assistant who acted as a model teacher.

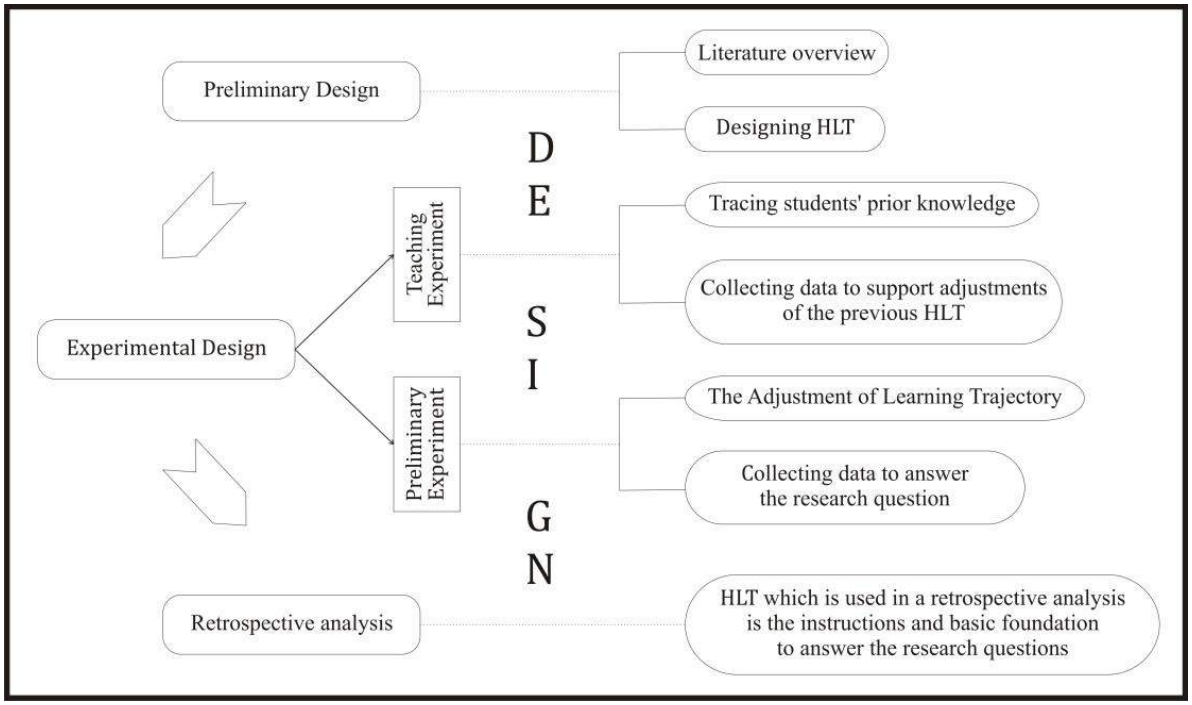


Figure 1. Phase of the design research (Prahmana et al., 2012).

RESULTS AND DISCUSSIONS

The learning activities start from making the same perceptions of the meaning of boxes containing something in that boxes to introduce the concept of multiplication. Furthermore, the students were trained to memorize the multiplication for 1 to 10 using several methods. Lastly, the teacher gave the evaluation to investigate the students' understanding of multiplication using mental arithmetic activity as one of assessment process in this learning activities and exercise by using the student evaluation sheet. As a result, the students were able to master the multiplication operation in Math GASING as shown from the results of the final evaluation and importantly, the learning of Math GASING could be detected from the comments from the students who wished to abandon the old way

of learning mathematics. The results of this study indicated that learning design of multiplication operation in Math GASING have a very important role as the starting point and improvement in the students' motivation in learning. For more details, the researchers will discuss the results of this study, which is divided into three stages that are called preliminary design, teaching experiments, and retrospective analysis.

Preliminary Design

At this stage, the researchers began to implement the idea of multiplication operation in Math GASING by reviewing the literature, conducting observations in matriculation class, and designing a sequence of instructional learning for the learning of multiplication to reach the goals formulated in Table 1 (adapted from Surya (2011)). A set of activities for learning multiplication in Math GASING has been designed based learning trajectory and thinking process of students who hypothesized. The instruction set of activities has been divided into six activities that have been completed in two meetings with a variety of fun activities that made the students interested and engaged in the learning process, and end with the evaluation process.

Table 1. Overview of the learning trajectory of multiplication (adapted from Surya (2011).

Sequence of activities	Goals	Descriptions
Playing some games using Math GASING learning aids	Understanding the multiplication concept	Students learn multiplication starting from understanding the basic concept of addition using the term of "box", for example 2 × 3 means there are 2 boxes containing 3 things in that box, and so on.
Using some method to memorize this part more easy	Memorizing the multiplication of numbers 1, 10, 9, 2 and 5	Students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students are able to master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on.
Using the patterns of two same numbers multiplication	Memorizing the multiplication of two same numbers, such as 1 × 1,2 × 2,…,10 × 10	Students learn about the same numbers of multiplication, such as 1 × 1, 2 × 2, 3 × 3, ... , 10x10.
Using multiplication characteristics as a commutative operation	Memorizing the multiplication of numbers 3 and 4	Students learn multiplication for 3 and 4 using a commutative operation.

Reducing some part in multiplication that already mastered	Memorizing the multiplication of numbers 8, 7 and 6	Students learn multiplication for 8, 7, and 6. Teacher teaches student by using reduce some part in multiplication that already mastered before.
Evaluation	Determining the student ability in learning multiplication	Teacher evaluates the student about multiplication problem in the formal and informal form.

Teaching Experiment

In teaching experiment, researchers test the learning activities that had been designed in the preliminary design stage. When the teacher models have started to see students do not get excited, then the teacher models provide educational games that make fun learning activities, because it is becoming one of characteristics in Math GASING learning process. There are five activities in this stage using whiteboard and presentation. First, teacher introduced the concept of multiplication by playing some games using Math GASING learning aids. In that games, students learn the concept of multiplication starting from understanding the basic concept of addition using the term of "box", for example 2 × 3 means there are 2 boxes containing 3 things in that box, and so on. Secondly, students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students are able to master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on. Different with the memorizing process of multiplication order in mathematics in general, students memorize the multiplication start from 1, 10, 9, 2 and 5. Furthermore, students learn about the same numbers of multiplication, such as 1 × 1, 2 × 2, 3 × 3, ... , 10x10. Fourthly, students learn multiplication for 3 and 4 using a commutative operation. Lastly, students learn multiplication for 8, 7, and 6. For this step, teacher teaches student by using reduce some part in multiplication that already mastered before. So, the students can memorize all multiplication concept from one to ten more easily. In the second meeting, teacher evaluates the student about multiplication problem in the formal and informal form. All activities can be shown in Figure 2.

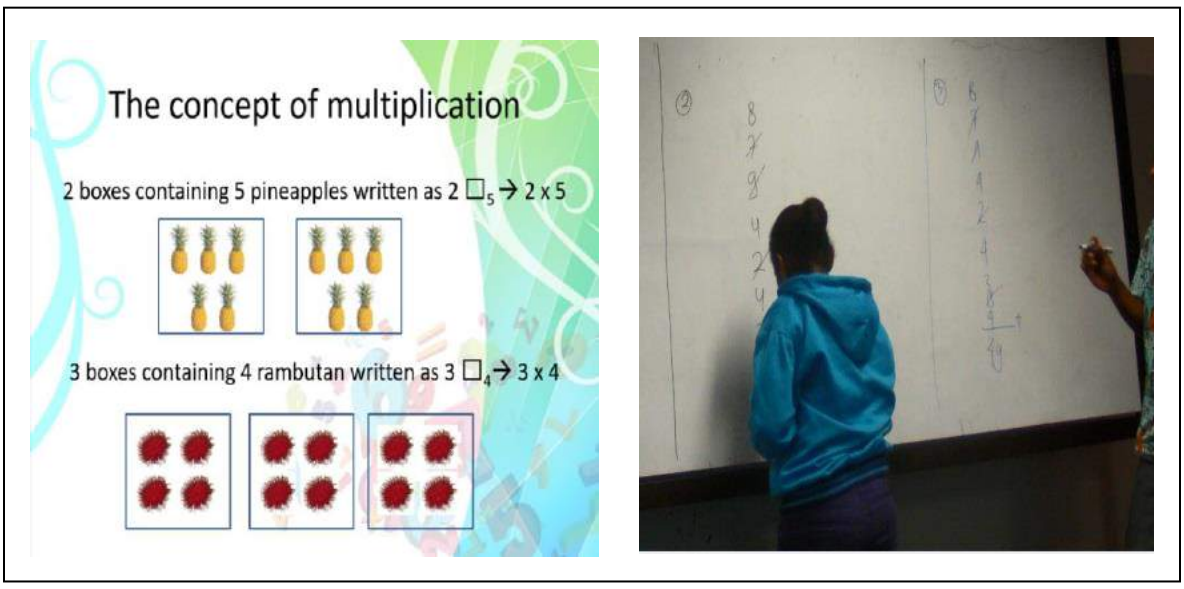


Figure 2. Several activities in teaching experiment phase.

Retrospective Analysis

Multiplication process in Math GASING is different with multiplication process in mathematics in general. As a result, all activities, which have been designed, can be used to answer the research question above. The activities are given as follows.

Learning trajectory that has been modeled in Table 1 are the activities undertaken in this study to guide students in mastering the multiplication operations. So that, researcher designed an activity using Math GASING aids. The goal is that students are able to understand the concrete form of multiplication using the understanding of boxes and something in there. Student must understand that multiplication in the form of repeated addition. The teacher used combination learning tools such as presentation and whiteboard to make learning process effective and efficiency (seen in Figure 2).

Furthermore, from these activities, teachers guided students toward the concept of multiplication as the form of repeated addition. Teacher uses several methods to memorize multiplication for one to ten more easily and meaningful. On the other hands, teacher make the order of memorize the multiplication with different order in mathematical in general. First, students memorize the multiplication for 1, 10, 9, 2 and 5. Next, students memorize the multiplication for the same numbers of multiplication, such as 1×1, 2×2, 3×3, ... , 10×10. After that, students memorize the multiplication for 3, 4, 8, 7, and 6. Finally, all students can memorize the multiplication form from one to ten and answer the teacher exercise directly using their mental arithmetic.

Based on all the activities above, it can be seen that the students have gone through the process of activity based on experience using their ability and math GASING learning aids, moving toward a more formal, the understanding of formal level from the critical point, and then reached into the formal level desired as the ultimate goal of this learning activities.

In the design of this study, the researchers used the learning steps of multiplication in Math GASING as shown in Table 1. When the activity takes place, the dialogue is very good in the process of introducing the basic concepts of multiplication operations. In the dialogue, it seems that students feel learning multiplication in Math GASING looks so easy and so much fun. As a result, the learning process can guide students in understanding multiplication. It can also be seen from the student evaluation of learning multiplication process given by the teacher to evaluate student understanding (Figure 3). As a result, students seemed to be able to apply multiplication operation process in solving each problem is given in terms of evaluation. Therefore, it can be seen that learning multiplication operation in Math GASING can use to raise students' understanding in integer multiplication

operations or in other words, the design of this study can be used as the starting point of learning multiplication.

Figure 3. Student evaluation process using student worksheet.

The retrospective analysis shown that one of the ways making student understanding in learning multiplication is make the learning process can be imaging for students. This results is in line with previous research stated in learning multiplication have several ways to master it (Caron, 2007; Ischebeck, et al. 2006). On the other hand, learning environment also can support the result of the learning process. Finally, all students can solve several problems and exercises regarding in multiplication operation.

CONCLUSION

The researchers can conclude that the learning of multiplication operation in Math GASING have a very important role as the starting point and improve students' motivation in learning multiplication. In addition, the activities that have been designed in such way those students find the concept of multiplication starting from understanding the concept of multiplication to mastering the multiplication concept of 1×1 to 10×10 which is the critical point in learning multiplication in Math GASING. The student can learn a variety of multiplication operation problems more easily after passing the critical point. Lastly, each student can do mental arithmetic for any given multiplication problem and resolve many multiplication questions very quickly and precisely where is both of this are one of assessment forms in Math GASING.

REFERENCES

Ahmad, N. S. B. (2010). Multiplication and the reference sum method. *Procedia-Social and Behavioral Sciences, 8*, 72-78.

Akker, J.V.D., Gravemeijer, K., McKenney, S., & Nieveen, N. (2006). *Education Design Research*. London: Routledge Taylor and Francis Group.

bin Syed Ismail, S. A. (2010). Multiplication with the Vedic method. *Procedia-Social and Behavioral Sciences, 8*, 129-133.

Caron, T. A. (2007). Learning multiplication: The easy way. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *80*(6), 278-282.

Chang, K. E., Sung, Y. T., Chen, Y. L., & Huang, L. H. (2008). Learning multiplication through computer-assisted learning activities. *Computers in Human Behavior*, *24*(6), 2904-2916.

Chung, I. (2004). A comparative assessment of constructivist and traditionalist approaches to establishing mathematical connections in learning multiplication. *Education*, *125*(2), 271-279.

Drews, D., Dudgeon, J., Hansen, A., Lawton, F., & Surtees, L. (Eds.). (2005). *Children's Errors in Mathematics: Understanding Common Misconceptions in Primary Schools*. SAGE.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K. (2004). Local instructional theories as means of support for teacher in reform mathematics education. *Mathematical Thinking and Learning*, *6*(2), 105-128.

Ischebeck, A., Zamarian, L., Siedentopf, C., Koppelstätter, F., Benke, T., Felber, S., & Delazer, M. (2006). How specifically do we learn? Imaging the learning of multiplication and subtraction. *Neuroimage, 30*(4), 1365-1375.

Kilian, L., Cahill, E., Ryan, C., Sutherland, D., & Taccetta, D. (1980). Errors that are common in multiplication. *The Arithmetic Teacher*, 22-25.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Prahmana, R. C. I. (2013). Designing division operation learning in the mathematics of GASING. *Proceeding in The First South East Asia Design/Development Research (SEA-DR) Conference 2013*, 391-398.

Prahmana, R. C. I. (2015). The hypothetical learning trajectory on addition in mathematics GASING. *Southeast Asian Mathematics Education Journal*, *5*(1), 49-61.

Prahmana, R. C. I., & Suwasti, P. (2014). Local instruction theory on division in mathematics GASING. *Journal on Mathematics Education*, *5*(1), 17-26.

Prahmana, R. C. I., Zulkardi, & Hartono, Y. (2012). Learning multiplication using Indonesian traditional game in third grade. *Journal on Mathematics Education*, *3*(2), 115-132.

Revina, S., Zulkardi, Darmawijoyo, & Galen, F. (2011). Spatial visualization tasks to support students' spatial structuring in learning volume measurement. *Journal on Mathematics Education*, *2*(2), 127-146.

Reys, R. E., Suydam, M. N., Lindquist, M. M., & Smith, N. L. (1984). *Helping Children Learn Mathematics*. Boston: Allyn and Bacon.

Shanty, N. O. , & Wijaya, S. (2012). Rectangular array model supporting students' spatial structuring in learning multiplication. *Journal on Mathematics Education*, *3*(2), 175-186.

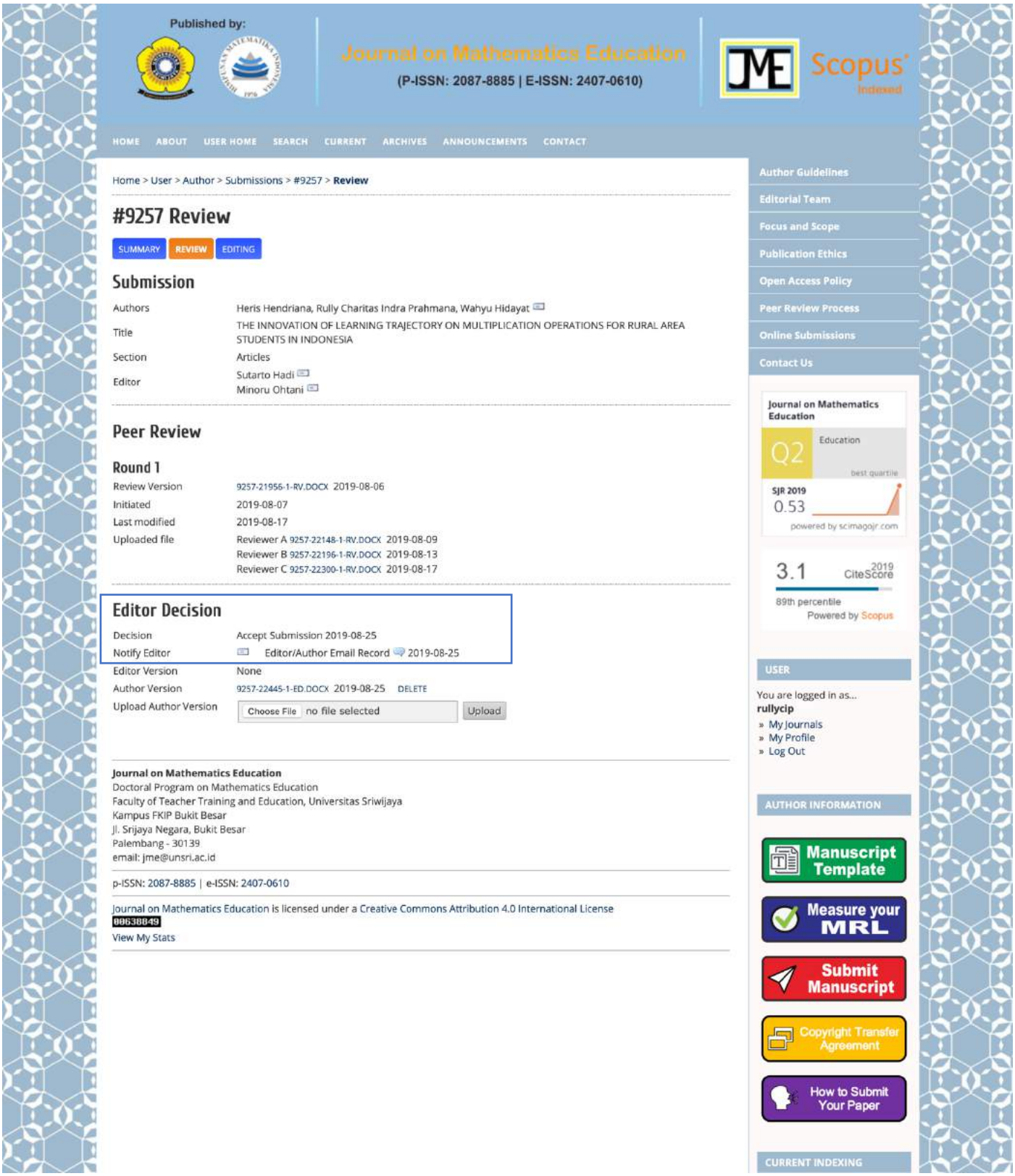
Surya, Y. (2011). *Petunjuk Guru: Dasar-Dasar Pintar Berhitung GASING*. Tangerang: PT. Kandel.

Surya, Y., & Moss, M. (2012). Mathematics education in rural Indonesia. *Proceeding in the 12th International Congress on Mathematics Education: Topic Study Group 30*. Korea National University of Education, Seoul, South Korea, 6223-6229.

Tanujaya, B., Prahmana, R. C. I., & Mumu, J. (2017). Mathematics instruction, problems, challenges and opportunities: A case study in Manokwari Regency, Indonesia. *World Transactions on Engineering and Technology Education*, *15*(3**)**, 287-291.

Unlu, M., & Ertekin, E. (2012). Why do pre-service teachers pose multiplication problems instead of division problems in fractions? *Procedia-Social and Behavioral Sciences*, *46*, 490-494.

Keputusan diterima pasca revisi pada tanggal 25 Agustus 2019.



Published by:

Journal on Mathematics Education
(P-ISSN: 2087-8885 | E-ISSN: 2407-0610)

Scopus Indexed

HOME ABOUT USER HOME SEARCH CURRENT ARCHIVES ANNOUNCEMENTS CONTACT

Home > User > Author > Submissions > #9257 > Review

#9257 Review

SUMMARY REVIEW EDITING

Submission

Authors	Heris Hendriana, Rully Charitas Indra Prahmana, Wahyu Hidayat
Title	THE INNOVATION OF LEARNING TRAJECTORY ON MULTIPLICATION OPERATIONS FOR RURAL AREA STUDENTS IN INDONESIA
Section	Articles
Editor	Sutarto Hadi Minoru Ohtani

Peer Review

Round 1

Review Version	9257-21956-1-RV.DOCX 2019-08-06
Initiated	2019-08-07
Last modified	2019-08-17
Uploaded file	Reviewer A 9257-22148-1-RV.DOCX 2019-08-09 Reviewer B 9257-22196-1-RV.DOCX 2019-08-13 Reviewer C 9257-22300-1-RV.DOCX 2019-08-17

Editor Decision

Decision	Accept Submission 2019-08-25
Notify Editor	Editor/Author Email Record 2019-08-25
Editor Version	None
Author Version	9257-22445-1-ED.DOCX 2019-08-25 DELETE
Upload Author Version	Choose File no file selected Upload

Journal on Mathematics Education
Doctoral Program on Mathematics Education
Faculty of Teacher Training and Education, Universitas Sriwijaya
Kampus FKIP Bukit Besar
Jl. Srijaya Negara, Bukit Besar
Palembang - 30139
email: jme@unsri.ac.id

p-ISSN: 2087-8885 | e-ISSN: 2407-0610

Author Guidelines
Editorial Team
Focus and Scope
Publication Ethics
Open Access Policy
Peer Review Process
Online Submissions
Contact Us

Journal on Mathematics Education
Q2 Education best quartile
SJR 2019 0.53 powered by scimagojr.com
3.1 CiteScore 2019
89th percentile
Powered by Scopus

USER
You are logged in as...
rullycip
» My Journals
» My Profile
» Log Out

AUTHOR INFORMATION
Manuscript Template
Measure your MRL
Submit Manuscript
Copyright Transfer Agreement
How to Submit Your Paper

CURRENT INDEXING

Artikel terbit di website Journal on Mathematics Education pada tanggal 28 Agustus 2019, dengan URL artikel sebagai berikut https://ejournal.unsri.ac.id/index.php/jme/article/view/9257

Published by:

Journal on Mathematics Education

(P-ISSN: 2087-8885 | E-ISSN: 2407-0610)

Scopus Indexed

THE INNOVATION OF LEARNING TRAJECTORY ON MULTIPLICATION OPERATIONS FOR RURAL AREA STUDENTS IN INDONESIA

Heris Hendriana, Rully Charitas Indra Prahmana, Wahyu Hidayat

Abstract

The rural area's student difficulties in learning the concept of number operation had been documented by several studies, especially for the case of multiplication. The teacher typically introduces the multiplication concepts using the formula without involving the concept itself. Furthermore, this study aims to design learning trajectory on multiplication operations in the Mathematics of GASING (Math GASING) by focusing more on the concept itself than the formula and by starting from the informal to a formal level of teaching. Design research used as the research method to solve this problem consisting of three phases, namely preliminary design, teaching experiment, and retrospective analysis. The research results show that the Math GASING has a real contribution for students to understanding and mastering in the concept of the multiplication operations. This research also explains the strategy and the model discovered by students in learning multiplication that the students used as a basic concept of multiplication. Finally, the students were able to understand the concept of multiplication more easily, and they showed interest in using this learning trajectory.

Keywords

multiplication; learning trajectory; design research; rural area

Full Text:

PDF

References

Ahmad, N.S.B. (2010). Multiplication and the reference sum method. Procedia-Social and Behavioral Sciences, 8, 72-78. https://doi.org/10.1016/j.sbspro.2010.12.010.

Akker, J.V.D., Gravemeijer, K., McKenney, S., & Nieveen, N. (2006). Education Design Research. London: Routledge Taylor and Francis Group.

bin Syed Ismail, S.A. (2010). Multiplication with the Vedic method. Procedia-Social and Behavioral Sciences, 8, 129-133. https://doi.org/10.1016/j.sbspro.2010.12.018.

Caron, T.A. (2007). Learning multiplication: The easy way. The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 80(6), 278-282. https://doi.org/10.3200/TCHS.80.6.278-282.

Chang, K. E., Sung, Y.T., Chen, Y.L., & Huang, L.H. (2008). Learning multiplication through computer-assisted learning activities. Computers in Hhuman Bbehavior, 24(6), 2904-2916. https://doi.org/10.1016/j.chb.2008.04.015.

Chung, I. (2004). A comparative assessment of constructivist and traditionalist approaches to establishing mathematical connections in learning multiplication. Education, 125(2), 271-279.

Drews, D., Dudgeon, J., Hansen, A., Lawton, F., & Surtees, L. (Eds.). (2005). Children's Errors in Mathematics: Understanding Common Misconceptions in Primary Schools. SAGE.

Freudenthal, H. (1973). Mathematics as an Educational Task. Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K. (2004). Local instructional theories as means of support for teacher in reform mathematics education. Mathematical Thinking and Learning, 6(2), 105-128. https://doi.org/10.1207/s15327833mtl0602_3.

Ischebeck, A., Zamarian, L., Siedentopf, C., Koppelstätter, F., Benke, T., Felber, S., & Delazer, M. (2006). How specifically do we learn? Imaging the learning of multiplication and subtraction. Neuroimage, 30(4), 1365-1375. https://doi.org/10.1016/j.neuroimage.2005.11.016.

Kilian, L., Cahill, E., Ryan, C., Sutherland, D., & Taccetta, D. (1980). Errors that are common in multiplication. The Arithmetic Teacher, 22-25.Kilian, L., Cahill, E., Ryan, C., Sutherland, D., & Taccetta, D. (1980). Errors that are common in multiplication. The Arithmetic Teacher, 27(5), 22-25.

National Council of Teachers of Mathematics (NCTM). (2000). Principles and Standards for School Mathematics. Reston, VA: National

Artikel terbit di Journal on Mathematics Education Vol. 10 No. 3, 397-408
[DOI: https://doi.org/10.22342/jme.10.3.9257.397-408]

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 3, September 2019, pp. 397-408

THE INNOVATION OF LEARNING TRAJECTORY ON MULTIPLICATION OPERATIONS FOR RURAL AREA STUDENTS IN INDONESIA

Heris Hendriana[1], Rully Charitas Indra Prahmana[2], Wahyu Hidayat[1]

[1]IKIP Siliwangi, Jalan Terusan Jenderal Sudirman, Cimahi 40526 Cimahi, Indonesia
[2]Universitas Ahmad Dahlan, Jl. Pramuka 42, Pandeyan, Umbulharjo 55161, Yogyakarta, Indonesia
Email: rully.indra@mpmat.uad.ac.id

Abstract

The rural area's student difficulties in learning the concept of number operation had been documented by several studies, especially for the case of multiplication. The teacher typically introduces the multiplication concepts using the formula without involving the concept itself. Furthermore, this study aims to design learning trajectory on multiplication operations in the Mathematics of GASING (Math GASING) by focusing more on the concept itself than the formula and by starting from the informal to a formal level of teaching. Design research used as the research method to solve this problem consisting of three phases, namely preliminary design, teaching experiment, and retrospective analysis. The research results show that the Math GASING has a real contribution for students to understanding and mastering in the concept of the multiplication operations. This research also explains the strategy and the model discovered by students in learning multiplication that the students used as a basic concept of multiplication. Finally, the students were able to understand the concept of multiplication more easily, and they showed interest in using this learning trajectory.

Keywords: multiplication, learning trajectory, design research, rural area

Abstrak

Kesulitan siswa di daerah pedesaan dalam mempelajari konsep operasi angka telah didokumentasikan oleh beberapa studi, terutama untuk kasus perkalian. Guru selalu memperkenalkan konsep perkalian menggunakan rumus tanpa melibatkan konsep itu sendiri. Selanjutnya, penelitian ini bertujuan untuk merancang lintasan pembelajaran pada operasi multiplikasi dalam Matematika GASING (Matematika GASING) lebih fokus pada konsep itu sendiri daripada rumus dan mulai dari tingkat informal ke formal. Desain penelitian digunakan sebagai metode penelitian untuk menyelesaikan masalah ini yang terdiri dari tiga fase yaitu desain pendahuluan, eksperimen mengajar, dan analisis retrospektif. Hasil penelitian menunjukkan bahwa GASING Matematika memiliki kontribusi nyata bagi siswa untuk memahami dan menguasai dalam konsep operasi multiplikasi. Penelitian ini juga menjelaskan strategi dan model yang ditemukan oleh siswa dalam mempelajari perkalian yang digunakan siswa untuk membantu pemahaman awal mereka tentang konsep perkalian. Akhirnya, siswa dapat memahami konsep perkalian dengan lebih mudah dan menyenangkan dengan menggunakan lintasan pembelajaran ini.

Kata kunci: perkalian, lintasan belajar, design research, daerah pedesaan

How to Cite: Hendriana, H., Prahmana, R.C.I., & Hidayat, W. (2019). The Innovation of Learning Trajectory on Multiplication Operations for Rural Area Students in Indonesia. *Journal on Mathematics Education, 10*(3), 397-408. https://doi.org/10.22342/jme.10.3.9257.397-408.

Learning number operations is important for almost all topics in Mathematics involving numbers (Ahmad, 2010; Freudenthal, 1973; NCTM, 2000; Prahmana, et al. 2012). It is because learning number operations involves an understanding of symbols, notation, and reference number (or other forms to represent) (NCTM, 2000), and it also plays an important role in determining students' performance in other related Mathematics topics (Ahmad, 2010). Therefore, learning number operations would be one of the prior knowledge that students must have to learn other topics in

mathematics.

The concept of number operations, especially in multiplication, is one of the students' difficulties in understanding mathematics concepts (Ahmad, 2010; bin Syed Ismail, 2010; Drews, et al. 2005; Kilian, et al. 1980; Tanujaya, et al. 2017; Unlu & Ertekin, 2012). Teachers usually teach multiplication operations using symbolic form or something abstract (Unlu & Ertekin, 2012). As a result, students learn multiplication operations more on the process of memorizing than understanding it (bin Syed Ismail, 2010). They also have several errors which reflected their lack of understanding of various mathematical concepts and also the long multiplication algorithm (Ahmad, 2010). On the other hands, they also have a poor understanding of the place value (tens and ones) concept concerning multiplication (Drews, et al. 2005; Kilian, et al. 1980).

The result of the previous research explored about number operations is in line with the preliminary classroom observation results of the rural area's students came from, namely Serui, Ambon, and Sorong Selatan. Teachers introduced the concept of division using the formula without involving the concept itself (Prahmana & Suwasti, 2014). Therefore, this research focuses on multiplication operation as one of the concept of number operation that students must be mastered to support their knowledge in learning another mathematics subject.

Several studies indicated that constructivism approach could improve students' understanding of learning multiplication (Ahmad, 2010; Prahmana, et al. 2012; Chang, et al. 2008; Chung, 2004). The mathematics of GASING (Math GASING) method is one of learning method using constructivism approach (Prahmana & Suwasti, 2014; Prahmana, 2015; Surya & Moss, 2012; Shanty & Wijaya, 2012; Prahmana, 2013). This method has been applied to students from rural areas in Indonesia starting from the introduction of integer number and number operations (Prahmana & Suwasti, 2014; Surya & Moss, 2012; Shanty & Wijaya, 2012; Prahmana, 2013). This situation underlies the researchers of this present study to try designing learning trajectory on number operations especially for multiplication operation in Math GASING for rural area students derived from Serui, Sorong Selatan, and Ambon, Indonesia. Therefore, the focus of this study is to describe the learning activities on students' performance to do multiplication in Math GASING. It is also because several researcher stated that Math GASING is the suitable method to use in teaching mathematics, especially number operation, more easy, fun, and meaningful.

Finally, the research question of this study is how the learning trajectory of multiplication in Math GASING is evolved the rural area's students' understanding in multiplication from informal to a formal level. Hopefully, the learning trajectory has a role in learning multiplication that makes the learning more easy, joyful, and meaningful for the students.

In this research, the literature on Math GASING and number operations are studied as basic knowledge to design sequential activities that will be passed by students ranging from concrete situations to abstract levels. All literature will be explained further in the next section.

Number Operations

Integer operations that we know are addition, subtraction, multiplication, and division, where the four operations have any connection with each other (Reys, et al. 1998). The following four relations operation that has a relationship with each other, and students must understand the relationships. Addition and subtraction are inverse operations. There are several ways to teach the concept of integer operations in the learning of mathematics. One of the ways to teach them is Math GASING, such as:

1. Multiplication and division are inverse operations
   4 x 6 = 24 ---------- 24 : 4 = 6
2. Multiplication can be seen as a repeated addition
   4 x 6 ---------- 6 + 6 + 6 + 6
3. Division can be seen as a repeated subtraction
   24 : 6 ---------- 24 - 6 - 6 - 6 - 6

Mathematics GASING

Surya and Moss (2012) stated that GASING has several basic premises. Firstly, there is no such thing as a child that cannot learn mathematics, only children that have not had the opportunity to learn mathematics in a fun and meaningful way. Secondly, mathematics is based on patterns, and these patterns make math understandable. Thirdly, a visual context to mathematical concepts should come before the symbolic notation. Lastly, mathematics is not memorization, but knowing basic facts comes easily with a conceptual and visual understanding. Memorization of basic math facts is easy if it is based on conceptual learning and visual representations.

The learning process makes students' learning easy (*GAmpang*), fun (*ASyIk*), and enjoyable (*menyenaNGkan*) in Math GASING (Shanty & Wijaya, 2012). Easy means the students are introduced to mathematical logic that is easy to learn and to remember — exciting means the students have motivation which comes from by them to learn mathematics (intrinsic factor). Fun is more in the direction of outside influences such as visual aids and games (extrinsic factor). On the other hand, Prahmana (2013) stated that Math GASING shows how to change a concrete sample into an abstract symbol so the students will be able to read a mathematical pattern, thus gain the conclusion by themselves.

Math GASING, as one of the innovations in learning mathematics, offers critical point in its learning process. The critical point of GASING means the condition that students must pass during the learning process and studying a topic in Math GASING. After reaching this critical point, students will not be difficult anymore to work on the problems in that topic (Surya & Moss, 2012). The critical point in learning multiplication is that students must master the multiplication concept of 1×1 to 10×10. Students could learn various problems of multiplication operations more easily after passing a critical point.

This research uses Math GASING to describe the learning outcomes of rural area's student in learning multiplication as a repeated addition and see student responses. Researchers conducted research on rural area's students because students have experience difficulties in multiplication operations based on the pre-evaluation results. In addition, students are less focused, less accurate in counting, and easy to forget. Therefore, this study could be solved the students' mathematical problem by using Math GASING.

Hypothetical Learning Trajectory

Hypothetical Learning Trajectory (HLT) is proposed as a term to identify and describe relevant aspects associated with a mathematics lesson plan, including: A description of the students' mathematical goals, the mathematical activities (including the tasks or problems, that students will work on to achieve the goals), and a hypothetical path that describes the students learning process (Revina, et al. 2011). Furthermore, Prahmana (2017) stated that HLT is a hypothesis or prediction of how students' thinking and understanding develop in a learning activity. The HLT in this study had several learning goals expected to be reached by the students during one phase.

METHOD

Design research is used as the research method of this study. Design research consists of five characteristics, such as interventionist nature, process-oriented, reflective component, cyclic character, and theory-oriented (Akker, et al. 2006; Gravemeijer, 2004; Prahmana, 2017). There are two important aspects related to design research namely Hypothetical Learning Trajectory (HLT) and Local Instruction Theory (LIT). The learning activities as learning trajectory taken by students in their learning activities must have HLT and LIT.

The HLT consists of three components (Gravemeijer, 2004). The first component is the purpose of mathematics teaching for the students. Secondly, it is the sequence activity that students must do during the learning process. Lastly, the conjecture is the various answers, strategies, and models that researcher expected from student understanding that emerge and develop when learning activities are carried out in class. Furthermore, there are three phases of design research, such as preliminary design, teaching experiment, and retrospective analysis that can be seen in Figure 1.

The research data came from various data sources. All data sources used aim is to get a visualization of mastery of the basic concepts of student multiplication operations. There are documentation (photos), video, student worksheets, and observation sheets. Furthermore, the data were analyzed retrospectively with HLT as a guide. This research was conducted and completed in 2 days. The research subjects are 11 matriculation teacher candidates at one of the College of Teacher Training and Education in Tangerang. All research subjects came from rural areas in Indonesia, such as Yapen, Ambon, South Sorong, Serui, and also a teacher model.

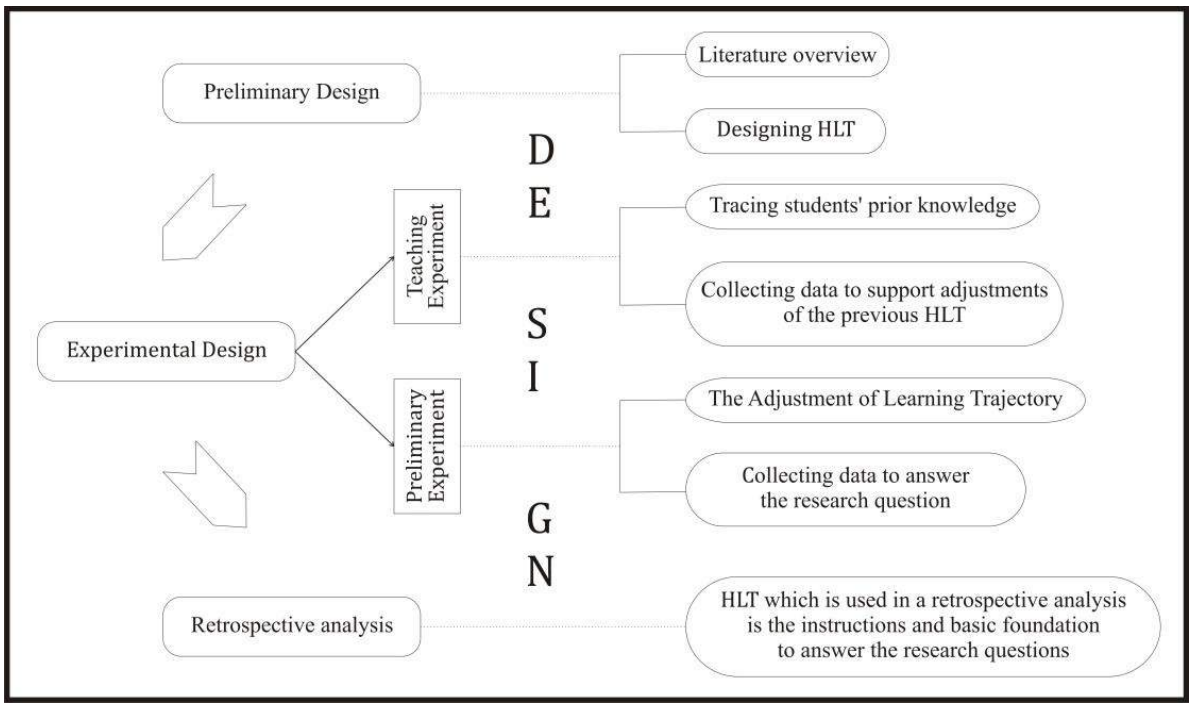


Figure 1. The phase of the design research (Prahmana, et al. 2012).

RESULTS AND DISCUSSIONS

The learning activities start from making the same perceptions of the meaning of boxes containing something in that boxes to introduce the concept of multiplication. Furthermore, the students were trained to memorize the multiplication for 1 to 10 using several methods. Lastly, the teacher provides an evaluation to study students' understanding of multiplication by using mental arithmetic activities namely *mencongak* as one of the evaluation processes in these learning activities and exercises using student worksheet and also evaluation sheets. The results show that students master the multiplication operations based on the final evaluation results. On the other hands, the important results is student would like to leave the old way in learning mathematics and change to the Math GASING way. Furthermore, another results indicate that the design of multiplication learning operations in Math GASING has a crucial role as a starting point and increases student motivation in learning. The details would be discussed in the further section.

Preliminary Design

The researchers start to do literature review, conduct observation, and design the learning trajectory as a sequence of instructional learning for the learning of multiplication to reach the goals formulated in Table 1 (adapted from Surya (2011)). The activities are designed by HLT consisting of six activities for two meetings through several easy, fun, and enjoyable activities. Students should be interested and engaged during the learning process. The last activity is evaluation process by using student worksheet and also evaluation sheet to measure the understanding of student in learning multiplication.

Table 1. Overview of the learning trajectory of multiplication (adapted from Surya (2011).

Sequence of activities	Goals	Descriptions
Playing some games using Math GASING learning aids	Understanding the multiplication concept	Students learn multiplication starting from understanding the basic concept of addition using the term of "box," for example 2 × 3 means 2 boxes are containing three things in that box, and so on.
Using some method to memorize this part easier	Memorizing the multiplication of numbers 1, 10, 9, 2 and 5	Students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students can master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on.
Using the patterns of two same numbers multiplication	Memorizing the multiplication of two same numbers, such as 1 × 1,2 × 2,…,10 × 10	Students learn about the same numbers of multiplication, such as 1 × 1, 2 × 2, 3 × 3, ... , 10x10.
Using multiplication characteristics as a commutative operation	Memorizing the multiplication of numbers 3 and 4	Students learn multiplication for 3 and 4 using a commutative operation.
Reducing some part in multiplication that already mastered	Memorizing the multiplication of numbers 8, 7 and 6	Students learn multiplication for 8, 7, and 6. The teacher teaches student by using reduce some part in multiplication that already mastered before.
Evaluation	Determining the student ability in learning multiplication	Teacher evaluates the student about the multiplication problem in the formal and informal form.

Teaching Experiment

Teaching experiment phase consists of several activities that already design in the preliminary stage. In these phases, researchers implement the learning activities using HLT as a teacher guide for the teacher model. The various educational games provided are to make teaching and learning activities more fun and enjoyable for students. This activity is one of the characteristics of learning Math GASING.

The five activities conduct using whiteboard and presentation. First, teacher introduced the concept of multiplication by playing some games using Math GASING learning aids. In that games, students learn the concept of multiplication starting from understanding the basic concept of addition using the term of "box," for example 2 × 3 means 2 boxes are containing three things in that box, and so on. Secondly, students learn multiplication for 1, 10, 9, 2 and 5 in various ways, so that students can master in the multiplication part, for example using finger method, sing a number song, pattern of multiplication numbers, and so on. Different from the memorizing process of multiplication order in mathematics in general, students memorize the multiplication start from 1, 10, 9, 2 and 5.

Furthermore, students learn about the same numbers of multiplication, such as 1 × 1, 2 × 2, 3 × 3, ... , 10x10. Fourthly, students learn multiplication for 3 and 4 using a commutative operation. Lastly, students learn multiplication for 8, 7, and 6. For this step, teacher teaches student by using reduce some part in multiplication that already mastered before. So, the students can memorize all multiplication concept from one to ten more easily. In the second meeting, teacher evaluates the student about multiplication problem in the formal and informal form. All activities can be shown in Figure 2.

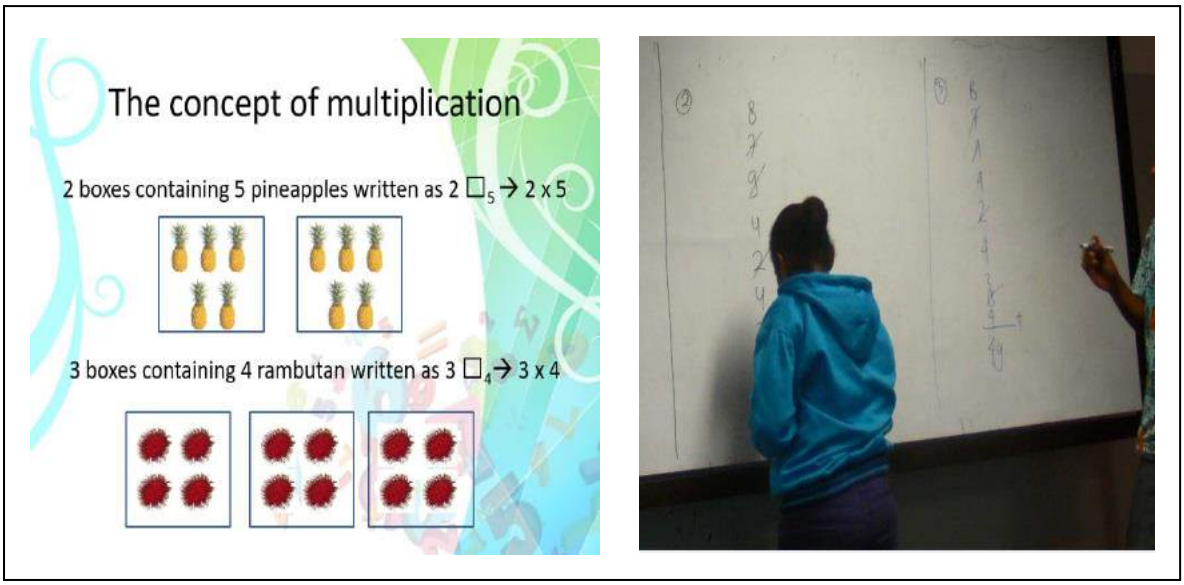


Figure 2. Several activities in teaching experiment phase.

Retrospective Analysis

There are some differences between the multiplication process in Math GASING and the multiplication process in general. These differences are the answer for the research question in this research. The difference is manifested in the learning trajectory to be analyzed retrospectively.

The designing learning trajectory seen in Table 1 is the student-guided activities to mastering the multiplication operations. Therefore, the researcher designed an activity using Math GASING aids. The goal is that students can understand the concrete form of multiplication using the understanding of boxes and something in there. The student must understand that multiplication in the form of repeated addition. The teacher used combination learning tools such as presentation and whiteboard to make learning process effective and efficiency that can be seen in Figure 2. Next, the teacher guides students lead the concept of multiplication as a form of repeated addition during this activity. The teacher uses several methods to remember doubling for one to ten more easily and meaningfully.

On the other hands, teacher makes the order of memorizing the multiplication with different order in mathematical in general. First, students memorize the multiplication for 1, 10, 9, 2 and 5. Next, students memorize the multiplication for the same numbers of multiplication, such as 1 × 1, 2 ×

2, 3 × 3, ... , 10 × 10. After that, students memorize the multiplication for 3, 4, 8, 7, and 6. Finally, all students can memorize the multiplication form from one to ten and answer the teacher exercise directly using their mental arithmetic.

The researchers used multiplication learning phases in the Math GASING (Table 1). The introduction activity in learning the basic concept of multiplication have several good discussion. During the discussion, students look like easy, fun, and enjoyable in learning multiplication in Math GASING. Therefore, the learning trajectory guides students to understand the concept of multiplication.

All activities describe the process of students understanding from informal to a formal level according to the multiplication concept. Their experience supported by the Math GASING learning aids can make students pass the critical point of multiplication so that students can master multiplication as a whole. Surya and Moss (2012) stated that student would be able to master the mathematics subject regarding in Math GASING after their pass the critical point of the subject.

The results show that the students can apply the multiplication in solving each problem is given in terms of evaluation. Therefore, it can be seen that learning multiplication operation in Math GASING can use to raise students' understanding in integer multiplication operations or other words, the design of this study can be used as the starting point of learning multiplication. In the last activities, teacher gives evaluation to measure the students' understanding in multiplication that can be seen in Figure 3.

Figure 3. Student evaluation process using student worksheet.

The retrospective analysis has shown that one of the ways making student understanding in learning multiplication makes the learning process can be imaging for students. This result is in line with previous research stated in learning multiplication have several ways to master it (Caron, 2007; Ischebeck, et al. 2006). On the other hand, learning environment also can support the result of the

learning process (Putri, et al. 2015; Nuari, et al. 2019). Finally, all students can solve several problems and exercises regarding multiplication operation.

CONCLUSION

The learning of multiplication operation in Math GASING have a significant role as the starting point and improve students' motivation in learning multiplication. Also, the designed students' activities find the multiplication concept. The activities are starting from understanding the concept of multiplication to mastering the multiplication concept of 1×1 to 10×10, which is the critical point in learning multiplication in Math GASING. The students solve several multiplication problems more easily after passing the critical point. Lastly, students can do mental arithmetic for any given multiplication problem and answer many multiplication questions very quickly and precisely. Both of evaluation is the characteristics of the assessment in Math GASING.

ACKNOWLEDGMENTS

Firstly, we thank to STKIP Surya for providing the opportunity to do this research and give facilitated until this research is completed. Then, we also thank to Petra Suwasti as a teacher model and all of students as a research subject in this research.

REFERENCES

Ahmad, N.S.B. (2010). Multiplication and the reference sum method. *Procedia-Social and Behavioral Sciences, 8*, 72-78. https://doi.org/10.1016/j.sbspro.2010.12.010.

Akker, J.V.D., Gravemeijer, K., McKenney, S., & Nieveen, N. (2006). *Education Design Research*. London: Routledge Taylor and Francis Group.

bin Syed Ismail, S.A. (2010). Multiplication with the Vedic method. *Procedia-Social and Behavioral Sciences*, *8*, 129-133. https://doi.org/10.1016/j.sbspro.2010.12.018.

Caron, T.A. (2007). Learning multiplication: The easy way. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *80*(6), 278-282. https://doi.org/10.3200/TCHS.80.6.278-282.

Chang, K. E., Sung, Y.T., Chen, Y.L., & Huang, L.H. (2008). Learning multiplication through computer-assisted learning activities. *Computers in Hhuman Bbehavior*, *24*(6), 2904-2916. https://doi.org/10.1016/j.chb.2008.04.015.

Chung, I. (2004). A comparative assessment of constructivist and traditionalist approaches to establishing mathematical connections in learning multiplication. *Education*, *125*(2), 271-279.

Drews, D., Dudgeon, J., Hansen, A., Lawton, F., & Surtees, L. (Eds.). (2005). *Children's Errors in Mathematics: Understanding Common Misconceptions in Primary Schools*. SAGE.

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K. (2004). Local instructional theories as means of support for teacher in reform mathematics education. *Mathematical Thinking and Learning*, *6*(2), 105-128.

https://doi.org/10.1207/s15327833mtl0602_3.

Ischebeck, A., Zamarian, L., Siedentopf, C., Koppelstätter, F., Benke, T., Felber, S., & Delazer, M. (2006). How specifically do we learn? Imaging the learning of multiplication and subtraction. *Neuroimage*, *30*(4), 1365-1375. https://doi.org/10.1016/j.neuroimage.2005.11.016.

Kilian, L., Cahill, E., Ryan, C., Sutherland, D., & Taccetta, D. (1980). Errors that are common in multiplication. The Arithmetic Teacher, 22-25.Kilian, L., Cahill, E., Ryan, C., Sutherland, D., & Taccetta, D. (1980). Errors that are common in multiplication. *The Arithmetic Teacher*, *27*(5), 22-25.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Nuari, L.F., Prahmana, R.C.I., & Fatmawati, I. (2019). Learning of division operation for mental retardations' student through Math GASING. *Journal on Mathematics Education*, *10*(1), 127-142. https://doi.org/10.22342/jme.10.1.6913.127-142.

Prahmana, R.C.I. (2013). Designing division operation learning in the mathematics of GASING. *Proceeding in The First South East Asia Design/Development Research (SEA-DR) Conference 2013*, 391-398.

Prahmana, R.C.I. (2015). The hypothetical learning trajectory on addition in mathematics GASING. *Southeast Asian Mathematics Education Journal*, *5*(1), 49-61.

Prahmana, R.C.I. (2017). *Design Research (Teori dan Implementasinya: Suatu Pengantar) [Design Research (Theory and Its Implementation: An Introduction)]*. Depok: Rajawali Pers.

Prahmana, R.C.I., & Suwasti, P. (2014). Local instruction theory on division in mathematics GASING. *Journal on Mathematics Education*, *5*(1), 17-26. https://doi.org/10.22342/jme.5.1.1445.17-26.

Prahmana, R. C. I., Zulkardi, & Hartono, Y. (2012). Learning multiplication using Indonesian traditional game in third grade. *Journal on Mathematics Education*, *3*(2), 115-132. https://doi.org/10.22342/jme.3.2.1931.115-132.

Putri, R.I.I., Dolk, M., & Zulkardi. (2015). Professional development of PMRI teachers for introducing social norms. *Journal on Mathematics Education*, *6*(1), 11-19. https://doi.org/10.22342/jme.6.1.1900.11-19.

Revina, S., Zulkardi, Darmawijoyo, & Galen, F. (2011). Spatial visualization tasks to support students' spatial structuring in learning volume measurement. *Journal on Mathematics Education*, *2*(2), 127-146. http://dx.doi.org/10.22342/jme.2.2.745.127-146.

Reys, R.E., Suydam, M.N., Lindquist, M.M., & Smith, N.L. (1998). *Helping Children Learn Mathematics*. Boston: Allyn and Bacon.

Shanty, N.O., & Wijaya, S. (2012). Rectangular array model supporting students' spatial structuring in learning multiplication. *Journal on Mathematics Education*, *3*(2), 175-186. http://dx.doi.org/10.22342/jme.3.2.603.175-186.

Surya, Y. (2011). *Petunjuk Guru: Dasar-Dasar Pintar Berhitung GASING [Teachers' Guide: Smart Basics of Counting's GASING]*. Tangerang: PT. Kandel.

Surya, Y., & Moss, M. (2012). Mathematics education in rural Indonesia. *Proceeding in the 12th International Congress on Mathematics Education: Topic Study Group 30*. Korea National

University of Education, Seoul, South Korea Selatan, 6223-6229.

Tanujaya, B., Prahmana, R. C. I., & Mumu, J. (2017). Mathematics instruction, problems, challenges and opportunities: A case study in Manokwari Regency, Indonesia. *World Transactions on Engineering and Technology Education*, *15*(3**)**, 287-291.

Unlu, M., & Ertekin, E. (2012). Why do pre-service teachers pose multiplication problems instead of division problems in fractions?. *Procedia-Social and Behavioral Sciences*, *46*, 490-494. https://doi.org/10.1016/j.sbspro.2012.05.148.

Profile artikel di database Scopus

Scopus Search Sources Lists SciVal KK

Document details

1 of 1

Text export Download Print E-mail Save to PDF Save to list More...
View at Publisher

Journal on Mathematics Education Open Access
Volume 10, Issue 3, 2019, Pages 397-408

The innovation of learning trajectory on multiplication operations for rural area students in Indonesia (Article) (Open Access)

Hendriana, H.[a], Prahmana, R.C.I.[b], Hidayat, W.[a]
View additional authors Save all to author list

[a]IKIP Siliwangi, Jalan Terusan Jenderal Sudirman, Cimahi, Cimahi, 40526, Indonesia
[b]Universitas Ahmad Dahlan, Jl. Pramuka 42, Pandeyan, Umbulharjo, Yogyakarta, 55161, Indonesia
View additional affiliations

Abstract

The rural area's student difficulties in learning the concept of number operation had been documented by several studies, especially for the case of multiplication. The teacher typically introduces the multiplication concepts using the formula without involving the concept itself. Furthermore, this study aims to design learning trajectory on multiplication operations in the Mathematics of GASING (Math GASING) by focusing more on the concept itself than the formula and by starting from the informal to a formal level of teaching. Design research used as the research method to solve this problem consisting of three phases, namely preliminary design, teaching experiment, and retrospective analysis. The research results show that the Math GASING has a real contribution for students to understanding and mastering in the concept of the multiplication operations. This research also explains the strategy and the model discovered by students in learning multiplication that the students used as a basic concept of multiplication. Finally, the students were able to understand the concept of multiplication more easily, and they showed interest in using this learning trajectory.

Author keywords

Design research | Learning trajectory | Multiplication | Rural area

ISSN: 20878885
Source Type: Journal
Original language: English
DOI: 10.22342/jme.10.3.9257.397-408
Document Type: Article
Publisher: Sriwijaya University

References (27)

View in search results format

All Export Print E-mail Save to PDF Create bibliography

1 Ahmad, N.S.B., Sivasubramaniam, P.
Multiplication and the reference sum method (Open Access)
(2010) *Procedia - Social and Behavioral Sciences*, 8, pp. 72-78. Cited 2 times.
http://www.sciencedirect.com/science/journal/18770428/1
doi: 10.1016/j.sbspro.2010.12.010
View at Publisher

2 Akker, J.V.D., Gravemeijer, K., McKenney, S., Nieveen, N.
(2006) *Education Design Research*. Cited 421 times.
London: Routledge Taylor and Francis Group

3 Ismail, S.A.B.S., Sivasubramniam, P.
Multiplication with the Vedic method (Open Access)
(2010) *Procedia - Social and Behavioral Sciences*, 8, pp. 129-133. Cited 6 times.
http://www.sciencedirect.com/science/journal/18770428/1
doi: 10.1016/j.sbspro.2010.12.018
View at Publisher

4 Caron, T.A.
Learning multiplication: The easy way
(2007) *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, 80 (6), pp. 278-282. Cited 9 times.

5 Chang, K.-E., Sung, Y.-T., Chen, Y.-L., Huang, L.-H.
Learning multiplication through computer-assisted learning activities
(2008) *Computers in Human Behavior*, 24 (6), pp. 2904-2916. Cited 11 times.
doi: 10.1016/j.chb.2008.04.015

Metrics

View all metrics

17 Citations in Scopus

Field-Weighted Citation Impact

Cited by 17 documents

Teaching higher-order thinking skills in mathematics classrooms: Gender differences
Sadijah, C. , Murtafiah, W. , Anwar, L.
(2021) Journal on Mathematics Education

Animated media design based on visual basic application microsoft powerpoint on the material build flat side spaces
Rohaeti, E.E. , Putra, H.D. , Purwandari, A.S.
(2020) Journal of Physics: Conference Series

Adversity quotient of prospective primary school teachers in making scratch-assisted math application
Ruqoyyah, S. , Ristiana, M.G.
(2020) Journal of Physics: Conference Series

View all 17 citing documents

Inform me when this document is cited in Scopus:

Set citation alert

Related documents

Find more related documents in Scopus based on:

Authors Keywords

Profile Jurnal di Website Scopus