Artikel di submit tanggal 17 Januari 2021

Published by:

Journal on Mathematics Education

(P-ISSN: 2087-8885 | E-ISSN: 2407-0610)

JME Scopus Indexed

HOME ABOUT USER HOME SEARCH CURRENT ARCHIVES ANNOUNCEMENTS CONTACT

Home > User > Author > Submissions > #13537 > **Summary**

#13537 Summary

SUMMARY REVIEW EDITING

Submission

Authors	Wahid Yunianto, Rully Charitas Indra Prahmana, Cosette Crisan
Title	INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS OF AREA AND PERIMETER OF RECTANGLE
Original file	13537-35162-2-SM.DOCX 2021-01-17
Supp. files	13537-35163-1-SP.PDF 2021-01-17
Submitter	Rully Charitas Indra Prahmana
Date submitted	January 17, 2021 - 12:13 AM
Section	Articles
Editor	Zulkardi Zulkardi
Author comments	

Dear Prof. Zulkardi, M.I.Komp., M.Sc.,
Editor-in-Chief of Journal on Mathematics Education

Greetings from Yogyakarta and wishing you a great day with happiness and healthy condition in this era COVID-19.

We as the research collaboration team are writing the manuscript entitled "Indonesian Mathematics Teachers' Knowledge of Content and Students: Predicting and Responding to Students' Responses to the Topic of Area and Perimeter" for consideration for publication in the Journal on Mathematics Education (JME). This manuscript was written using the author guidelines of JME mentioned on the website.

This paper provides comprehensive studies about predicting and responding to students' responses to the topic of area and perimeter from Indonesian mathematics teachers' Knowledge of Content and Students (KCS). As we know that teachers' quality is essential to the teaching and learning process, and it influences the quality of school education. Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. However, to some extent of teacher assessment has not yet completely covered the full range of teacher skills and competencies. This study investigates mathematics teachers' Pedagogical Content Knowledge (PCK) on the topic of area-perimeter through their designed lesson plans. It focused only on one aspect of PCK called Knowledge of Content and Students (KCS). Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items to assess mathematics teacher knowledge are dominated by subject matter knowledge. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers PCK through lesson plans developed by the teachers. Mathematics teachers attended a professional development activity and voluntarily participated in this study. The lesson plan focused on the area-perimeter topic for grade 7. Content analysis of the lesson plan and semi-structured interviews were conducted, and the data analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible methods and mistakes, these teachers were poorly prepared to support student learning. Further studies are needed to explore other aspects of PCK involving lesson plans.

This paper also describes **our original work** and is **not under consideration** by any **other journal**. All authors approved the manuscript and this submission. The three co-authors do not have any conflict of interest regarding this manuscript. This document was reported as the result of the research we conducted as one of the requirements of our responsibility as a researcher in our university. This year, we didn't get funding for our research publication because of the COVID-19 Pandemic disease case in our country, so I would like to waive all article processing charges if our paper is accepted. Lastly, we do hope that this article can be published in this journal so that we can contribute our research results in this journal.

Thank you for receiving our manuscript and considering it for review. We do really appreciate your time and look forward to seeing your response.

Best Wishes,

Assoc. Prof. Dr. Rully Charitas Indra Prahmana
Department of Master Program on Mathematics Education
Faculty of Teacher Training and Education
Universitas Ahmad Dahlan, Yogyakarta, Indonesia

Abstract Views	83

Status

Status	Published Vol 12, No 2 (2021)
Initiated	2021-03-25
Last modified	2021-04-12

Submission Metadata

Authors

Name	Wahid Yunianto
Affiliation	SEAMEO QITEP in Mathematics, Yogyakarta
Country	Indonesia
Bio Statement	—

Name Rully Charitas Indra Prahmana
Affiliation Universitas Ahmad Dahlan, Yogyakarta
Country Indonesia
Bio Statement —
Principal contact for editorial correspondence.
Name Cosette Crisan
Affiliation University College London, London
Country United Kingdom
Bio Statement —

ERIC
Institute of Education Sciences

KEYWORDS

Addition Asian Games Curriculum Design Research Design research Ethnomathematics Geometry Integers Mathematical Literacy Mental models Metacognition Metaphors Microworlds PISA PMRI Realistic Mathematics Education Subtraction design research learning trajectory mathematical proof multiplication

Title and Abstract

Title INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS OF AREA AND PERIMETER OF RECTANGLE

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. Research has shown teachers competencies and skills influence students' performances. Previous studies explored teachers' knowledge through testing. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items or tasks to assess mathematics teacher knowledge in the previous studies were dominated by subject matter knowledge problems. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' Knowledge of Content and Students (KCS) through lesson plans developed by the teachers. To accommodate the gap in the previous studies, this study focuses on KCS on the topic of area-perimeter through their designed lesson plans. Twenty-nine mathematics teachers attended a professional development activity voluntarily participated in this study. Two teachers were selected to be the focus of this case study. Content analysis of the lesson plan and semi-structured interviews were conducted, and then data were analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible strategies and mistakes, these teachers were poorly prepared to support student learning.

Indexing

Keywords Knowledge of Content and Students; Mathematics Teacher; Area and Perimeter; Teachers' Skills and Competencies

Language en

Supporting Agencies

Agencies Ministry of Education of Republic of Indonesia, Planning and Cooperation of Foreign Affairs, and SEAMEO QITEP in Mathematics

References

References

An, S., Kulm, G., & Wu, Z. (2004). The Pedagogical Content Knowledge of Middle School, Mathematics Teachers in China and the U.S. Journal of Mathematics Teacher Education, 7, 145-172. https://doi.org/10.1023/b:jmte.0000021943.35739.1c

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? Journal of Teacher Education, 59(5), 389-407. https://doi.org/10.1177/0022487108324554

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. Educational Studies in Mathematics, 31, 235-268. https://doi.org/10.1007/BF00376322

Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project (pp. 25-48). Boston: Springer. https://doi.org/10.1007/978-1-4614-5149-5_2

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., ... Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. American Educational Research Journal, 47(1), 133-180. https://doi.org/10.3102/0002831209345157

Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' Thinking About Instruction. Remedial and Special Education, 11(6), 40-49. https://doi.org/10.1177/074193259001100609

Burns, R. B., & Lash, A. A. (1988). Nine Seventh-Grade Teachers' Knowledge and Planning of Problem-Solving Instruction. The Elementary School Journal, 88(4), 369-386. https://doi.org/10.1086/461545

Butt, G. (2008). Lesson Planning 3rd Edition. London: Bloomsbury Publishing.

Carle, S. M. (1993). Student held misconceptions regarding area and perimeter of rectangles. Critical and Creative Thinking Capstones Collection, 46. http://scholarworks.umb.edu/cct_capstone/46

Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), Annual Conference of the Mathematics Education Research Group of Australasia (pp. 136–143). Adelaide: Australian Association of Mathematics Teachers.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. Handbook of research on teaching. New York: MacMillan

Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from Lessons: Studying the Construction of Teacher Knowledge Catalysed by Purposefully-Designed Experimental Mathematics Lessons. Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia (pp. 165-172). Sunshine Coast: MERGA

Denscombe, M. (2010). The Good Research Guide For Small Scale Research Projects. Berkshire: Open University Press.

Fauskanger, J. (2015). Challenges in measuring teachers' knowledge. Educational Studies in Mathematics, 90, 57-73. https://doi.org/10.1007/s10649-015-9612-4

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. Journal for Research in Mathematics Education, 39(4), 372-400. https://www.jstor.org/stable/pdf/40539304

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. American Educational Research Journal, 42(2), 371-406. https://doi.org/10.3102/00028312042002371

John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. Journal of Curriculum Studies, 38(4), 483-498. https://doi.org/10.1080/00220270500363620

Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In Learning to Teach Mathematics in the Secondary School: A Companion to School Experience: 3rd Edition (pp. 79–100). London: Routledge Taylor & Francis Group. https://doi.org/10.4324/9780203844120

Korkmaz, H. I., & ?ahin, Ö. (2019). Preservice Preschool Teachers' Pedagogical Content Knowledge on Geometric Shapes in Terms of Children's Mistakes. Journal of Research in Childhood Education, 34(3), 385-405. https://doi.org/10.1080/02568543.2019.1701150

Kow, K., & Yeo, J. (2008). Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action. Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia.

Kristanto, Y. D., Panuluh, A. H., & Atmajati, E. D. (2020). Development and validation of a test instrument to measure pre-service mathematics teachers' content knowledge and pedagogical content knowledge. Journal of Physics: Conference Series, 1470(1), 012008. https://doi.org/10.1088/1742-6596/1470/1/012008

Lee, Y., Capraro, R. M., & Capraro, M. M. (2018). Mathematics Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge in Problem Posing. International Electronic Journal of Mathematics Education, 13(2), 75-90. https://doi.org/10.12973/iejme/2698

Nakahara, T., & Koyama, M. (2000). Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1.
Özerem, A. (2012). Misconceptions In Geometry And Suggested Solutions For Seventh Grade Students. Procedia - Social and Behavioral Sciences, 55, 720-729. https://doi.org/10.1016/j.sbspro.2012.09.557
Superfine, A. M. C. (2008). Planning for Mathematics Instruction: A Model of Experienced Teachers' Planning Processes in the Context of a Reform Mathematics Curriculum. The Mathematics Educator, 18(2), 11-22. https://ojs01.galib.uga.edu/tme/article/view/1925/1830
Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical Content Knowledge of Mathematics Pre-service Teachers: Do they know their students? Journal of Physics: Conference Series, 1097(1), 012098. https://doi.org/10.1088/1742-6596/1097/1/012098
Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. Educational Researcher, 15(2), 4-14. https://doi.org/10.3102/0013189X015002004
Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. Journal for Research in Mathematics Education, 26(2), 114-145. https://doi.org/10.2307/749205
Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., ... Rowley, G. (2012). Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). Amsterdam: International Association for the Evaluation of Educational Achievement.
Turnuklu, E., & Yesildere, S. (2007). The Pedagogical Content Knowledge in Mathematics: Pre-Service Primary Mathematics Teachers' Perspectives in Turkey. Issues in the Undergraduate Mathematics Preparation of School Teachers, 1, 1-13.
Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating Preservice Mathematics Teachers' Pedagogical Content Knowledge through Microteaching. REDIMAT-Journal of Research in Mathematics Education, 9(1), 62-87. http://dx.doi.org/10.17583/redimat.2020.3353
Watson, A., Jones, K., & Pratt, D. (2013). Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19 (1st ed.). Oxford: Oxford University Press.
White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2012). Teachers learning from teachers. In Third International Handbook of Mathematics Education (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13
Widodo, & Tamimudin, M. (2014). Three Training Strategies for Improving Mathematics Teacher Competences in Indonesia. Electronic Proceedings of the 19th Asian Technology Conference in Mathematics. Yogyakarta: Mathematics and Technology, LLC. Retrieved from http://atcm.mathandtech.org/EP2014/index.html
Yeo, K. K. Y. (2008). Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action. Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia (pp. 621-627). Brisbane: The University of Queensland. Retrieved from https://repository.nie.edu.sg/bitstream/10497/14397/1/MERGA-2008-621-YeoKK_a.pdf
Yin, R. K. (2014). Case study research: Design and methods. Thousand Oaks: SAGE Publications.
Yunianto, W. (2015). Supporting Students' Understanding of Area Measurement Through Verknippen Applet. Southeast Asian Mathematics Education Journal, 5(1), 73-82. https://doi.org/10.46517/seamej.v5i1.34
Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. Review of Science, Mathematics and ICT Education, 6(2), 41–62. https://doi.org/10.26220/rev.1627

Journal on Mathematics Education
Doctoral Program on Mathematics Education
Faculty of Teacher Training and Education, Universitas Sriwijaya
Kampus FKIP Bukit Besar
Jl. Srijaya Negara, Bukit Besar
Palembang - 30139
email: jme@unsri.ac.id

p-ISSN: 2087-8885 | e-ISSN: 2407-0610

Blind Review Artikel yang di submit pada tanggal 17 Januari 2021 dengan judul awal

“Indonesian Mathematics Teachers’ Knowledge of Content and Students: Predicting and Responding to Students’ Responses to the Topic of Area and Perimeter”

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume xx, No. x, January xxxx, pp. x-xx

INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS: PREDICTING AND RESPONDING TO STUDENTS' RESPONSES TO THE TOPIC OF AREA AND PERIMETER

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. However, to some extent of teacher assessment has not yet completely covered the full range of teacher skills and competencies. This study investigates focuses on Knowledge of Content and Students (KCS) on the topic of area-perimeter through their designed lesson plans. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items to assess mathematics teacher knowledge are dominated by subject matter knowledge. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' KCS through lesson plans developed by the teachers. Mathematics teachers attended a professional development activity and voluntarily participated in this study. Content analysis of the lesson plan and semi-structured interviews were conducted, and the data analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible methods and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Namun, dalam beberapa hal, penilaian guru belum sepenuhnya mencakup seluruh keterampilan dan kompetensi guru. Fokus penelitian ini adalah menyelidiki *Knowledge of Content and Students* (KCS) pada topik luas dan keliling melalui rancangan rencana pembelajaran mereka. Pengetahuan guru tentang topik luas dan keliling dan strategi pengajaran telah dinilai melalui pengujian. Secara umum, materi untuk menilai pengetahuan guru matematika didominasi oleh materi pengetahuan. Dengan demikian, penilaian tersebut tampaknya belum sepenuhnya mencakup seluruh pengetahuan dan kompetensi guru. Dalam penelitian ini, peneliti menginvestigasi KCS guru matematika melalui RPP yang dikembangkan oleh guru. Guru matematika mengikuti kegiatan pengembangan profesional dan secara sukarela berpartisipasi dalam penelitian ini. Analisis isi RPP dan wawancara semi-terstruktur dilakukan, dan data dianalisis. Hasil penelitian ini mengungkapkan bahwa guru yang berpartisipasi ditantang ketika membuat prediksi kemungkinan tanggapan siswa. Mereka tampaknya tidak menyadari strategi siswa biasa, yang digunakan untuk menyelesaikan memaksimalkan luas dari keliling tertentu. Dengan pengetahuan yang terbatas tentang kemungkinan metode dan kesalahan siswa, para guru ini kurang siap untuk mendukung pembelajaran siswa.

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

How to Cite: Authors. (2021). Indonesian Mathematics Teachers' Knowledge of Content and Students: Predicting and Responding to Students' Responses to the Topic of Area and Perimeter. *Journal on Mathematics Education, x* (x), xx-xx.

Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research has shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related

to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

> *Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2013, p.394).*

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

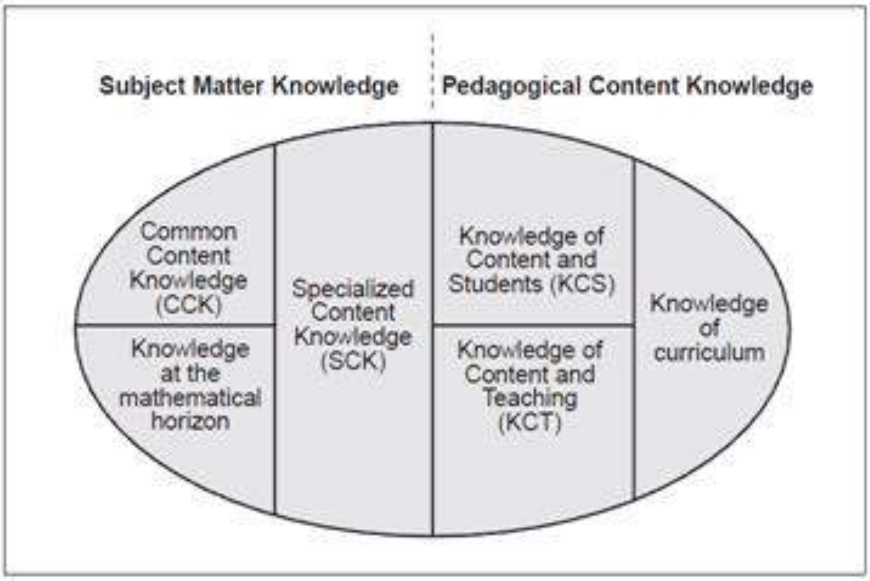


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). Pre-service teachers have challenges with student thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COCATIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin H,

2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Lesson planning is considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers who read it can easily understand it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called 'Bansho', it anticipates student mathematical thinking and student thinking schema for solving given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). Student engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom behaviour is influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

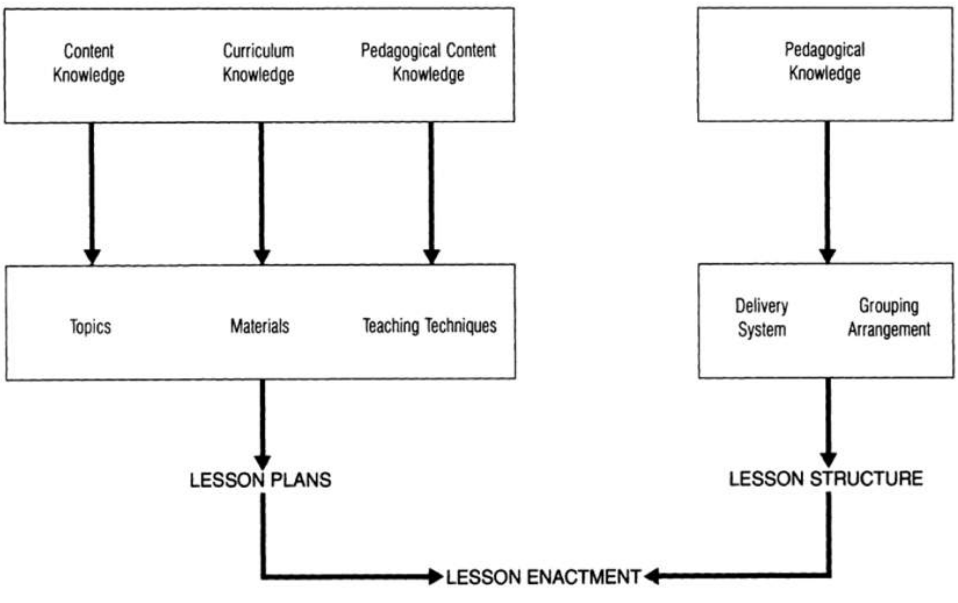


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zacahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013).

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason, (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Yeo, 2008).

This study intends to focus on a part of PCK pedagogical content knowledge, the KCS within lesson plans on the topic of area-perimeter. It is necessary to obtain a fuller insight into teacher knowledge. How mathematics teachers in Yogyakarta prepare their lesson plans and how is PCK integrated in their lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved human and had been through research ethics approval by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk). This study administrated a case study approach. This approach suits this study as it doesn't seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). Through this approach, the researchers examined two selected lesson plans of the mathematics teachers. The sample was chosen from twenty-nine teachers who attended a Professional Development (PD) session, and two teachers were selected for the lesson plan analysis and interview. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan. The topic that would be taught was area and perimeter for grade 7. The "Gold Rush/Mining" task was selected. This task has several ways to be solved (see

Figure ***3***).

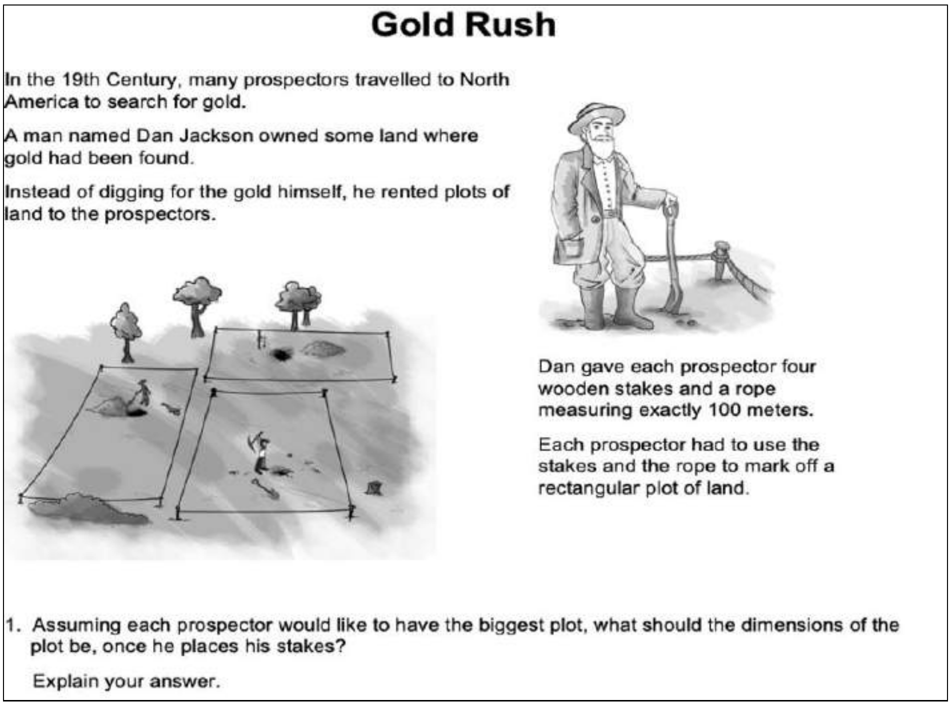


Figure 3. The Gold Rush problem *(https://www.map.mathshell.org/download.php?fileid=1637)*

To analyze the lesson plans, the researchers used content analysis. This method has the 'potential to disclose many hidden aspects of what is being communicated through the written text' (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers' knowledge of students' conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find the themes. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers included information about what students would do to the task. The data were presented descriptively.

Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing

2. The ability to predict what students will find interesting and motivating when choosing a task
3. The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4. The ability to hear and interpret students' emerging and incomplete thinking

The two lesson plans were coded and analyzed. There were three types of instructions to refer to the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3).

Table 3. Proportions of the instructions

Instruction	T1	T2
GI	8 (35%)	6 (31.6%)

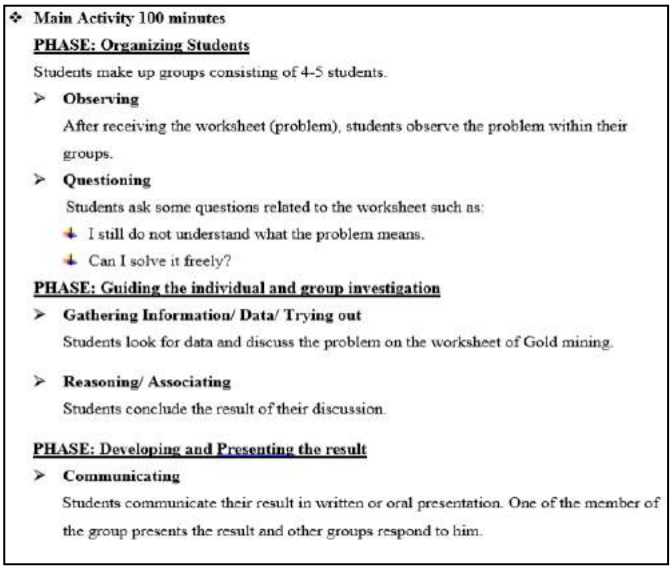
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	**23 (100%)**	**19 (100%)**

Indonesian teachers follow the prescribed template of a lesson plan. The template consists of three main parts namely; introduction, main and closure. Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2's SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	**Introduction**		**Main**		**Closure**	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	**12**	**3**	**7**	**11**	**4**	**5**

T1 put more details of what students would ask to her on her lesson plan. For instance: 'Can I solve it freely?' has been put on her lesson plan. In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4).


❖ **Main Activity 100 minutes**

<u>PHASE: Organizing Students</u>

Students make up groups consisting of 4-5 students.

➢ **Observing**

After receiving the worksheet (problem), students observe the problem within their groups.

➢ **Questioning**

Students ask some questions related to the worksheet such as:

- I still do not understand what the problem means.
- Can I solve it freely?

<u>PHASE: Guiding the individual and group investigation</u>

➢ **Gathering Information/ Data/ Trying out**

Students look for data and discuss the problem on the worksheet of Gold mining.

➢ **Reasoning/ Associating**

Students conclude the result of their discussion.

<u>PHASE: Developing and Presenting the result</u>

➢ **Communicating**

Students communicate their result in written or oral presentation. One of the member of the group presents the result and other groups respond to him.

Figure 4. Teacher 1 Lesson Plan

The T2 lesson plan depicted detailed information about a possible student strategy. Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

Main Activity

- ☑ Teacher divide students into groups
- ☑ Teacher delivers the worksheet to be discussed
- ☑ Teacher facilitates the learning processes

- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
  for instance p=10, l=… m then the area = …
  p = 15 m, l = ...m, then the area = ….
  Students determine the largest area by themselves
- for the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
  Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
  How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan

After finding the largest area, students had to find the largest area by joining two miners' ropes. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

- Students evaluate and make generalisation into questioning.
  - ☑ Teacher asks students to present in front of the class
  - ☑ Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Teachers T1 and T2 have more than five-years teaching experience each. However, their schools are different in terms of location and students. These teachers themselves employed different abilities in solving the Gold Mining problem (

Figure ***3***). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : Are there other ways T1?
T1 : Yesterday, I just did that one.
T1 : …just let students find the ways to solve it …. Then, I will let them know that there are some ways to solve it. I give that opportunity to students

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

*T2 : To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*
R : So, you (T2), induce them by using the table?
T2 : Yes, by the table.
R : What do you think, how many ways to solve it?
*T2 : To me, I did one way I know it directly it would be a square**. I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and.., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

*T2 : I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*
T2 : However, I have thought only one strategy, which is global to solve a task. … I, I... know at least I understand my students' characteristic so that it will be difficult for my students. … It is not possible to come up if I let them to do it freely. … I am so careful to give it the various strategies because students would get confuse

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

T2 : By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.
T2 : To me, I did one way I know it directly it would be a square. I knew it already
T2 : …instead of table, we can make the variable x, then I will be a quadratic function,
R : Are there other ways to solve it?
T2 : For the time being, not yet, making rectangles and to the square

R : Do you think there are still other ways to solve that problem?
T2 : I could use the graph …

To some extent, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task. However, by giving the students the strategy, he inadvertently is making the student dependent on him. Whereas, T1 is helping the students to make decisions themselves. From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

*T1 : The task has missing information, it should be more, and some students would think that. So that they **have not thought** yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
*T1 : 100. Maybe **they thought that** that's the only think they know.*
R : … So, they would answer it 100, possibly
T1 : Yeah, possibly
*T2 : … for those who did not understand, **they would not know what 100** m rope is to with the perimeter. So that the concept of perimeter, for those who understood, they already make it but later **they would not think** the rectangles can be varied.*
*T2 : Students **would confuse** the meaning of maximum, which is the largest, **they have not thought about it.** So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7).

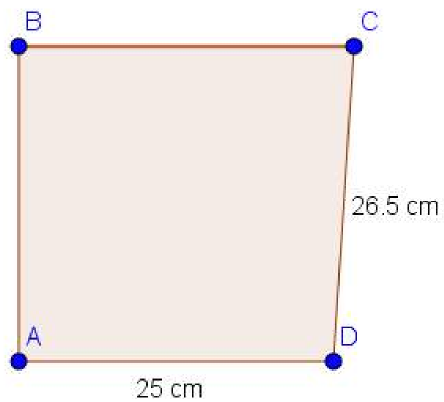


Figure 7. A student's possible mistake proposed by the researcher

If faced with a student mistake that they not have thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R : If it happens if you see this (showing)

T1 : I would ask students back to try it then you calculate it as what being asked to you
R : They have not yet known the result!
T1 Try, try it, by trialing they would know that it is different, this one is more, and that one is like that, ...
R : T2, what if your students did this? what would you do?
T2 : I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...

In this study, the lesson plans facilitated an insight into teachers' knowledge. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Yeo, 2008).

Teachers have different ways of supporting students to solve tasks (Yei, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task. However, these results are not generalizable. The sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared. Developing higher order thinking and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment.

ACKNOWLEDGMENTS

The authors wish to thank to Ministry of Education of Republic of Indonesia; Planning and Cooperation of Foreign Affairs for the scholarship. The authors also would like to thank SEAMEO QITEP in Mathematics for its endless support.

REFERENCES

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, *7*, 145-172. https://doi.org/10.1023/b:jmte.0000021943.35739.1c

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics, 31*, 235-268. https://doi.org/10.1007/BF00376322

Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project* (pp. 25-48). https://doi.org/10.1007/978-1-4614-5149-5_2

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., … Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*(1), 133-180. https://doi.org/10.3102/0002831209345157

Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' thinking about instruction. *Remedial and Special Education*, *11*(6), 40-49. https://doi.org/10.1177/074193259001100609

Burns, R. B., & Lash, A. A. (1988). Nine seventh-grade teachers' knowledge and planning of problem-solving instruction. *The Elementary School Journal*, *8*(4), 369-386. https://doi.org/10.1086/461545

Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.

Carle, S. M. (1993). *Student held misconceptions regarding area and perimeter of rectangles* (University of Massachusetts Boston). Retrieved from https://scholarworks.umb.edu/cgi/viewcontent.cgi?article=1045&context=cct_capstone

Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves,

& T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136–143). Adelaide: Australian Association of Mathematics Teachers.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching, 3rd Edition* (pp. 255-296). New York: Macmillan.

Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from lessons: Studying the construction of teacher knowledge catalysed by purposefully-designed experimental mathematics lessons. In M. Marshman, V. Geiger, & A Bennison (Eds.), *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165–172). Sunshine Coast: MEGA.

Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371-406. https://doi.org/10.3102/00028312042002371

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. Journal for *Research in Mathematics Education, 39*, 372-400. Retrieved from https://www.jstor.org/stable/40539304

John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies, 38*(4), 483-498. https://doi.org/10.1080/00220270500363620

Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In S. Johnston-Wilder, C. Lee, & D. Pimm (Eds.), *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience, 3rd Edition* (pp. 79-100). London: Routledge. https://doi.org/10.4324/9780203844120

Korkmaz, H. I., & Şahin, Ö. (2019). Preservice preschool teachers' pedagogical content knowledge on geometric shapes in terms of children's mistakes. *Journal of Research in Childhood Education*, *34*(3), 385-405. https://doi.org/10.1080/02568543.2019.1701150

Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1.* Retrieved from https://files.eric.ed.gov/fulltext/ED452031.pdf

Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *Procedia - Social and Behavioral Sciences, 55,* 720-729. https://doi.org/10.1016/j.sbspro.2012.09.557

Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical content knowledge of mathematics pre-service teachers: Do they know their students? *Journal of Physics: Conference Series*, *1097*(1), 012098. https://doi.org/10.1088/1742-6596/1097/1/012098

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *2*(15), 4-14. https://doi.org/10.3102/0013189X015002004

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, *26*(2), 114-145. https://doi.org/10.2307/749205

Superfine, A. C. (2008). Planning for mathematics instruction: A model of experienced teachers' planning processes in the context of a reform mathematics curriculum. *Mathematics Educator*, *18*(2), 11-22. Retrieved from http://tme.journals.libs.uga.edu/index.php/tme/article/view/198

Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., … Rowley, G. (2012). Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). In *International Association for the Evaluation of Educational Achievement*. Retrieved from https://files.eric.ed.gov/fulltext/ED542380.pdf

Turnuklu, E., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, *1*, 1-13. Retrieved from http://www.k-12prep.math.ttu.edu/journal/1.contentknowledge/yesildere01/article.pdf

Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating preservice mathematics teachers' pedagogical content knowledge through microteaching. *REDIMAT-Journal of Research in Mathematics Education*, *9*(1), 62-87. https://doi.org/http://dx.doi.org/10.17583/redimat.2020.3353

Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.

White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2013). Teachers learning from teachers. In M.A. Clements, A.J. Bishop, C. Keitel, J. Kilpatrick, & F.K.S. Leung (Eds.), *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13

Widodo, & Tamimudin, H. M. (2014). Three training strategies for improving mathematics teacher competence in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Retrieved from http://atcm.mathandtech.org/EP2014/index.html

Yeo, K. K. Y. (2008). Teaching area and perimeter : Mathematics-pedagogical-content knowledge-in-action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*. Retrieved from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.471.9965&rep=rep1&type=pdf

Yin, R. K. (2014). Case Study Research: Design and Methods (5th ed.). *Thousand Oaks: SAGE Publications*.

Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, *6*(2), 41-62. https://doi.org/10.26220/rev.1627

Keputusan diterima dengan revisi pada pada tanggal 8 Maret 2021

[JME] Editor Decision

Inbox ×

Jme Fkip Matematika <jme@unsri.ac.id>
to Wahid, me, Cosette

Mon, Mar 8, 8:45 PM

Dear Wahid Yunianto, Rully Charitas Indra Prahmana, and Cosette Crisan,

It is my great pleasure to inform you that your paper entitled "Indonesian Mathematics Teachers' Knowledge of Content and Students: Predicting and Responding to Students' Responses to the Topic of Area and Perimeter" has been Accepted with Major Revision and will be published in the Journal on Mathematics Education (JME). Your paper will be published for forthcoming issues after suitable revision and fulfil the JME's standard.

Authors are encouraged to carefully consider the reviewers' comments and suggestions for improvement of your manuscript, such as:

Reviewer A:
The research question seems to address how teachers prepare lessons, while the question is whether lesson plans can be used to evaluate teachers' PCK. Improving the question is a minor detail. I have a bit more concerns with the method and the scope of the conclusions. The procedure of performing the task for the teachers is not clear to me. To what extent were these two teachers aware of the purpose of the task and how was that related to the PD course they were attending? This is relevant for interpreting the results. E.g. I missed the teachers' reflections on intentions for the lesson plan (learning about area? Eliciting misconceptions? Learning about problem solving? Etc.). Since the study is a case study with two teachers, the risk is to generalize too quickly without understanding the context. This context and a detailed account of the method are needed to describe limitations of the study and possible generalizations or theoretical reflections. Detailed information about my concerns can be found in the document.

Reviewer B:
The research topic is intriguing and important in mathematics education. Examining and developing math teachers' knowledge in general and KCS in specific is challenging. The current research attempted to address this. I must say that the attempt is already good but still insufficient to comprehensively address the teachers' KCS. I found some issues;
Firstly, the abstract and introduction need a rework if this manuscript will be accepted. An essential missing part of the introduction is what did the study offer compared to the prior relevant studies. Furthermore, they have not been logically and sequentially arranged.
Secondly, the first issue is a technical one; it can be revised. However, the second issue is principle one- the research procedures were not able to fully reveal the teachers' KCS. This review is based on my knowledge on the topic. Others might have different standpoints.
Here is my argumentation for the second point;

After reading the introduction with its limitations, I was wondering how the indicators of KCS were examined in this study. So, I decided to jump into Table 1 (The indicators of KCS). Well, that is great; four indicators. Then, the question popped up to my mind was how the research procedures facilitate the intended indicators.

Initially, the researchers asked the teachers to devise a lesson plan with the Gold Rush task. The teachers did it and two selected lesson plans (it has not been explained why they chose these two) were examined to reveal teachers' KCS (Table 1). I was thrilled to read the findings; how the content analysis revealed the teachers' KCS. I tried carefully and repeatedly reading the findings to make sure that I did not miss the details. What I found is that the findings do not comprehensively represent the indicators of KCS. The findings of the content analysis on the lessons plan are;
1. Types of instructions (Table 2-3)
2. The delivery of the lessons (Figure 4-6 along with its interpretations and interview scripts)
In my opinion which refer to the indicators of KCS, those findings are not KCS. Finding 2 tends to be teachers' pedagogical knowledge instead of KCS

Scrolling to the next pages of Findings, all of a sudden, I found that the researchers provided a possible students' answer on the task. This procedure has not been explained in the method. Technically, this can be revised. However, the data (interview transcripts) were yet able to reveal the teachers' KCS.

In short, KCS, which is clearly shown in the indicators, is about teachers' mathematical content knowledge and how they utilize this knowledge to facilitate students' understanding of the task or the construction of mathematical knowledge in general. In this case, a typical procedure of research on this topic is
1. How the teachers' knowledge to solve the task
2. Teachers' solution is confronted with various possible students' answers on the task
3. Using point 2, the teachers plan to facilitate the students' understanding on the tasks
In my perspective, the current research missed some of the points.

To conclude, the procedures of the current research were not able to fully reveal the teachers' KCS.
Referring to point 2, I am a bit sceptic whether the manuscript meets the expectation or standard of the journal or not. However, in my personal view, the research need to be extended to address the topic. That is to say, as a reviewer, I decline the manuscript.

Reviewer C:
The manuscript covers an interesting topic. However, there are many changes to be made to improve the readability and cohesiveness, especially for the excerpts of interview transcripts. In relation, the author should place a more in- depth discussion with support from the literature.

I want to suggest that the editor and author make sure the article's English writing level is at least par before sending it to any reviewers. By saying this, I recommend the author to use a proofreader service to get a better quality manuscript.

Reviewer D:
This paper is worthed to be published. However, several additional information is needed and the representation of the data analysis need more effort to make it more comprehensive relate those to the theory about knowledge

Reviewer E:
Ask English expert to consult the paper. Please see the reviewer's comments on the manuscript.

Inbox 406
Starred
Snoozed
Sent
Drafts
Academia dan Res... 19
IConProCS 204
Notes
Publons 2
Rejected Email
Research Gate 38
Sejarah Matematik... 24
More
Meet
New meeting
My meetings
Hangouts
Rully
No recent chats
Start a new one
Inbox 406
Starred
Snoozed
Sent
Drafts
Academia dan Res... 19
IConProCS 204
Notes
Publons 2
Rejected Email
Research Gate 38
Sejarah Matematik... 24
More

Meet

New meeting

My meetings

Hangouts

Rully

No recent chats
Start a new one

Inbox 406
Starred
Snoozed
Sent
Drafts
Academia dan Res... 19
IConProCS 204
Notes
Publons 2
Rejected Email
Research Gate 38
Sejarah Matematik... 24
More

Meet
New meeting
My meetings
Hangouts
Rully
No recent chats
Start a new one
Inbox 406
Starred
Snoozed
Sent
Drafts
Academia dan Res... 19
IConProCS 204
Notes
Publons 2
Rejected Email
Inbox 406
Starred
Snoozed
Sent
Drafts
Academia dan Res... 19
IConProCS 204
Notes
Publons 2
Rejected Email
Research Gate 38
Sejarah Matematik... 24
More
Meet
New meeting
My meetings
Hangouts
Rully
No recent chats
Start a new one

Reviewer F:

I suggest accepting the article with minor changes. However, before accepting the editor should look at the following remark regarding plagiarism.

Many parts of the article are identical to a master thesis by Wahid Yunianto. On "Research Gate," this thesis has the remarkable title "Dissertation Cover Sheet PART 1 -TO BE COMPLETED BY THE STUDENT Student Name."

This article refers both to one author (researcher) and more authors (researchers). If there are more authors a reference to this thesis is needed at all sentences that come from this thesis. If Wahid was the only author, there is the question of self-plagiarism. What is the policy of the journal regarding self-plagiarism? I would think that a remark that this article is an adaptation/modification of that thesis is needed.

If Wahid is not one of the authors, then there is a serious issue with plagiarism and the article should be refused.

I would suggest the author (authors) to use a good spelling and grammar checker to have a better look at the text. It will improve the readability of the article. They could try Grammarly and trinka (or both).

Furthermore, the author must make sure all references have DOI and follow this guideline to ensure that your final file is complete including the blind references from authors and in the correct format (https://bit.ly/33GvXT3) for preparing their paper strictly. You can follow the paper that was already published in JME. The fluent, comprehensible, and correct use of English is the main criterion in publishing proven by proofread certificate from a reputable proof reader (attached in revision submission process). Lastly, the recapitulation of the contents of the revised article and similarity check result file must be attached as a supplementary file in the revision submission process.

To support the cost of wide-open access dissemination of research results, to manage the various costs associated with handling and editing of the submitted manuscripts, and the Journal management and publication in general, the authors or the author's institution is requested to pay the Article Processing Charge (APC).

* Publication fee : USD 250
* Layout & Editing fee : USD 35

---

* Total : USD 285

The above publication fee covers the standard twelve (12) pages manuscript. For every additional page, an extra fee of 30 USD per page will be charged. The Publication fee could be paid by bank transfer to:

Bank Account name (please be exact): Dr. Zulkardi, M.Ikom., M.Sc
Address: BNI, Palembang, Indonesia
Bank Account No. : 0216647385
Bank Name: BANK NEGARA INDONESIA - PT (PERSERO)
City: Palembang
Country: Indonesia
Bank SWIFT Code: BNINIDJAPBG

Please submit the final revised paper along with your payment receipt, Copyright Transfer Agreement, the recapitulation of the contents of the revised article, similarity check result file, and proofread certificate from a reputable proof reader to email: jme@unsri.ac.id or via your JME OJS Account as an Author Version. The revised manuscript should be submitted by March 17, 2021; if you anticipate that you will be unable to meet this deadline, please notify the Editorial Office. Should you have any questions, please do not hesitate to contact us.

Thank you very much for your cooperation. I do really appreciate it.

Kind regards,

Prof. Dr. Zulkardi, M.Ikom., M.Sc.
Editor in Chief
Journal on Mathematics Education

IMPORTANT:

"Please revise and give a comment in the attached file and it is not permitted to send a new article file, other than the revised results of the attached file"

7 Attachments

- 13537-35584-1-RV...
- 13537-35933-1-RV...
- 13537-36158-1-RV...
- 13537-35552-1-RV...
- Recapitulation the ...
- Author_Guidelines_...
- JME_Copyright Tra...

Received, thank you. | Noted with thanks. | Thank you for the feedback.

Reply | Reply all | Forward

Hasil review dari 6 reviewer dengan 4 diantaranya memberikan catatan pada artikel nya secara langsung, yaitu Reviewer A, B, D, dan E

[Paper ID: 13537]

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume xx, No. x, January xxxx, pp. x-xx

INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS: PREDICTING AND RESPONDING TO STUDENTS' RESPONSES TO THE TOPIC OF AREA AND PERIMETER

Abstract

Measuring teachers' skills and competencies is necessary to ensure teachers' quality and contribute to education quality. However, to some extent of teacher assessment has not yet completely covered the full range of teacher skills and competencies. This study investigates focuses on Knowledge of Content and Students (KCS) on the topic of area-perimeter through their designed lesson plans. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items to assess mathematics teacher knowledge are dominated by subject matter knowledge. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' KCS through lesson plans developed by the teachers. Mathematics teachers attended a professional development activity and voluntarily participated in this study. Content analysis of the lesson plan and semi-structured interviews were conducted, and the data analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible methods and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Namun, dalam beberapa hal, penilaian guru belum sepenuhnya mencakup seluruh keterampilan dan kompetensi guru. Fokus penelitian ini adalah menyelidiki *Knowledge of Content and Students* (KCS) pada topik luas dan keliling melalui rancangan rencana pembelajaran mereka. Pengetahuan guru tentang topik luas dan keliling dan strategi pengajaran telah dinilai melalui pengujian. Secara umum, materi untuk menilai pengetahuan guru matematika didominasi oleh materi pengetahuan. Dengan demikian, penilaian tersebut tampaknya belum sepenuhnya mencakup seluruh pengetahuan dan kompetensi guru. Dalam penelitian ini, peneliti menginvestigasi KCS guru matematika melalui RPP yang dikembangkan oleh guru. Guru matematika mengikuti kegiatan pengembangan profesional dan secara sukarela berpartisipasi dalam penelitian ini. Analisis isi RPP dan wawancara semi-terstruktur dilakukan, dan data dianalisis. Hasil penelitian ini mengungkapkan bahwa guru yang berpartisipasi ditantang ketika membuat prediksi kemungkinan tanggapan siswa. Mereka tampaknya tidak menyadari strategi siswa biasa, yang digunakan untuk menyelesaikan memaksimalkan luas dari keliling tertentu. Dengan pengetahuan yang terbatas tentang kemungkinan metode dan kesalahan siswa, para guru ini kurang siap untuk mendukung pembelajaran siswa.

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

How to Cite: Authors. (2021). Indonesian Mathematics Teachers' Knowledge of Content and Students: Predicting and Responding to Students' Responses to the Topic of Area and Perimeter. *Journal on Mathematics Education, x* (x), xx-xx.

Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research have shown that students' achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of

Commented [KW1]: This requires students' works

Commented [KW2]: Overall, the author(s) needs to rework the abstract, from the use of standardized English to the completion of some missing elements, such as how the data, both lesson plans and interview were analyzed; were they using the same content analysis? Furthermore, you need highlight the subjects, were they primary or secondary teachers?

Commented [KW3]: It is better to be consistent using teachers' knowledge

Commented [KW4]: I am not sure which contribute to education quality in this sentence, either teachers' knowledge or teacher's quality. If you mean teachers' quality then it should be 'which' not 'and' before the word 'contribute'

Commented [KW5]: Be careful! The author(s) has full responsibility on the content of this manuscript including English. You have to make sure that the English is standard

Commented [KW6]: In order to be more effective and avoid redundancy, it is better to make this as the opening sentence, merging with the second sentence

Commented [KW7]: Redundant

Commented [KW8]: What do you mean? It is yet unclear

Commented [KW9]: Be consistent! It is better to use strategies

Commented [KW10]: This has a negative tone and multi-interpretative. I, as a reader, might understand this as your PD program poorly prepare them. It is better to have a 'positive tone' for the findings

Commented [KW11]: After carefully reading this introduction, as a reader, I found some notes that might be beneficial for revisions Firstly, there are frequent grammatical errors. I just revised some. Poor English will hinder readability
Secondly, in terms of organization, it is a bit hard to follow the Introduction. Let's take a look what I have got from this and this might happen to other readers.

The research examined (in-services) mathematics' teachers KCS, which is part of PCK using their lessons plan. The rationale for this research is that large assessments have not focused on KCS. To this point, it is great. However, I was wondering, within the prior studies, what this study offers? If it is only to understand teachers' knowledge on content and students, some prior studies have addressed this. They even offer complete pictures of the topic (e.g., Hill, Ball, & Schilling, 2008) since the studies involved planning and real actions in the classroom.

Deleted: has

instruction relates to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

> *Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2013, p.394).*

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

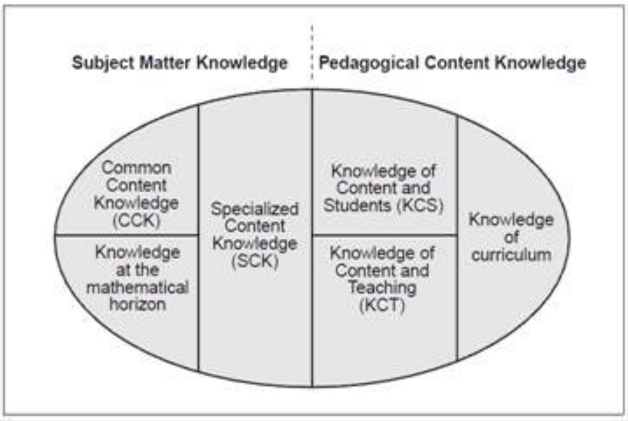


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). Pre-service teachers have challenges with student thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COCATIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin H,

Deleted: is related

Formatted: Font: Not Bold, Not Expanded by / Condensed by

Deleted: Figure 1

Commented [KW12]: I do not why the author(s) thinks this is necessary since this manuscript is about KCS

2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Lesson plans are considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instructions so that teachers can easily understand when reading it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned called 'Bansho', which anticipates student mathematical thinking and student thinking schema for solving the given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed that the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). The students' engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom behaviour is influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions. Therefore, arguably it is possible to look at the teachers' lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

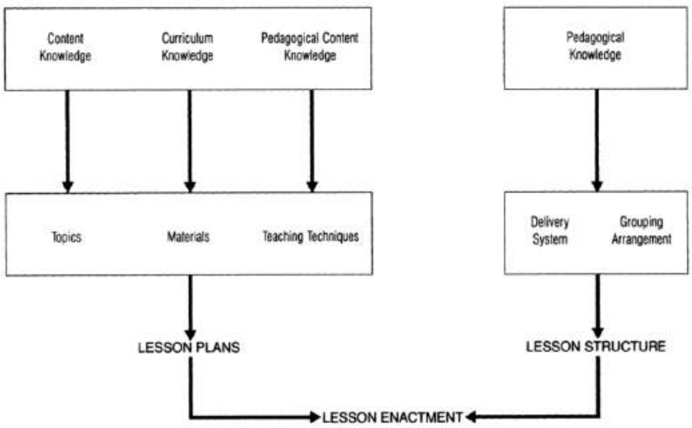


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Deleted: planning
Deleted: is
Commented [KW13]: Are your sure about this diction?
Deleted: who read it
Deleted: it
Deleted: .
Deleted: C
Deleted: it
Deleted: S
Commented [KW14]: it think the word 'practices' is more representative
Formatted: Font: Not Bold, Font color: Black
Formatted: Font: Not Bold, Not Italic, Font color: Black, Check spelling and grammar
Deleted: Figure 2

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zacahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013).

It has been shown that SMK and PCK of mathematics teachers influenced students' performance (Baumert et al., 2010). Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Yeo, 2008).

This study intends to focus on a part of PCK, the KCS within lesson plans on the topic of area-perimeter. It is necessary to obtain a fuller insight into teacher knowledge. How mathematics teachers in Yogyakarta prepare their lesson plans and how is PCK integrated in their lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved human and had been through research ethics approval by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk). This study administrated a case study approach. This approach suits this study as it doesn't seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). Through this approach, the researchers examined two selected lesson plans of the mathematics teachers. The sample was chosen from twenty-nine teachers who attended a Professional Development (PD) session, and two teachers were selected

Deleted: ,

Deleted: pedagogical content knowledge

Commented [KW15]: After reading the introduction with its limitations, I was wondering how the indicators of KCS were examined in this study. So, I decided to jump into Table 1. Well, that is great; four indicators. Then, the question popped up to my mind was how the research procedures facilitate the intended indicators.

Initially, the researchers asked the teachers to devise a lesson plan with the Gold Rush task. The teachers did it and two selected lessons plan (it has not been explicated why chose these two) were examined to reveal teachers' KCS (Table 1). I was thrilled to read the findings; how the content analysis revealed the teachers' KCS. I tried carefully and repeatedly reading the findings to make sure that I did not miss the details. What I found is that the findings do not comprehensively represent the indicators of KCS. The findings of the content analysis on the lessons plan are;

1. Types of instructions (Table 2-3)
2. The delivery of the lessons (Figure 4-6 along with its interpretations and interview scripts)

In my opinion which refer to the indicators of KCS, those findings are not KCS. Finding 2 tends to be teachers' pedagogical knowledge instead of KCS

Scrolling to the next pages of Findings, all of a sudden, I found that the researchers provided a possible students' answer on the task. This procedure has not been explained in the method. Technically, this can be revised. However, the data (interview transcripts) were yet able to reveal the teachers' KCS.

In short, KCS, which is clearly shown in the indicators, is about teachers' mathematical content knowledge and how they utilize this knowledge to facilitate students' understanding of the task or the construction of mathematical knowledge in general. In this case, a typical procedure of research on this topic is

1. How the teachers' knowledge to solve the task
2. Teachers' solution is confronted with various possible students' answers on the task
3. Using point 2, the teachers plan to facilitate the students' understanding on the tasks

In my perspective, the current research missed some of the points.

To conclude, the procedures of the current research were not able to fully reveal the teachers' KCS.

for the lesson plan analysis and interview. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan. The topic that would be taught was area and perimeter for grade 7. The "Gold Rush/Mining" task was selected. This task has several ways to be solved (see

Figure 3).

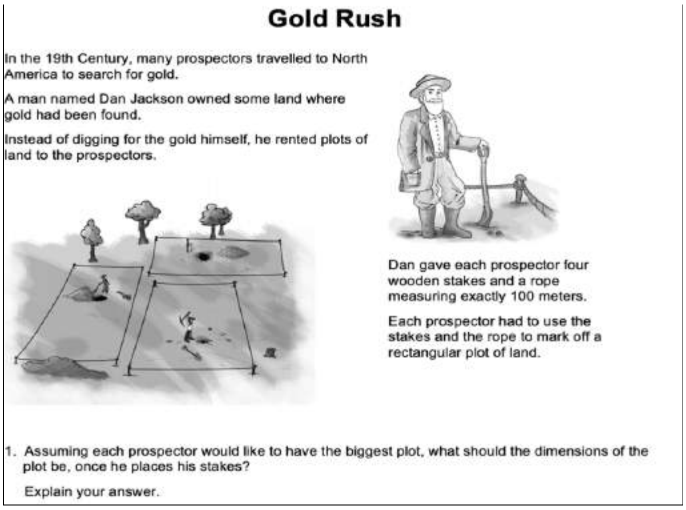


Figure 3. The Gold Rush problem *(https://www.map.mathshell.org/download.php?fileid=1637)*

To analyze the lesson plans, the researchers used content analysis. This method has the 'potential to disclose many hidden aspects of what is being communicated through the written text' (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers' knowledge of students' conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find the themes. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers included information about what students would do to the task. The data were presented descriptively.

Formatted: Font: 11 pt, Not Bold

Formatted: Font: Not Bold

Deleted: Figure 3

Commented [KW16]: pre-conceptions…?

Formatted: Highlight

Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students' emerging and incomplete thinking

The two lesson plans were coded and analyzed. There were three types of instructions to refer to the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3).

Table 3. Proportions of the instructions

Instruction	T1	T2
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	**23 (100%)**	**19 (100%)**

Indonesian teachers follow the prescribed template of a lesson plan. The template consists of three main parts namely; introduction, main and closure. Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2's SID, there were several instructions that provided information relating to PCK (Table 4).

Formatted: Font: Not Bold

Formatted: Font: Not Bold, Not Italic, Check spelling and grammar

Deleted: Table 4

Table 4. Comparison of Instructions

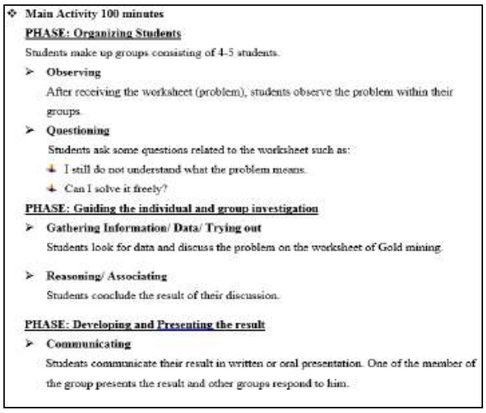
Code	Introduction		Main		Closure	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	**12**	**3**	**7**	**11**	**4**	**5**

T1 put more details of what students would ask to her on her lesson plan. For instance: 'Can I solve it freely?' has been put on her lesson plan. In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4).

Formatted: Font: Not Bold

Formatted: Font: Not Bold, Not Italic, Check spelling and grammar

Deleted: Figure 4



- **Main Activity 100 minutes**

PHASE: Organizing Students

Students make up groups consisting of 4-5 students.

- **Observing**
  After receiving the worksheet (problem), students observe the problem within their groups.
- **Questioning**
  Students ask some questions related to the worksheet such as:
  - I still do not understand what the problem means.
  - Can I solve it freely?

PHASE: Guiding the individual and group investigation

- **Gathering Information/ Data/ Trying out**
  Students look for data and discuss the problem on the worksheet of Gold mining.
- **Reasoning/ Associating**
  Students conclude the result of their discussion.

PHASE: Developing and Presenting the result

- **Communicating**
  Students communicate their result in written or oral presentation. One of the member of the group presents the result and other groups respond to him.

Figure 4. Teacher 1 Lesson Plan

The T2 lesson plan depicted detailed information about a possible student strategy. Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

Formatted: Font: Not Bold

Formatted: Font: Not Bold, Not Italic, Check spelling and grammar

Deleted: Figure 5

Formatted: Highlight

Formatted: Highlight

Main Activity
- ☑ Teacher divide students into groups
- ☑ Teacher delivers the worksheet to be discussed
- ☑ Teacher facilitates the learning processes

- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
  for instance p=10, l=… m then the area = …
  p = 15 m, l = ...m, then the area = ….
  Students determine the largest area by themselves
- for the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
  Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
  How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan

After finding the largest area, students had to find the largest area by joining two miners' ropes. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

Deleted: Figure 6

Formatted: Font: Not Bold

Formatted: Font: Not Bold, Not Italic, Check spelling and grammar

- Students evaluate and make generalisation into questioning.
  - ☑ Teacher asks students to present in front of the class
  - ☑ Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Teachers T1 and T2 have more than five-years teaching experience each. However, their schools are different in terms of location and students. These teachers themselves employed different abilities in solving the Gold Mining problem (

Figure 3). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

Formatted: Font: 11 pt, Not Bold

Formatted: Font: Not Bold

Deleted: Figure 3

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : *Are there other ways T1?*
T1 : *Yesterday, I just did that one.*
T1 : *...just let students find the ways to solve it .... Then, I will let them know that there are some ways to solve it. I give that opportunity to students*

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

T2 : *To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*
R : *So, you (T2), induce them by using the table?*
T2 : *Yes, by the table.*
R : *What do you think, how many ways to solve it?*
T2 : *To me, I did one way I know it directly it would be a square. **I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and.., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

T2 : *I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*
T2 : *However, I have thought only one strategy, which is global to solve a task. ... I, I... know at least I understand my students' characteristic so that it will be difficult for my students. ... It is not possible to come up if I let them to do it freely. ... I am so careful to give it the various strategies because students would get confuse*

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

T2 : By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.
T2 : To me, I did one way I know it directly it would be a square. I knew it already
T2 : …instead of table, we can make the variable x, then I will be a quadratic function,
R : Are there other ways to solve it?
T2 : For the time being, not yet, making rectangles and to the square
R : Do you think there are still other ways to solve that problem?
T2 : I could use the graph …

To some extent, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task. However, by giving the students the strategy, he inadvertently is making the student dependent on him. Whereas, T1 is helping the students to make decisions themselves. From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

*T1 : The task has missing information, it should be more, and some students would think that. So that they **have not thought** yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
*T1 : 100. Maybe **they thought that** that's the only think they know.*
R : … So, they would answer it 100, possibly
T1 : Yeah, possibly
*T2 : … for those who did not understand, **they would not know what 100** m rope is to with the perimeter. So that the concept of perimeter, for those who understood, they already make it but later **they would not think** the rectangles can be varied.*
*T2 : Students **would confuse** the meaning of maximum, which is the largest, **they have not thought about it.** So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7).

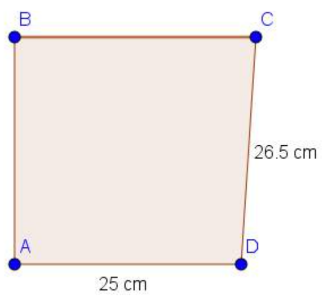


Figure 7. A student's possible mistake proposed by the researcher

Formatted: Font: Not Bold
Formatted: Font: Not Bold, Not Italic, Check spelling and grammar
Deleted: Figure 7

If faced with a student mistake that they not have thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R : If it happens if you see this (showing)
T1 : I would ask students back to try it then you calculate it as what being asked to you
R : They have not yet known the result!
T1 Try, try it, by trialing they would know that it is different, this one is more, and that one is like that, ...
R : T2, what if your students did this? what would you do?
T2 : I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...

In this study, the lesson plans facilitated an insight into teachers' knowledge. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Yeo, 2008).

Teachers have different ways of supporting students to solve tasks (Yei, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task. However, these results are not generalizable. The sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared. Developing higher order thinking and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment.

ACKNOWLEDGMENTS

The authors wish to thank to Ministry of Education of Republic of Indonesia; Planning and Cooperation of Foreign Affairs for the scholarship. The authors also would like to thank SEAMEO QITEP in Mathematics for its endless support.

REFERENCES

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, *7*, 145-172. https://doi.org/10.1023/b:jmte.0000021943.35739.1c

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics, 31*, 235-268. https://doi.org/10.1007/BF00376322

Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project* (pp. 25-48). https://doi.org/10.1007/978-1-4614-5149-5_2

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., … Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*(1), 133-180. https://doi.org/10.3102/0002831209345157

Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' thinking about instruction. *Remedial and Special Education*, *11*(6), 40-49. https://doi.org/10.1177/074193259001100609

Burns, R. B., & Lash, A. A. (1988). Nine seventh-grade teachers' knowledge and planning of problem-

solving instruction. *The Elementary School Journal*, *8*(4), 369-386. https://doi.org/10.1086/461545

Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.

Carle, S. M. (1993). *Student held misconceptions regarding area and perimeter of rectangles* (University of Massachusetts Boston). Retrieved from https://scholarworks.umb.edu/cgi/viewcontent.cgi?article=1045&context=cct_capstone

Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136–143). Adelaide: Australian Association of Mathematics Teachers.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching, 3rd Edition* (pp. 255-296). New York: Macmillan.

Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from lessons: Studying the construction of teacher knowledge catalysed by purposefully-designed experimental mathematics lessons. In M. Marshman, V. Geiger, & A Bennison (Eds.), *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165–172). Sunshine Coast: MEGA.

Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371-406. https://doi.org/10.3102/00028312042002371

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. Journal for *Research in Mathematics Education, 39*, 372-400. Retrieved from https://www.jstor.org/stable/40539304

John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies, 38*(4), 483-498. https://doi.org/10.1080/00220270500363620

Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In S. Johnston-Wilder, C. Lee, & D. Pimm (Eds.), *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience, 3rd Edition* (pp. 79-100). London: Routledge. https://doi.org/10.4324/9780203844120

Korkmaz, H. I., & Şahin, Ö. (2019). Preservice preschool teachers' pedagogical content knowledge on geometric shapes in terms of children's mistakes. *Journal of Research in Childhood Education*, *34*(3), 385-405. https://doi.org/10.1080/02568543.2019.1701150

Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1*. Retrieved from https://files.eric.ed.gov/fulltext/ED452031.pdf

Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *Procedia - Social and Behavioral Sciences, 55*, 720-729. https://doi.org/10.1016/j.sbspro.2012.09.557

Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical content knowledge of mathematics pre-service teachers: Do they know their students? *Journal of Physics: Conference Series*, *1097*(1), 012098. https://doi.org/10.1088/1742-6596/1097/1/012098

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *2*(15), 4-14. https://doi.org/10.3102/0013189X015002004

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, *26*(2), 114-145. https://doi.org/10.2307/749205

Superfine, A. C. (2008). Planning for mathematics instruction: A model of experienced teachers' planning processes in the context of a reform mathematics curriculum. *Mathematics Educator, 18*(2), 11-22. Retrieved from http://tme.journals.libs.uga.edu/index.php/tme/article/view/198

Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., … Rowley, G. (2012). Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). In *International Association for the Evaluation of Educational Achievement*. Retrieved from https://files.eric.ed.gov/fulltext/ED542380.pdf

Turnuklu, E., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, *1*, 1-13. Retrieved from http://www.k-12prep.math.ttu.edu/journal/1.contentknowledge/yesildere01/article.pdf

Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating preservice mathematics teachers' pedagogical content knowledge through microteaching. *REDIMAT-Journal of Research in Mathematics Education*, *9*(1), 62-87. https://doi.org/http://dx.doi.org/10.17583/redimat.2020.3353

Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.

White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2013). Teachers learning from teachers. In M.A. Clements, A.J. Bishop, C. Keitel, J. Kilpatrick, & F.K.S. Leung (Eds.), *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13

Widodo, & Tamimudin, H. M. (2014). Three training strategies for improving mathematics teacher competence in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Retrieved from http://atcm.mathandtech.org/EP2014/index.html

Yeo, K. K. Y. (2008). Teaching area and perimeter : Mathematics-pedagogical-content knowledge-in-action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*. Retrieved from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.471.9965&rep=rep1&type=pdf

Yin, R. K. (2014). Case Study Research: Design and Methods (5th ed.). *Thousand Oaks: SAGE Publications*.

Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, *6*(2), 41-62. https://doi.org/10.26220/rev.1627

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume xx, No. x, January xxxx, pp. x-xx

INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS: PREDICTING AND RESPONDING TO STUDENTS' RESPONSES TO THE TOPIC OF AREA AND PERIMETER

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. However, to some extent of teacher assessment has not yet completely covered the full range of teacher skills and competencies. This study investigates focuses on Knowledge of Content and Students (KCS) on the topic of area-perimeter through their designed lesson plans. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items to assess mathematics teacher knowledge are dominated by subject matter knowledge. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' KCS through lesson plans developed by the teachers. Mathematics teachers attended a professional development activity and voluntarily participated in this study. Content analysis of the lesson plan and semi-structured interviews were conducted, and the data analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible methods and mistakes, these teachers were poorly prepared to support student learning.

Commented [A1]: Is this true? In many cases also portfolios and reflections on lesson designs and lessons play important roles. Reference?

Commented [A2]: Repetition of second sentence in this abstract.

Commented [A3]: How many? What context? (e.g. primary, lower, upper secondary, …)

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Namun, dalam beberapa hal, penilaian guru belum sepenuhnya mencakup seluruh keterampilan dan kompetensi guru. Fokus penelitian ini adalah menyelidiki *Knowledge of Content and Students* (KCS) pada topik luas dan keliling melalui rancangan rencana pembelajaran mereka. Pengetahuan guru tentang topik luas dan keliling dan strategi pengajaran telah dinilai melalui pengujian. Secara umum, materi untuk menilai pengetahuan guru matematika didominasi oleh materi pengetahuan. Dengan demikian, penilaian tersebut tampaknya belum sepenuhnya mencakup seluruh pengetahuan dan kompetensi guru. Dalam penelitian ini, peneliti menginvestigasi KCS guru matematika melalui RPP yang dikembangkan oleh guru. Guru matematika mengikuti kegiatan pengembangan profesional dan secara sukarela berpartisipasi dalam penelitian ini. Analisis isi RPP dan wawancara semi-terstruktur dilakukan, dan data dianalisis. Hasil penelitian ini mengungkapkan bahwa guru yang berpartisipasi ditantang ketika membuat prediksi kemungkinan tanggapan siswa. Mereka tampaknya tidak menyadari strategi siswa biasa, yang digunakan untuk menyelesaikan memaksimalkan luas dari keliling tertentu. Dengan pengetahuan yang terbatas tentang kemungkinan metode dan kesalahan siswa, para guru ini kurang siap untuk mendukung pembelajaran siswa.

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

How to Cite: Authors. (2021). Indonesian Mathematics Teachers' Knowledge of Content and Students: Predicting and Responding to Students' Responses to the Topic of Area and Perimeter. *Journal on Mathematics Education, x* (x), xx-xx.

Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research has shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related

to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

Deleted: Figure 1

Formatted: Font: Not Bold, Not Expanded by / Condensed by

Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2013, p.394).

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

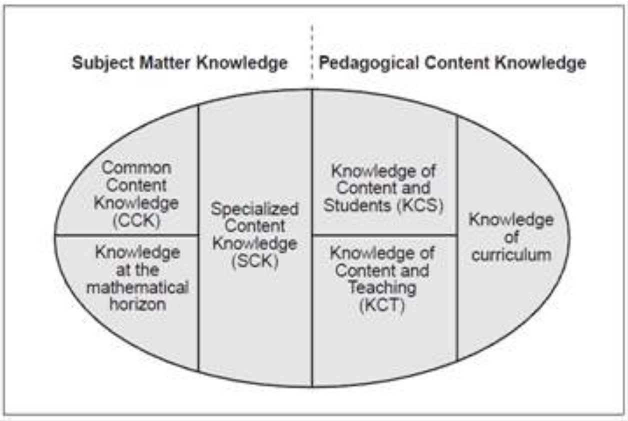


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). Pre-service teachers have challenges with student thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COCATIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

Commented [A4]: How does this connect to the previous sentence? Is this a critique for assessing with micro teaching?

Commented [A5]: Why? In what direction? (that helps to understand the problem that you try to tackle.)

The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin H,

Commented [A6]: Is this the only measure to evaluate teachers? Maybe include references to initiatives in many countries for securing content knowledge of (future) teachers.

2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Lesson planning is considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers who read it can easily understand it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called 'Bansho', it anticipates student mathematical thinking and student thinking schema for solving given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). Student engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom behaviour is influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

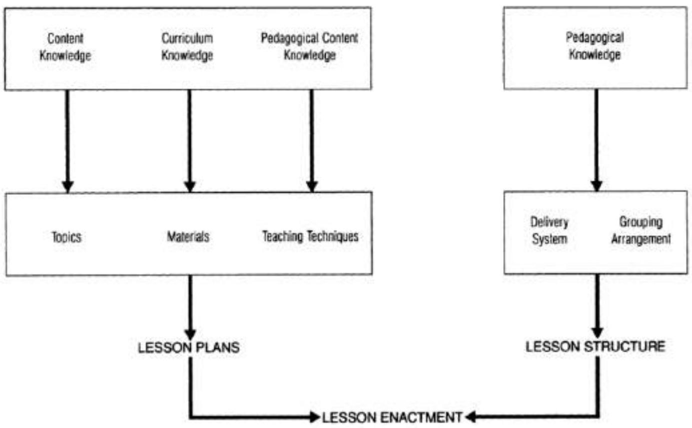


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Commented [A7]: ...and tries to elicit...

Formatted: Font: Not Bold, Font color: Black

Deleted: Figure 2

Formatted: Font: Not Bold, Not Italic, Font color: Black, Check spelling and grammar

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zacahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013).

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason, (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Yeo, 2008).

This study intends to focus on a part of PCK pedagogical content knowledge, the KCS within lesson plans on the topic of area-perimeter. It is necessary to obtain a fuller insight into teacher knowledge. How mathematics teachers in Yogyakarta prepare their lesson plans and how is PCK integrated in their lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved humans and has been approved by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk). This study administrated a case study approach. This approach suits this study as it doesn't seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). Through this approach, the researchers examined two selected lesson plans of two mathematics teachers. The sample was chosen from twenty-nine teachers who attended a Professional Development (PD) session, and two teachers were selected for the lesson plan

Commented [A8]: It would be helpful when you can provide a more strong argument to use this domain for this study (e.g. these misconceptions and potential starting points for learning make it a rich domain for…)

Commented [A9]: The RQ could more explicitly focus on whether this method can be used to evaluate teachers' PCK.

Deleted: d

Deleted: through research ethics approval

Formatted: Normal, Justified, Indent: First line: 1 cm, Line spacing: single

Deleted: the

Commented [A10]: How?

analysis and interview. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan. The topic that would be taught was area and perimeter for grade 7. The "Gold Rush/Mining" task was selected. This task has several ways to be solved (see

Figure 3).

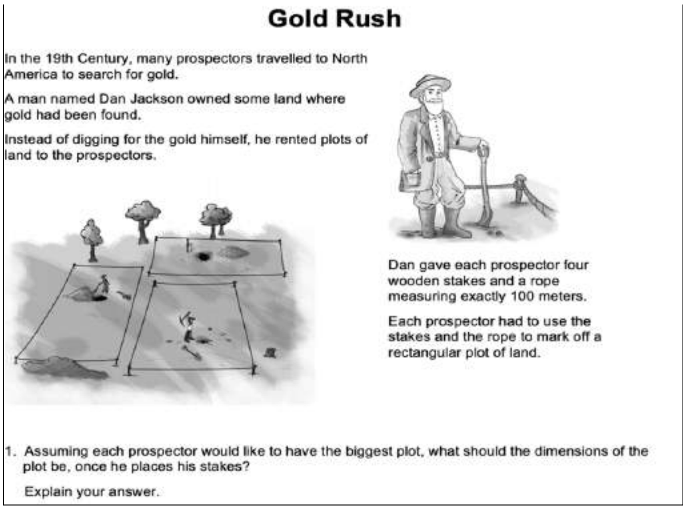


Figure 3. The Gold Rush problem *(https://www.map.mathshell.org/download.php?fileid=1637)*

To analyze the lesson plans, the researchers used content analysis. This method has the 'potential to disclose many hidden aspects of what is being communicated through the written text' (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers' knowledge of students' conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find the themes. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers included information about what students would do to the task. The data were presented descriptively.

Commented [A11]: Since this is a case study with only two selected teachers it would be helpful to also provide information about the rest of the group to have an understanding to what extent these two are representative for the whole group.

Commented [A12]: I am not sure about the whole procedure of the study. Were the teachers first following a PD and at the end asked to provide a lesson plan, or was at the start, or at the middle of the course? How could the participating teachers have interpreted this request of designing a lesson plan and what experiences within the course were input for their design?

Formatted: Font: 11 pt, Not Bold

Formatted: Font: Not Bold

Deleted: Figure 3

Commented [A13]: How was the code book developed? Based on theory or through a bottom up approach? And how reliable was the coding? Please add information on a reliability check.

Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students' emerging and incomplete thinking

The two lesson plans were coded and analyzed. There were three types of instructions to refer to with the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

Commented [A14]: How?

RESULTS AND DISCUSSION

Commented [A15]: I prefer to separate a presentation of the results from a discussion of the results.

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3).

Table 3. Proportions of the instructions

Instruction	T1	T2
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	**23 (100%)**	**19 (100%)**

Indonesian teachers follow the prescribed template of a lesson plan. The template consists of three main parts namely; introduction, main and closure. Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2's SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	Introduction		Main		Closure	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	**12**	**3**	**7**	**11**	**4**	**5**

T1 put more details of what students would ask to her on her lesson plan. For instance: 'Can I solve it freely?' has been put on her lesson plan. In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4).

❖ **Main Activity 100 minutes**

PHASE: Organizing Students

Students make up groups consisting of 4-5 students.

- **Observing**

  After receiving the worksheet (problem), students observe the problem within their groups.

- **Questioning**

  Students ask some questions related to the worksheet such as:
  - I still do not understand what the problem means.
  - Can I solve it freely?

PHASE: Guiding the individual and group investigation

- **Gathering Information/ Data/ Trying out**

  Students look for data and discuss the problem on the worksheet of Gold mining.

- **Reasoning/ Associating**

  Students conclude the result of their discussion.

PHASE: Developing and Presenting the result

- **Communicating**

  Students communicate their result in written or oral presentation. One of the member of the group presents the result and other groups respond to him.

Figure 4. Teacher 1 Lesson Plan

Commented [A16]: All? Part of instruction in teacher education?

Commented [A17]: Did the template require them to formulate learning goals? (or misconceptions being elicited and solved?) What is the teacher's purpose for organizing a lesson with the problem?

Formatted: Font: Not Bold, Not Italic, Check spelling and grammar

Deleted: Table 4

Formatted: Font: Not Bold

Deleted: Figure 4

Formatted: Font: Not Bold

Formatted: Font: Not Bold, Not Italic, Check spelling and grammar

The T2 lesson plan depicted detailed information about a possible student strategy. Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

Main Activity

- ☑ Teacher divide students into groups
- ☑ Teacher delivers the worksheet to be discussed
- ☑ Teacher facilitates the learning processes

- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
  for instance p=10, l=... m then the area = ...
  p = 15 m, l = ...m, then the area = ...
  Students determine the largest area by themselves
- for the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
  Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
  How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan

After finding the largest area, students had to find the largest area by joining two miners' ropes. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

- Students evaluate and make generalisation into questioning.
  - ☑ Teacher asks students to present in front of the class
  - ☑ Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Formatted: Font: Not Bold

Formatted: Font: Not Bold, Not Italic, Check spelling and grammar

Deleted: Figure 5

Formatted: Font: Not Bold

Formatted: Font: Not Bold, Not Italic, Check spelling and grammar

Deleted: Figure 6

Teachers T1 and T2 have more than five-years teaching experience each. However, their schools are different in terms of location and students. These teachers themselves employed different abilities in solving the Gold Mining problem (

Figure 3). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : Are there other ways T1?
T1 : Yesterday, I just did that one.
T1 : ...just let students find the ways to solve it .... Then, I will let them know that there are some ways to solve it. I give that opportunity to students

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

T2 : To me, I could do it directly because ***I already knew it*** *but to students if I want to students to learn it,* ***I make a table for them****. If the table is not made, students will find it difficult to solve it for students in my school.*
R : So, you (T2), induce them by using the table?
T2 : Yes, by the table.
R : What do you think, how many ways to solve it?
*T2 : To me, I did one way I know it directly it would be a square****. I knew it already****. But for students,* ***with table****, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and.., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

*T2 : I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (****metaphor****).*
T2 : However, I have thought only one strategy, which is global to solve a task. ... I, I... know at least I understand my students' characteristic so that it will be difficult for my students. ... It is not possible to come up if I let them to do it freely. ... I am so careful to give it the various strategies because students would get confuse

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

Formatted: Font: 11 pt, Not Bold

Formatted: Font: Not Bold

Deleted: Figure 3

T2 : *By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.*
T2 : *To me, I did one way I know it directly it would be a square. I knew it already*
T2 : *…instead of table, we can make the variable x, then I will be a quadratic function,*
R : *Are there other ways to solve it?*
T2 : *For the time being, not yet, making rectangles and to the square*
R : *Do you think there are still other ways to solve that problem?*
T2 : *I could use the graph …*

To some extent, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task. However, by giving the students the strategy, he inadvertently is making the students dependent on him. Whereas, T1 is helping the students to make decisions themselves. From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

Commented [A18]: Well, is that visible in the lesson plan? How to guide the students from West to East? This problem is also designed to create opportunities for discussing and developing inquiry skills in mathematics: the importance of first trying some cases, try to see what happens, organize calculations, and organize them systematically to find a pattern. This only works when you address these issues explicitly. A teacher modelling a solution strategy (e.g. by providing a table) can have a role in this learning process, as long as (s)he is aware of this process (next step might be to give students the responsibility in organizing calculations).

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

T1 : *The task has missing information, it should be more, and some students would think that. So that they **have not thought** yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
T1 : *100. Maybe **they thought that** that's the only think they know.*
R : *… So, they would answer it 100, possibly*
T1 : *Yeah, possibly*
T2 : *… for those who did not understand, **they would not know what 100** m rope is to with the perimeter. So that the concept of perimeter, for those who understood, they already make it but later **they would not think** the rectangles can be varied.*
T2 : *Students **would confuse** the meaning of maximum, which is the largest, **they have not thought about it.** So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7).

Formatted: Font: Not Bold
Deleted: Figure 7
Formatted: Font: Not Bold, Not Italic, Check spelling and grammar

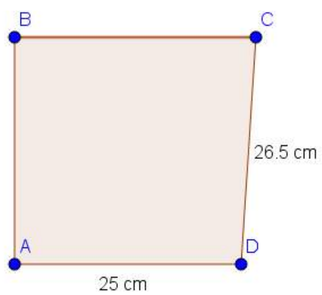


Figure 7. A student's possible mistake proposed by the researcher

If faced with a student mistake that they not have thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R : *If it happens if you see this (showing)*
T1 : *I would ask students back to try it then you calculate it as what being asked to you*
R : *They have not yet known the result!*
T1 *Try, try it, by trialing they would know that it is different, this one is more, and that one is like that, ...*
R : *T2, what if your students did this? what would you do?*
T2 : *I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...*

In this study, the lesson plans facilitated an insight into teachers' knowledge. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Yeo, 2008).

Commented [A19]: The section below seems to be a discussion of the results. New heading?

Commented [A20]: Well, for me the interview was more informative than the only the lesson plan. And still not fully clear to me what the student teachers' intentions were with their lesson plans for this particular problem.

Teachers have different ways of supporting students to solve tasks (Yei, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task. However, these results are not generalizable. The sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared. Developing higher order thinking and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment.

Commented [A21]: Please be more modest (e.g. aspects of KCS in the domain of area during a PD course… etc.).

Commented [A22]: Yes!

Commented [A23]: For designing lesson plans? Or for teaching? These conclusions seem to generalize (too) quick…

Commented [A24]: What is this? Not part of the paper…

Commented [A25]: Somewhere in a learning trajectory it can be helpful to model as a teacher a particular way. Don't suggest that this is always wrong… (or provide a more clear argument).

ACKNOWLEDGMENTS

The authors wish to thank to Ministry of Education of Republic of Indonesia; Planning and Cooperation of Foreign Affairs for the scholarship. The authors also would like to thank SEAMEO QITEP in Mathematics for its endless support.

REFERENCES

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, *7*, 145-172. https://doi.org/10.1023/b:jmte.0000021943.35739.1c

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics, 31*, 235-268. https://doi.org/10.1007/BF00376322

Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project* (pp. 25-48). https://doi.org/10.1007/978-1-4614-5149-5_2

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., … Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*(1), 133-180. https://doi.org/10.3102/0002831209345157

Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' thinking about instruction. *Remedial and Special Education*, *11*(6), 40-49. https://doi.org/10.1177/074193259001100609

Burns, R. B., & Lash, A. A. (1988). Nine seventh-grade teachers' knowledge and planning of problem-

solving instruction. *The Elementary School Journal*, *8*(4), 369-386. https://doi.org/10.1086/461545

Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.

Carle, S. M. (1993). *Student held misconceptions regarding area and perimeter of rectangles* (University of Massachusetts Boston). Retrieved from https://scholarworks.umb.edu/cgi/viewcontent.cgi?article=1045&context=cct_capstone

Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136–143). Adelaide: Australian Association of Mathematics Teachers.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching, 3rd Edition* (pp. 255-296). New York: Macmillan.

Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from lessons: Studying the construction of teacher knowledge catalysed by purposefully-designed experimental mathematics lessons. In M. Marshman, V. Geiger, & A Bennison (Eds.), *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165–172). Sunshine Coast: MEGA.

Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371-406. https://doi.org/10.3102/00028312042002371

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. Journal for *Research in Mathematics Education, 39*, 372-400. Retrieved from https://www.jstor.org/stable/40539304

John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies, 38*(4), 483-498. https://doi.org/10.1080/00220270500363620

Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In S. Johnston-Wilder, C. Lee, & D. Pimm (Eds.), *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience, 3rd Edition* (pp. 79-100). London: Routledge. https://doi.org/10.4324/9780203844120

Korkmaz, H. I., & Şahin, Ö. (2019). Preservice preschool teachers' pedagogical content knowledge on geometric shapes in terms of children's mistakes. *Journal of Research in Childhood Education*, *34*(3), 385-405. https://doi.org/10.1080/02568543.2019.1701150

Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1*. Retrieved from https://files.eric.ed.gov/fulltext/ED452031.pdf

Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *Procedia - Social and Behavioral Sciences, 55*, 720-729. https://doi.org/10.1016/j.sbspro.2012.09.557

Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical content knowledge of mathematics pre-service teachers: Do they know their students? *Journal of Physics: Conference Series*, *1097*(1), 012098. https://doi.org/10.1088/1742-6596/1097/1/012098

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *2*(15), 4-14. https://doi.org/10.3102/0013189X015002004

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, *26*(2), 114-145. https://doi.org/10.2307/749205

Superfine, A. C. (2008). Planning for mathematics instruction: A model of experienced teachers' planning processes in the context of a reform mathematics curriculum. *Mathematics Educator, 18*(2), 11-22. Retrieved from http://tme.journals.libs.uga.edu/index.php/tme/article/view/198

Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., … Rowley, G. (2012). Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). In *International Association for the Evaluation of Educational Achievement*. Retrieved from https://files.eric.ed.gov/fulltext/ED542380.pdf

Turnuklu, E., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, *1*, 1-13. Retrieved from http://www.k-12prep.math.ttu.edu/journal/1.contentknowledge/yesildere01/article.pdf

Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating preservice mathematics teachers' pedagogical content knowledge through microteaching. *REDIMAT-Journal of Research in Mathematics Education*, *9*(1), 62-87. https://doi.org/http://dx.doi.org/10.17583/redimat.2020.3353

Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.

White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2013). Teachers learning from teachers. In M.A. Clements, A.J. Bishop, C. Keitel, J. Kilpatrick, & F.K.S. Leung (Eds.), *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13

Widodo, & Tamimudin, H. M. (2014). Three training strategies for improving mathematics teacher competence in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Retrieved from http://atcm.mathandtech.org/EP2014/index.html

Yeo, K. K. Y. (2008). Teaching area and perimeter : Mathematics-pedagogical-content knowledge-in-action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*. Retrieved from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.471.9965&rep=rep1&type=pdf

Yin, R. K. (2014). Case Study Research: Design and Methods (5th ed.). *Thousand Oaks: SAGE Publications*.

Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, *6*(2), 41-62. https://doi.org/10.26220/rev.1627

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume xx, No. x, January xxxx, pp. x-xx

INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS: PREDICTING AND RESPONDING TO STUDENTS' RESPONSES TO THE TOPIC OF AREA AND PERIMETER

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. However, to some extent of teacher assessment has not yet completely covered the full range of teacher skills and competencies. This study investigates focuses on Knowledge of Content and Students (KCS) on the topic of area-perimeter through their designed lesson plans. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items to assess mathematics teacher knowledge are dominated by subject matter knowledge. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' KCS through lesson plans developed by the teachers. Mathematics teachers attended a professional development activity and voluntarily participated in this study. Content analysis of the lesson plan and semi-structured interviews were conducted, and the data analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible methods and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Namun, dalam beberapa hal, penilaian guru belum sepenuhnya mencakup seluruh keterampilan dan kompetensi guru. Fokus penelitian ini adalah menyelidiki *Knowledge of Content and Students* (KCS) pada topik luas dan keliling melalui rancangan rencana pembelajaran mereka. Pengetahuan guru tentang topik luas dan keliling dan strategi pengajaran telah dinilai melalui pengujian. Secara umum, materi untuk menilai pengetahuan guru matematika didominasi oleh materi pengetahuan. Dengan demikian, penilaian tersebut tampaknya belum sepenuhnya mencakup seluruh pengetahuan dan kompetensi guru. Dalam penelitian ini, peneliti menginvestigasi KCS guru matematika melalui RPP yang dikembangkan oleh guru. Guru matematika mengikuti kegiatan pengembangan profesional dan secara sukarela berpartisipasi dalam penelitian ini. Analisis isi RPP dan wawancara semi-terstruktur dilakukan, dan data dianalisis. Hasil penelitian ini mengungkapkan bahwa guru yang berpartisipasi ditantang ketika membuat prediksi kemungkinan tanggapan siswa. Mereka tampaknya tidak menyadari strategi siswa biasa, yang digunakan untuk menyelesaikan memaksimalkan luas dari keliling tertentu. Dengan pengetahuan yang terbatas tentang kemungkinan metode dan kesalahan siswa, para guru ini kurang siap untuk mendukung pembelajaran siswa.

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

How to Cite: Authors. (2021). Indonesian Mathematics Teachers' Knowledge of Content and Students: Predicting and Responding to Students' Responses to the Topic of Area and Perimeter. *Journal on Mathematics Education, x* (x), xx-xx.

Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research has shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related

to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

> *Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2013, p.394).*

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

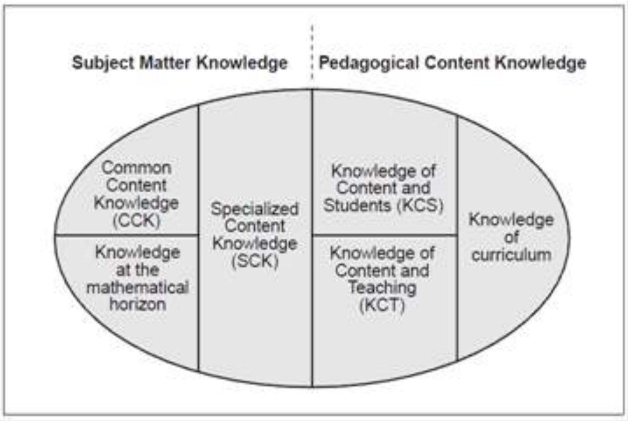


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). Pre-service teachers have challenges with student thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COCATIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin H,

2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Lesson planning is considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers who read it can easily understand it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called 'Bansho', it anticipates student mathematical thinking and student thinking schema for solving given problems.

Commented [Ma1]: Author needs to explain current research in PCK..there are several research in relation to PCK and describe the gap

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). Student engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom behaviour is influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

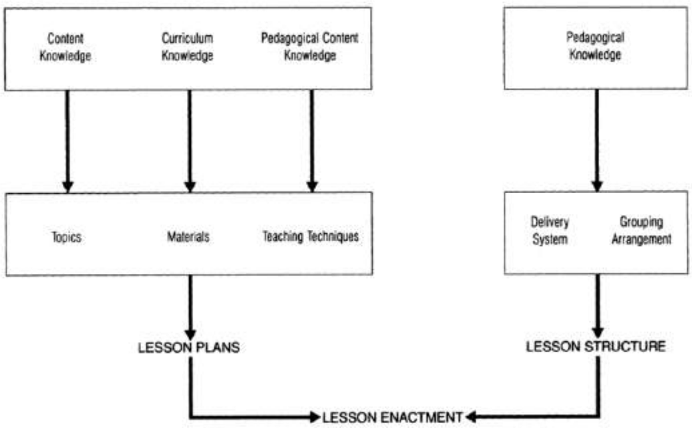


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zacahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013).

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason, (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Yeo, 2008).

This study intends to focus on a part of PCK pedagogical content knowledge, the KCS within lesson plans on the topic of area-perimeter. It is necessary to obtain a fuller insight into teacher knowledge. How mathematics teachers in Yogyakarta prepare their lesson plans and how is PCK integrated in their lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved human and had been through research ethics approval by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk). This study administrated a case study approach. This approach suits this study as it doesn't seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). Through this approach, the researchers examined two selected lesson plans of the mathematics teachers. The sample was chosen from twenty-nine teachers who attended a Professional Development (PD) session, and two teachers were selected

for the lesson plan analysis and interview. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan. The topic that would be taught was area and perimeter for grade 7. The "Gold Rush/Mining" task was selected. This task has several ways to be solved (see

Figure ***3***).

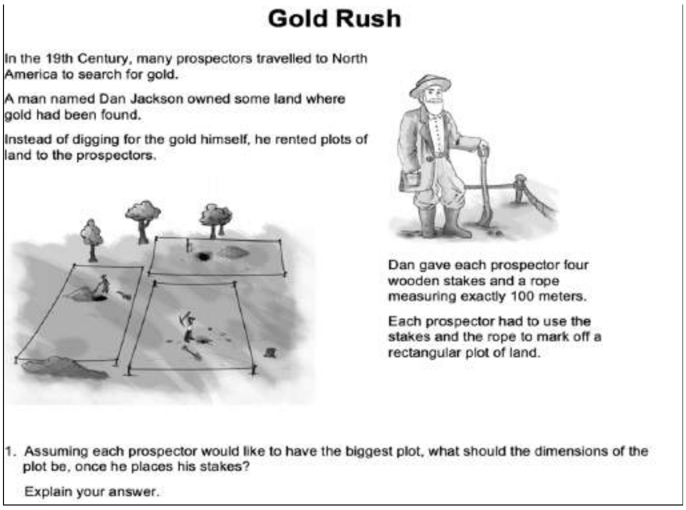


Figure 3. The Gold Rush problem *(https://www.map.mathshell.org/download.php?fileid=1637)*

To analyze the lesson plans, the researchers used content analysis. This method has the 'potential to disclose many hidden aspects of what is being communicated through the written text' (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers' knowledge of students' conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find the themes. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers included information about what students would do to the task. The data were presented descriptively.

Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students' emerging and incomplete thinking

Commented [Ma2]: How this part is coded in the coding for instruction

The two lesson plans were coded and analyzed. There were three types of instructions to refer to the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3).

Table 3. Proportions of the instructions

Instruction	T1	T2
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	**23 (100%)**	**19 (100%)**

Commented [Ma3]: How the percentage come out?

Indonesian teachers follow the prescribed template of a lesson plan. The template consists of three main parts namely; introduction, main and closure. Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2's SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	Introduction		Main		Closure	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	**12**	**3**	**7**	**11**	**4**	**5**

T1 put more details of what students would ask to her on her lesson plan. For instance: 'Can I solve it freely?' has been put on her lesson plan. In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4).

- **Main Activity 100 minutes**

  PHASE: Organizing Students

  Students make up groups consisting of 4-5 students.

  - **Observing**

    After receiving the worksheet (problem), students observe the problem within their groups.
  - **Questioning**

    Students ask some questions related to the worksheet such as:
    - I still do not understand what the problem means.
    - Can I solve it freely?

  PHASE: Guiding the individual and group investigation

  - **Gathering Information/ Data/ Trying out**

    Students look for data and discuss the problem on the worksheet of Gold mining.
  - **Reasoning/ Associating**

    Students conclude the result of their discussion.

  PHASE: Developing and Presenting the result

  - **Communicating**

    Students communicate their result in written or oral presentation. One of the member of the group presents the result and other groups respond to him.

Figure 4. Teacher 1 Lesson Plan

The T2 lesson plan depicted detailed information about a possible student strategy. Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

Main Activity

- ☑ Teacher divide students into groups
- ☑ Teacher delivers the worksheet to be discussed
- ☑ Teacher facilitates the learning processes

- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
  for instance p=10, l=… m then the area = …
  p = 15 m, l = ...m, then the area = ….
  Students determine the largest area by themselves
- for the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
  Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
  How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan

After finding the largest area, students had to find the largest area by joining two miners' ropes. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

- Students evaluate and make generalisation into questioning.
  - ☑ Teacher asks students to present in front of the class
  - ☑ Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Teachers T1 and T2 have more than five-years teaching experience each. However, their schools are different in terms of location and students. These teachers themselves employed different abilities in solving the Gold Mining problem (

Figure ***3***). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : Are there other ways T1?
T1 : Yesterday, I just did that one.
T1 : ...just let students find the ways to solve it .... Then, I will let them know that there are some ways to solve it. I give that opportunity to students

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

*T2 : To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*
R : So, you (T2), induce them by using the table?
T2 : Yes, by the table.
R : What do you think, how many ways to solve it?
*T2 : To me, I did one way I know it directly it would be a square**. I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and.., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

*T2 : I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*
T2 : However, I have thought only one strategy, which is global to solve a task. ... I, I... know at least I understand my students' characteristic so that it will be difficult for my students. ... It is not possible to come up if I let them to do it freely. ... I am so careful to give it the various strategies because students would get confuse

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

T2 : *By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.*
T2 : *To me, I did one way I know it directly it would be a square. I knew it already*
T2 : *…instead of table, we can make the variable x, then I will be a quadratic function,*
R : *Are there other ways to solve it?*
T2 : *For the time being, not yet, making rectangles and to the square*
R : *Do you think there are still other ways to solve that problem?*
T2 : *I could use the graph …*

To some extent, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task. However, by giving the students the strategy, he inadvertently is making the student dependent on him. Whereas, T1 is helping the students to make decisions themselves. From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

T1 : *The task has missing information, it should be more, and some students would think that. So that they* ***have not thought*** *yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
T1 : *100. Maybe* ***they thought that*** *that's the only think they know.*
R : *… So, they would answer it 100, possibly*
T1 : *Yeah, possibly*
T2 : *… for those who did not understand,* ***they would not know what 100*** *m rope is to with the perimeter. So that the concept of perimeter, for those who understood, they already make it but later* ***they would not think*** *the rectangles can be varied.*
T2 : *Students* ***would confuse*** *the meaning of maximum, which is the largest,* ***they have not thought about it.*** *So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7).

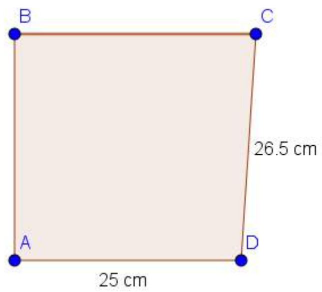


Figure 7. A student's possible mistake proposed by the researcher

If faced with a student mistake that they not have thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R : *If it happens if you see this (showing)*
T1 : *I would ask students back to try it then you calculate it as what being asked to you*
R : *They have not yet known the result!*
T1 *Try, try it, by trialing they would know that it is different, this one is more, and that one is like that, ...*
R : *T2, what if your students did this? what would you do?*
T2 : *I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...*

In this study, the lesson plans facilitated an insight into teachers' knowledge. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Yeo, 2008).

Teachers have different ways of supporting students to solve tasks (Yei, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task. However, these results are not generalizable. The sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

Commented [Ma4]: This result is important and can be related to the framework PCK

CONCLUSION

This study indicates that it is possible to assess teachers' KCS through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared. Developing higher order thinking and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment.

ACKNOWLEDGMENTS

The authors wish to thank to Ministry of Education of Republic of Indonesia; Planning and Cooperation of Foreign Affairs for the scholarship. The authors also would like to thank SEAMEO QITEP in Mathematics for its endless support.

REFERENCES

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, *7*, 145-172. https://doi.org/10.1023/b:jmte.0000021943.35739.1c

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics, 31*, 235-268. https://doi.org/10.1007/BF00376322

Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project* (pp. 25-48). https://doi.org/10.1007/978-1-4614-5149-5_2

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., … Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*(1), 133-180. https://doi.org/10.3102/0002831209345157

Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' thinking about instruction. *Remedial and Special Education*, *11*(6), 40-49. https://doi.org/10.1177/074193259001100609

Burns, R. B., & Lash, A. A. (1988). Nine seventh-grade teachers' knowledge and planning of problem-

solving instruction. *The Elementary School Journal*, *8*(4), 369-386. https://doi.org/10.1086/461545

Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.

Carle, S. M. (1993). *Student held misconceptions regarding area and perimeter of rectangles* (University of Massachusetts Boston). Retrieved from https://scholarworks.umb.edu/cgi/viewcontent.cgi?article=1045&context=cct_capstone

Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136–143). Adelaide: Australian Association of Mathematics Teachers.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching, 3rd Edition* (pp. 255-296). New York: Macmillan.

Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from lessons: Studying the construction of teacher knowledge catalysed by purposefully-designed experimental mathematics lessons. In M. Marshman, V. Geiger, & A Bennison (Eds.), *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165–172). Sunshine Coast: MEGA.

Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371-406. https://doi.org/10.3102/00028312042002371

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. Journal for *Research in Mathematics Education, 39*, 372-400. Retrieved from https://www.jstor.org/stable/40539304

John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies, 38*(4), 483-498. https://doi.org/10.1080/00220270500363620

Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In S. Johnston-Wilder, C. Lee, & D. Pimm (Eds.), *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience, 3rd Edition* (pp. 79-100). London: Routledge. https://doi.org/10.4324/9780203844120

Korkmaz, H. I., & Şahin, Ö. (2019). Preservice preschool teachers' pedagogical content knowledge on geometric shapes in terms of children's mistakes. *Journal of Research in Childhood Education*, *34*(3), 385-405. https://doi.org/10.1080/02568543.2019.1701150

Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1*. Retrieved from https://files.eric.ed.gov/fulltext/ED452031.pdf

Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *Procedia - Social and Behavioral Sciences, 55*, 720-729. https://doi.org/10.1016/j.sbspro.2012.09.557

Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical content knowledge of mathematics pre-service teachers: Do they know their students? *Journal of Physics: Conference Series*, *1097*(1), 012098. https://doi.org/10.1088/1742-6596/1097/1/012098

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *2*(15), 4-14. https://doi.org/10.3102/0013189X015002004

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, *26*(2), 114-145. https://doi.org/10.2307/749205

Superfine, A. C. (2008). Planning for mathematics instruction: A model of experienced teachers' planning processes in the context of a reform mathematics curriculum. *Mathematics Educator, 18*(2), 11-22. Retrieved from http://tme.journals.libs.uga.edu/index.php/tme/article/view/198

Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., … Rowley, G. (2012). Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). In *International Association for the Evaluation of Educational Achievement*. Retrieved from https://files.eric.ed.gov/fulltext/ED542380.pdf

Turnuklu, E., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, *1*, 1-13. Retrieved from http://www.k-12prep.math.ttu.edu/journal/1.contentknowledge/yesildere01/article.pdf

Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating preservice mathematics teachers' pedagogical content knowledge through microteaching. *REDIMAT-Journal of Research in Mathematics Education*, *9*(1), 62-87. https://doi.org/http://dx.doi.org/10.17583/redimat.2020.3353

Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.

White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2013). Teachers learning from teachers. In M.A. Clements, A.J. Bishop, C. Keitel, J. Kilpatrick, & F.K.S. Leung (Eds.), *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13

Widodo, & Tamimudin, H. M. (2014). Three training strategies for improving mathematics teacher competence in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Retrieved from http://atcm.mathandtech.org/EP2014/index.html

Yeo, K. K. Y. (2008). Teaching area and perimeter : Mathematics-pedagogical-content knowledge-in-action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*. Retrieved from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.471.9965&rep=rep1&type=pdf

Yin, R. K. (2014). Case Study Research: Design and Methods (5th ed.). *Thousand Oaks: SAGE Publications*.

Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, *6*(2), 41-62. https://doi.org/10.26220/rev.1627

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume xx, No. x, January xxxx, pp. x-xx

INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS: PREDICTING AND RESPONDING TO STUDENTS' RESPONSES TO THE TOPIC OF AREA AND PERIMETER

Commented [a1]: Of what: rectangles, squares, parallelogram, rhombus?

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. However, to some extent of teacher assessment has not yet completely covered the full range of teacher skills and competencies. This study investigates focuses on Knowledge of Content and Students (KCS) on the topic of area-perimeter through their designed lesson plans. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items to assess mathematics teacher knowledge are dominated by subject matter knowledge. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' KCS through lesson plans developed by the teachers. Mathematics teachers attended a professional development activity and voluntarily participated in this study. Content analysis of the lesson plan and semi-structured interviews were conducted, and the data analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible methods and mistakes, these teachers were poorly prepared to support student learning.

Commented [a2]: Overall this abstract is not clear:
- The background and the study's aim seem not relate to each other.
- Research method, research subject(number and characteristics of subjects are not clear), instrument and data collection are not clear
- Results are also not clear and are not related to aim of the study.

Commented [a3]: This sentence is not effective.

Commented [a4]: What is the relation between this sentence and the previous one?

Commented [a5]: This aim should appear based on the background in previous sentences.

Commented [a6]: What are the contents of the items?

Commented [a7]: How many did teachers involve in this study?

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Namun, dalam beberapa hal, penilaian guru belum sepenuhnya mencakup seluruh keterampilan dan kompetensi guru. Fokus penelitian ini adalah menyelidiki *Knowledge of Content and Students* (KCS) pada topik luas dan keliling melalui rancangan rencana pembelajaran mereka. Pengetahuan guru tentang topik luas dan keliling dan strategi pengajaran telah dinilai melalui pengujian. Secara umum, materi untuk menilai pengetahuan guru matematika didominasi oleh materi pengetahuan. Dengan demikian, penilaian tersebut tampaknya belum sepenuhnya mencakup seluruh pengetahuan dan kompetensi guru. Dalam penelitian ini, peneliti menginvestigasi KCS guru matematika melalui RPP yang dikembangkan oleh guru. Guru matematika mengikuti kegiatan pengembangan profesional dan secara sukarela berpartisipasi dalam penelitian ini. Analisis isi RPP dan wawancara semi-terstruktur dilakukan, dan data dianalisis. Hasil penelitian ini mengungkapkan bahwa guru yang berpartisipasi ditantang ketika membuat prediksi kemungkinan tanggapan siswa. Mereka tampaknya tidak menyadari strategi siswa biasa, yang digunakan untuk menyelesaikan memaksimalkan luas dari keliling tertentu. Dengan pengetahuan yang terbatas tentang kemungkinan metode dan kesalahan siswa, para guru ini kurang siap untuk mendukung pembelajaran siswa.

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

How to Cite: Authors. (2021). Indonesian Mathematics Teachers' Knowledge of Content and Students: Predicting and Responding to Students' Responses to the Topic of Area and Perimeter. *Journal on Mathematics Education, x* (x), xx-xx.

Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research has shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related

to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

Commented [a8]: This sentence is difficult to understand.

> *Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2013, p.394).*

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

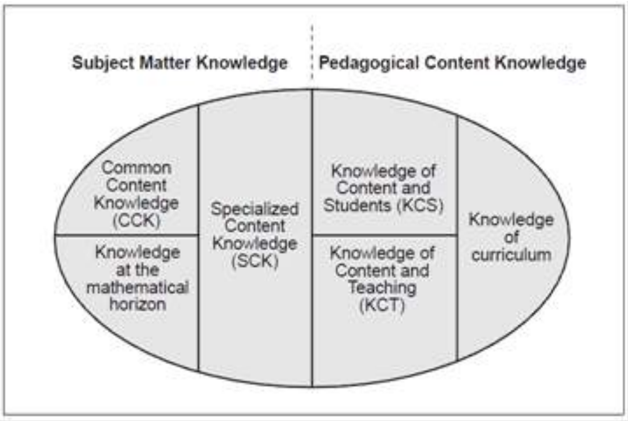


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

Commented [a9]: Authors should provide a consent from the owner of the figure/picture

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). Pre-service teachers have challenges with student thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COCATIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

Commented [a10]: This is not clear.

The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin H,

2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Lesson planning is considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers who read it can easily understand it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called 'Bansho', it anticipates student mathematical thinking and student thinking schema for solving given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). Student engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom behaviour is influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

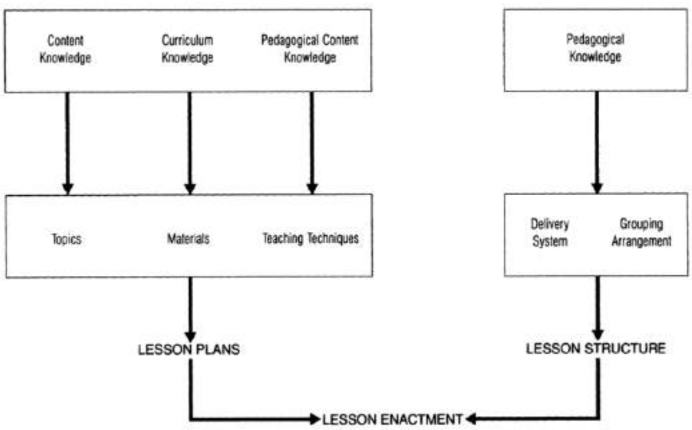


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zacahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013).

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason, (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Yeo, 2008).

This study intends to focus on a part of PCK pedagogical content knowledge, the KCS within lesson plans on the topic of area-perimeter. It is necessary to obtain a fuller insight into teacher knowledge. How mathematics teachers in Yogyakarta prepare their lesson plans and how is PCK integrated in their lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved human and had been through research ethics approval by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk). This study administrated a case study approach. This approach suits this study as it doesn't seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). Through this approach, the researchers examined two selected lesson plans of the mathematics teachers. The sample was chosen from twenty-nine teachers who attended a Professional Development (PD) session, and two teachers were selected

Commented [a11]: It is difficult to understand how a specific name just appear here.

Commented [a12]: For this section, overall, we suggest authors:
- Rewrite this section: from a more general background to a more specific case (from world situation to the specific case of Yogyakarta).
- Please indicate clearly the research problem(s)/research gap/urgency for conducting this research;
- Rewrite this section so that it clearly presents: from research background, previous relevant studies, research gap, aim of the study, and its corresponding theoretical framework.
- Please reduce the use of abbreviations to improve readability of this section.

Commented [a13]: Overall, we suggest authors to rewrite this section by giving sub-headings:
-Research approach and its corresponding reasons
- Participants
--Instruments
- Data collection
- Data analysis

Commented [a14]: It is not clear the relation between this study and University College London.

Commented [a15]: Does not?

Commented [a16]: Please inform how authors selected: two lesson plans; what criteria for this selection?

for the lesson plan analysis and interview. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan. The topic that would be taught was area and perimeter for grade 7. The "Gold Rush/Mining" task was selected. This task has several ways to be solved (see

Figure ***3***).

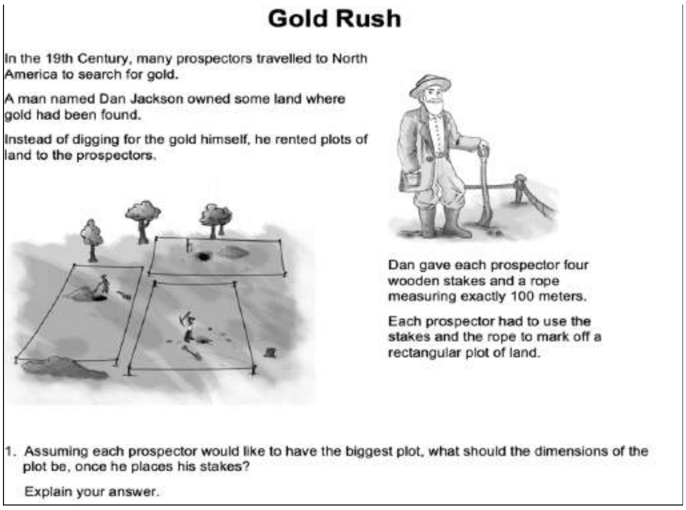


Figure 3. The Gold Rush problem *(https://www.map.mathshell.org/download.php?fileid=1637)*

To analyze the lesson plans, the researchers used content analysis. This method has the 'potential to disclose many hidden aspects of what is being communicated through the written text' (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers' knowledge of students' conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find the themes. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers included information about what students would do to the task. The data were presented descriptively.

Commented [a17]: Please inform how the interview was conducted: the validity of instrument, the time for conducting interview (how long an interview was conducted for each teacher); whether the interview was video-taped; how to transcribe the interview data; and how to analyze the data.

Commented [a18]: Please inform more characteristics for each subject more clearly: a number of years of teaching experience; the selection process of the teachers; criteria for selection; etc.

Commented [a19]: Please inform the validity of this problem; also the selection criteria for this problem.

Please also inform the relation between this task and the lesson plan more clearly.

Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students' emerging and incomplete thinking

The two lesson plans were coded and analyzed. There were three types of instructions to refer to the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Commented [a20]: Please reduce the use of abbreviations (or tell what it stands for).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Commented [a21]: Please provide sub-headings, so this section is clearer between findings and discussion.

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3).

Table 3. Proportions of the instructions

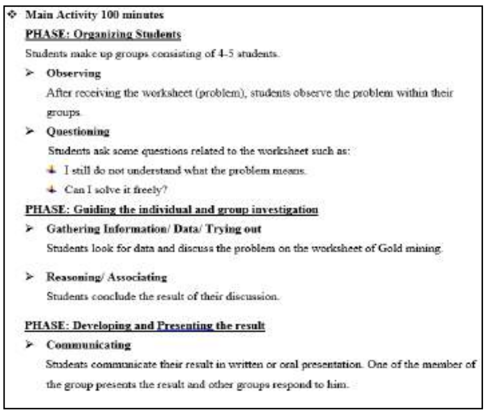
Instruction	T1	T2
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	**23 (100%)**	**19 (100%)**

Indonesian teachers follow the prescribed template of a lesson plan. The template consists of three main parts namely; introduction, main and closure. Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2's SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	Introduction		Main		Closure	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	**12**	**3**	**7**	**11**	**4**	**5**

T1 put more details of what students would ask to her on her lesson plan. For instance: 'Can I solve it freely?' has been put on her lesson plan. In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4).



❖ **Main Activity 100 minutes**

PHASE: Organizing Students

Students make up groups consisting of 4-5 students.

➢ **Observing**

After receiving the worksheet (problem), students observe the problem within their groups.

➢ **Questioning**

Students ask some questions related to the worksheet such as:

- I still do not understand what the problem means.
- Can I solve it freely?

PHASE: Guiding the individual and group investigation

➢ **Gathering Information/ Data/ Trying out**

Students look for data and discuss the problem on the worksheet of Gold mining.

➢ **Reasoning/ Associating**

Students conclude the result of their discussion.

PHASE: Developing and Presenting the result

➢ **Communicating**

Students communicate their result in written or oral presentation. One of the member of the group presents the result and other groups respond to him.

Figure 4. Teacher 1 Lesson Plan

The T2 lesson plan depicted detailed information about a possible student strategy. Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

Commented [a22]: This part is not clear, for which area this task is intended: rectangle, rhombus, square?

Main Activity

- ☑ Teacher divide students into groups
- ☑ Teacher delivers the worksheet to be discussed
- ☑ Teacher facilitates the learning processes

- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
  for instance p=10, l=… m then the area = …
  p = 15 m, l = ...m, then the area = ….
  Students determine the largest area by themselves
- for the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
  Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
  How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan

Commented [a23]: This part is not clear, for which area this task is intended: rectangle, rhombus, square?

After finding the largest area, students had to find the largest area by joining two miners' ropes. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

Commented [a24]: Of what?

- Students evaluate and make generalisation into questioning.
  - ☑ Teacher asks students to present in front of the class
  - ☑ Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Commented [a25]: We suggest to put this information within the text.

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Teachers T1 and T2 have more than five-years teaching experience each. However, their schools are different in terms of location and students. These teachers themselves employed different abilities in solving the Gold Mining problem (

Figure ***3***). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : Are there other ways T1?
T1 : Yesterday, I just did that one.
T1 : ...just let students find the ways to solve it .... Then, I will let them know that there are some ways to solve it. I give that opportunity to students

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

T2 : To me, I could do it directly because ***I already knew it*** *but to students if I want to students to learn it,* ***I make a table for them****. If the table is not made, students will find it difficult to solve it for students in my school.*
R : So, you (T2), induce them by using the table?
T2 : Yes, by the table.
R : What do you think, how many ways to solve it?
*T2 : To me, I did one way I know it directly it would be a square****. I knew it already****. But for students,* ***with table****, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and.., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

*T2 : I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (****metaphor****).*
T2 : However, I have thought only one strategy, which is global to solve a task. ... I, I... know at least I understand my students' characteristic so that it will be difficult for my students. ... It is not possible to come up if I let them to do it freely. ... I am so careful to give it the various strategies because students would get confuse

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

Commented [a26]: Does this matter? Please inform regarding this information in the method section.

Commented [a27]: Interview?

T2 : By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.
T2 : To me, I did one way I know it directly it would be a square. I knew it already
T2 : …instead of table, we can make the variable x, then I will be a quadratic function,
R : Are there other ways to solve it?
T2 : For the time being, not yet, making rectangles and to the square
R : Do you think there are still other ways to solve that problem?
T2 : I could use the graph …

To some extent, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task. However, by giving the students the strategy, he inadvertently is making the student dependent on him. Whereas, T1 is helping the students to make decisions themselves. From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

*T1 : The task has missing information, it should be more, and some students would think that. So that they **have not thought** yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
*T1 : 100. Maybe **they thought that** that's the only think they know.*
R : … So, they would answer it 100, possibly
T1 : Yeah, possibly
*T2 : … for those who did not understand, **they would not know what 100** m rope is to with the perimeter. So that the concept of perimeter, for those who understood, they already make it but later **they would not think** the rectangles can be varied.*
*T2 : Students **would confuse** the meaning of maximum, which is the largest, **they have not thought about it.** So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7).

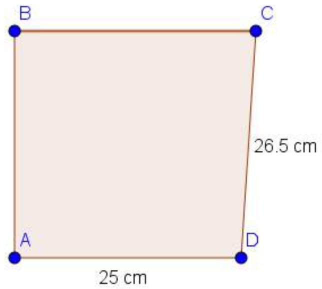


Figure 7. A student's possible mistake proposed by the researcher

Commented [a28]: Please inform and explain about the students' mistakes. Also please explain this figure.

If faced with a student mistake that they not have thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R	:	*If it happens if you see this (showing)*
T1	:	*I would ask students back to try it then you calculate it as what being asked to you*
R	:	*They have not yet known the result!*
T1		*Try, try it, by trialing they would know that it is different, this one is more, and that one is like that, ….*
R	:	*T2, what if your students did this? what would you do?*
T2	:	*I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...*

In this study, the lesson plans facilitated an insight into teachers' knowledge. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Yeo, 2008).

Commented [a29]: In this discussion section, please inform clearly the points of findings. Next, please address each finding and compare/contrast with other relevant studies.

Teachers have different ways of supporting students to solve tasks (Yei, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task. However, these results are not generalizable. The sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared. Developing higher order thinking and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment.

Commented [a30]: Please rewrite this section so it is clear main points of the conclusion drawn from the results and discussion section.

Please also add:
- Limitations of this study
- Recommendation for future research
- Implications of this study.

ACKNOWLEDGMENTS

The authors wish to thank to Ministry of Education of Republic of Indonesia; Planning and Cooperation of Foreign Affairs for the scholarship. The authors also would like to thank SEAMEO QITEP in Mathematics for its endless support.

REFERENCES

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, *7*, 145-172. https://doi.org/10.1023/b:jmte.0000021943.35739.1c

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics, 31*, 235-268. https://doi.org/10.1007/BF00376322

Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project* (pp. 25-48). https://doi.org/10.1007/978-1-4614-5149-5_2

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., … Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*(1), 133-180. https://doi.org/10.3102/0002831209345157

Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' thinking about instruction. *Remedial and Special Education*, *11*(6), 40-49. https://doi.org/10.1177/074193259001100609

Burns, R. B., & Lash, A. A. (1988). Nine seventh-grade teachers' knowledge and planning of problem-

solving instruction. *The Elementary School Journal*, *8*(4), 369-386. https://doi.org/10.1086/461545

Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.

Carle, S. M. (1993). *Student held misconceptions regarding area and perimeter of rectangles* (University of Massachusetts Boston). Retrieved from https://scholarworks.umb.edu/cgi/viewcontent.cgi?article=1045&context=cct_capstone

Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136–143). Adelaide: Australian Association of Mathematics Teachers.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching, 3rd Edition* (pp. 255-296). New York: Macmillan.

Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from lessons: Studying the construction of teacher knowledge catalysed by purposefully-designed experimental mathematics lessons. In M. Marshman, V. Geiger, & A Bennison (Eds.), *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165–172). Sunshine Coast: MEGA.

Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371-406. https://doi.org/10.3102/00028312042002371

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. Journal for *Research in Mathematics Education, 39*, 372-400. Retrieved from https://www.jstor.org/stable/40539304

John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies, 38*(4), 483-498. https://doi.org/10.1080/00220270500363620

Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In S. Johnston-Wilder, C. Lee, & D. Pimm (Eds.), *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience, 3rd Edition* (pp. 79-100). London: Routledge. https://doi.org/10.4324/9780203844120

Korkmaz, H. I., & Şahin, Ö. (2019). Preservice preschool teachers' pedagogical content knowledge on geometric shapes in terms of children's mistakes. *Journal of Research in Childhood Education*, *34*(3), 385-405. https://doi.org/10.1080/02568543.2019.1701150

Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1*. Retrieved from https://files.eric.ed.gov/fulltext/ED452031.pdf

Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *Procedia - Social and Behavioral Sciences, 55*, 720-729. https://doi.org/10.1016/j.sbspro.2012.09.557

Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical content knowledge of mathematics pre-service teachers: Do they know their students? *Journal of Physics: Conference Series*, *1097*(1), 012098. https://doi.org/10.1088/1742-6596/1097/1/012098

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *2*(15), 4-14. https://doi.org/10.3102/0013189X015002004

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, *26*(2), 114-145. https://doi.org/10.2307/749205

Superfine, A. C. (2008). Planning for mathematics instruction: A model of experienced teachers' planning processes in the context of a reform mathematics curriculum. *Mathematics Educator, 18*(2), 11-22. Retrieved from http://tme.journals.libs.uga.edu/index.php/tme/article/view/198

Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., … Rowley, G. (2012). Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). In *International Association for the Evaluation of Educational Achievement*. Retrieved from https://files.eric.ed.gov/fulltext/ED542380.pdf

Turnuklu, E., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, *1*, 1-13. Retrieved from http://www.k-12prep.math.ttu.edu/journal/1.contentknowledge/yesildere01/article.pdf

Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating preservice mathematics teachers' pedagogical content knowledge through microteaching. *REDIMAT-Journal of Research in Mathematics Education*, *9*(1), 62-87. https://doi.org/http://dx.doi.org/10.17583/redimat.2020.3353

Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.

White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2013). Teachers learning from teachers. In M.A. Clements, A.J. Bishop, C. Keitel, J. Kilpatrick, & F.K.S. Leung (Eds.), *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13

Widodo, & Tamimudin, H. M. (2014). Three training strategies for improving mathematics teacher competence in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Retrieved from http://atcm.mathandtech.org/EP2014/index.html

Yeo, K. K. Y. (2008). Teaching area and perimeter : Mathematics-pedagogical-content knowledge-in-action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*. Retrieved from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.471.9965&rep=rep1&type=pdf

Yin, R. K. (2014). Case Study Research: Design and Methods (5th ed.). *Thousand Oaks: SAGE Publications*.

Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, *6*(2), 41-62. https://doi.org/10.26220/rev.1627

Hasil revisi di kirim pada tanggal 22 Maret 2021 dengan perubahan signifikan pada konten isi dan judul artikel

Published by:

Journal on Mathematics Education

(P-ISSN: 2087-8885 | E-ISSN: 2407-0610)

JME Scopus Indexed

HOME ABOUT USER HOME SEARCH CURRENT ARCHIVES ANNOUNCEMENTS CONTACT

Home > User > Author > Submissions > #13537 > **Review**

#13537 Review

SUMMARY REVIEW EDITING

Submission

Authors	Wahid Yunianto, Rully Charitas Indra Prahmana, Cosette Crisan
Title	INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS OF AREA AND PERIMETER OF RECTANGLE
Section	Articles
Editor	Zulkardi Zulkardi

Peer Review

Round 1

Review Version	13537-35164-1-RV.DOCX 2021-01-17
Initiated	2021-01-19
Last modified	2021-03-08
Uploaded file	Reviewer A 13537-35584-1-RV.DOCX 2021-02-02 Reviewer E 13537-36158-1-RV.DOCX 2021-02-28 Reviewer D 13537-35933-1-RV.DOCX 2021-02-16 Reviewer B 13537-35552-1-RV.DOCX 2021-02-02

Editor Decision

Decision	Accept Submission 2021-03-22
Notify Editor	Editor/Author Email Record 2021-03-08
Editor Version	None
Author Version	13537-36624-1-ED.DOCX 2021-03-22 DELETE
Upload Author Version	Choose File no file selected Upload

Journal on Mathematics Education
Doctoral Program on Mathematics Education
Faculty of Teacher Training and Education, Universitas Sriwijaya
Kampus FKIP Bukit Besar
Jl. Srijaya Negara, Bukit Besar
Palembang - 30139
email: jme@unsri.ac.id

p-ISSN: 2087-8885 | e-ISSN: 2407-0610

Author Guidelines
Editorial Team
Focus and Scope
Publication Ethics
Open Access Policy
Peer Review Process
Online Submissions
Contact Us

Journal on Mathematics Education
Q2 Education best quartile
SJR 2019 0.53
powered by scimagojr.com

3.1 CiteScore 2019
89th percentile
Powered by Scopus

USER
You are logged in as...
rullycip
- My Journals
- My Profile
- Log Out

AUTHOR INFORMATION

Manuscript Template
Measure your MRL
Submit Manuscript
Copyright Transfer Agreement
How to Submit Your Paper

Paper hasil revisi dengan judul,

"Indonesian Mathematics Teachers' Knowledge of Content and Students of Area and Perimeter of Rectangle"

[Paper ID: 13537]

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 12, No. 2, May 2021, pp. xxx-xxx

INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS OF AREA AND PERIMETER OF RECTANGLE

Wahid Yunianto[1], Rully Charitas Indra Prahmana[2], Cosette Crisan[3]

[1]SEAMEO QITEP in Mathematics, Jl. Kaliurang, Condongcatur, Depok, Sleman, Yogyakarta 55281, Indonesia
[2]Universitas Ahmad Dahlan, Jl. Pramuka 42, Pandeyan, Umbulharjo, Yogyakarta, Indonesia
[3]University College London, Gower St, Bloomsbury, London WC1E 6BT, United Kingdom
Email: rully.indra@mpmat.ud.ac.id

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. Research has shown teachers Competencies and skills influence students' performances. Previous studies explored teachers' knowledge through testing. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items or tasks to assess mathematics teacher knowledge in the previous studies were dominated by subject matter knowledge problems. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' KCS through lesson plans developed by the teachers. To accommodate the gap in the previous studies, this study focuses on Knowledge of Content and Students (KCS) on the topic of area-perimeter through their designed lesson plans. Twenty-nine mathematics teachers attended a professional development activity and voluntarily participated in this study. Two teachers were selected to be the focus of this case study. Content analysis of the lesson plan and semi-structured interviews were conducted, and then data were analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible strategies and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Namun, dalam beberapa hal, penilaian guru belum sepenuhnya mencakup seluruh keterampilan dan kompetensi guru. Fokus penelitian ini adalah menyelidiki *Knowledge of Content and Students* (KCS) pada topik luas dan keliling melalui rancangan rencana pembelajaran mereka. Pengetahuan guru tentang topik luas dan keliling dan strategi pengajaran telah dinilai melalui pengujian. Secara umum, materi untuk menilai pengetahuan guru matematika didominasi oleh materi pengetahuan. Dengan demikian, penilaian tersebut tampaknya belum sepenuhnya mencakup seluruh pengetahuan dan kompetensi guru. Dalam penelitian ini, peneliti menginvestigasi KCS guru matematika melalui RPP yang dikembangkan oleh guru. Guru matematika mengikuti kegiatan pengembangan profesional dan secara sukarela berpartisipasi dalam penelitian ini. Analisis isi RPP dan wawancara semi-terstruktur dilakukan, dan data dianalisis. Hasil penelitian ini mengungkapkan bahwa guru yang berpartisipasi ditantang ketika membuat prediksi kemungkinan tanggapan siswa. Mereka tampaknya tidak menyadari strategi siswa biasa, yang digunakan untuk menyelesaikan memaksimalkan luas dari keliling tertentu. Dengan pengetahuan yang terbatas tentang kemungkinan metode dan kesalahan siswa, para guru ini kurang siap untuk mendukung pembelajaran siswa.

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

How to Cite: Yunianto, W., Prahmana, R.C.I., & Crisan, C. (2021). Indonesian Mathematics Teachers' Knowledge of Content and Students of Area and Perimeter of Rectangle. *Journal on Mathematics Education*, *12*(2), xx-xx. http://doi.org/10.22342/jme.12.2.13537.xxx-xxx

Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research have shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

> *Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2013, p.394).*

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Loewenberg Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

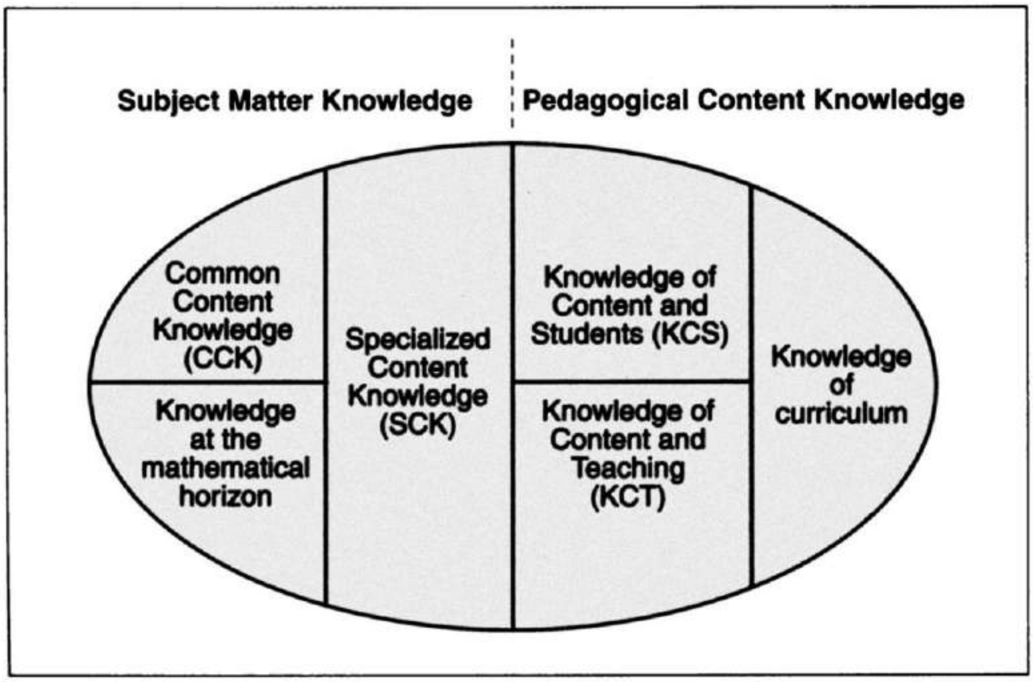


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). In the case of pre-service teachers, they have challenges with student's thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018;

Ünver et al., 2020). It makes sense as they have limited teaching experiences or even have not taught yet. For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COACTIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

As one of the ways, testing is used to assess teachers. The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin H, 2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Another study using testing; faced challenges in measuring teachers' knowledge (Fauskanger, 2015). An interesting finding of a study of pre-service teachers is that they possessed higher PCK scores than SMK from limited test items (Dwi Kristanto, Hariwangsa Panuluh, & Dian Atmajati, 2020). A case study in South Korea revealed that teachers with sufficient SMK of a certain competence/ topic faced challenges in incorporating KCS and KCT of that topic (Lee, Capraro, & Capraro, 2018). Therefore, testing to measure teachers' knowledge still face challenges.

Lesson plans are considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers can easily understand it when reading it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called 'Bansho', which anticipates and tries to elicit student mathematical thinking and student thinking schema for solving the given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed that the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). The students' engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; "Planning for Mathematics Instruction: A Model of Experienced Teachers' Planning Processes in the Context of a Reform Mathematics Curriculum," 2008). In other words, teacher classroom practices are influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions

Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

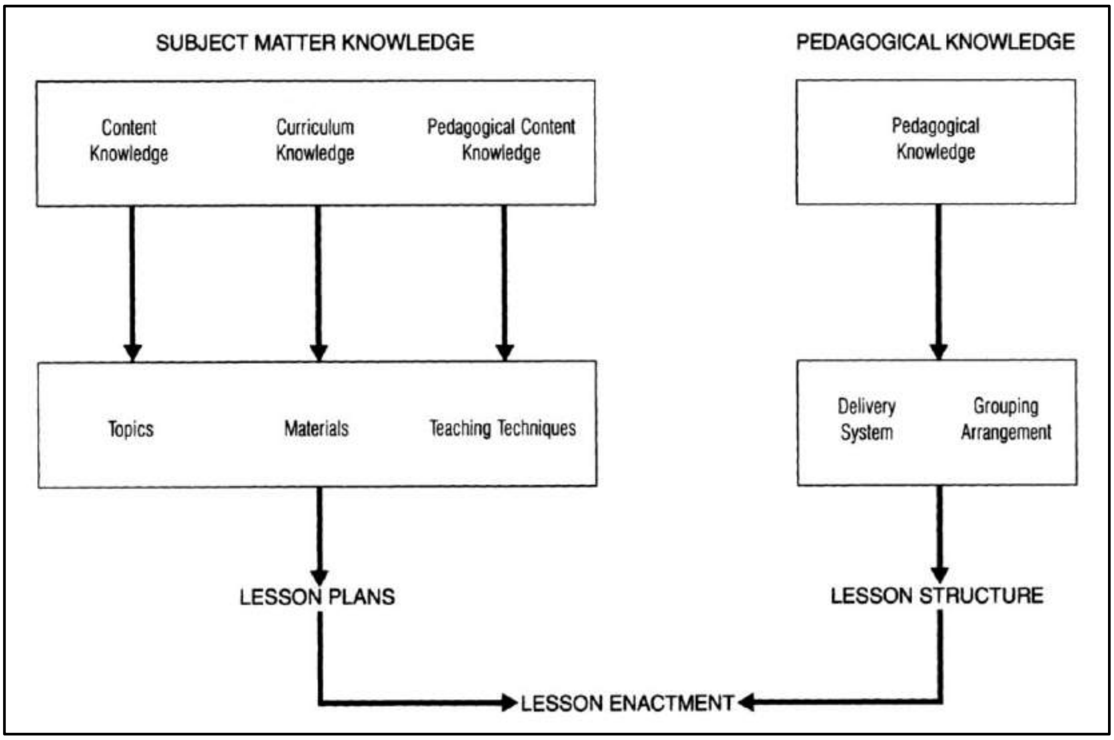


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zacahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013). It seems to make sense as they could manipulate and observe what changes happen by reshaping a figure. (Yunianto, 2015)

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Thus, we should not expect teachers to deliver mathematics well if they do not have mastered it and do not understand how to teach it. Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason, (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Kow & Yeo, 2008).

This study intends to focus on a part of PCK, the KCS within lesson plans on the topic of area-perimeter. It is necessary to obtain a fuller insight into teacher knowledge as it influence students' performance. Beside testing, there might be alternative way such as lesson plans to investigate teachers' knowledge. How mathematics teachers prepare their lesson plans and how is PCK integrated in their lesson plans? How are the KCS integrated in the lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved humans and has been approved by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk) as this is a part of completion of the first author's dissertation. This study administrated a case study approach. This approach suits this study as it does not seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). The research subjects were the mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan as part of the whole PD. It was done somewhere in the middle of all complete sessions. As it is a case study, the researchers examined two selected lesson plans of two mathematics teachers. The remaining lesson plans have not been analyzed due to time limitation. The sample was chosen from twenty-nine teachers who attended a professional development (PD) session, and two teachers were selected for the lesson plan analysis and interview. Additionally. these teachers were selected based on their teaching experience; at least five years. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The topic that would be taught was area and perimeter for grade 7. The "Gold Rush/Mining" task was selected. This task was chosen because it is a problem-solving task and has several ways to be solved on area-perimeter of a rectangle (see

Figure ***3***). Additionally, the complete Gold Rush activity showed the mistakes that students might do. Thus, it is considered as a good activity to be explored to understand how teachers prepare this activity.

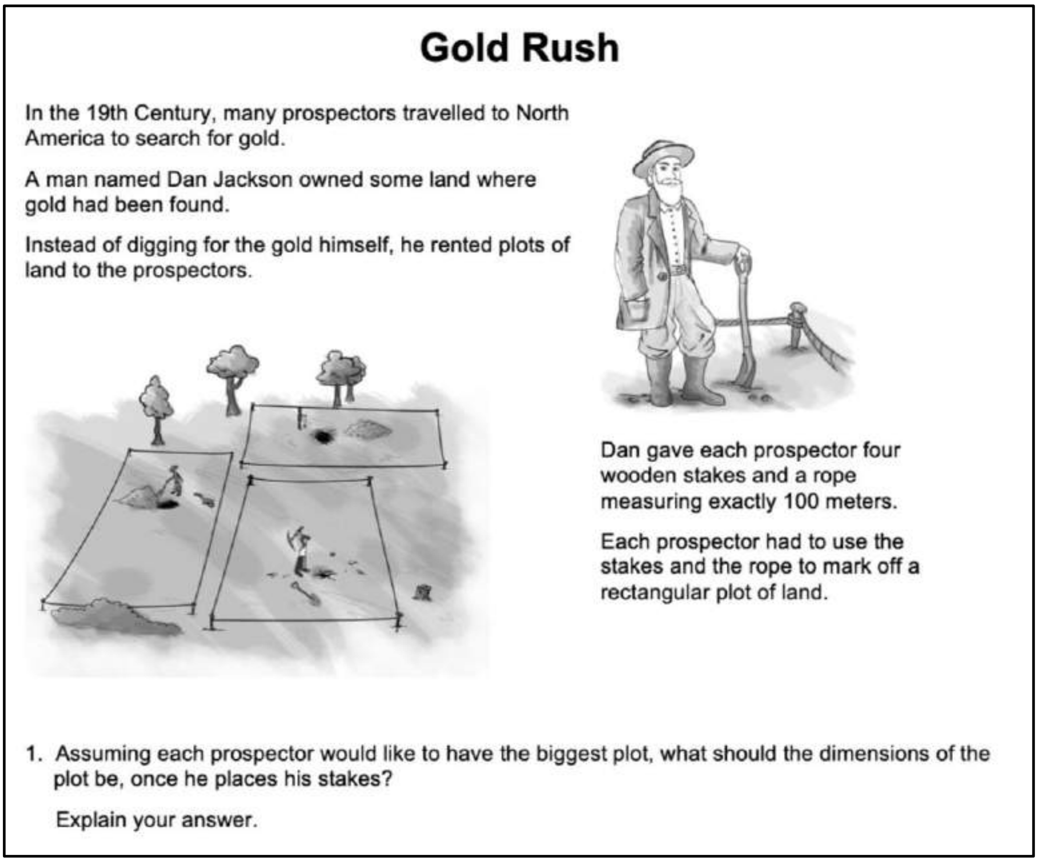


Figure 3. The Gold Rush problem (https://www.map.mathshell.org/download.php?fileid=1637)

To analyze the lesson plans, the researchers used content analysis. This method has the 'potential to disclose many hidden aspects of what is being communicated through the written text' (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers' knowledge of students' conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find themes by classifying instructions and KCS integrated in the lesson plans.

Table 1. Knowledge of Content and Student (KCS) (Loewenberg Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students' emerging and incomplete thinking

By using Table 2, it is easy to differentiate instructions' categories. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers

included information about what students would do to the task (KCS). The data were presented descriptively.

The two lesson plans were coded and analyzed. There were three types of instructions to refer to with the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. “what did you do and How did you do it? How are you convinced with your strategies?	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3). The percentage is from type of instruction per total instructions written on the lesson plans.

Indonesian teachers follow the prescribed template of a lesson plan by MoEC. The template consists of three main parts namely; introduction, main and closure. It also consists learning goals and how teachers and students would do in the classroom.

Table 3. Proportions of the instructions

Instruction	T1	T2

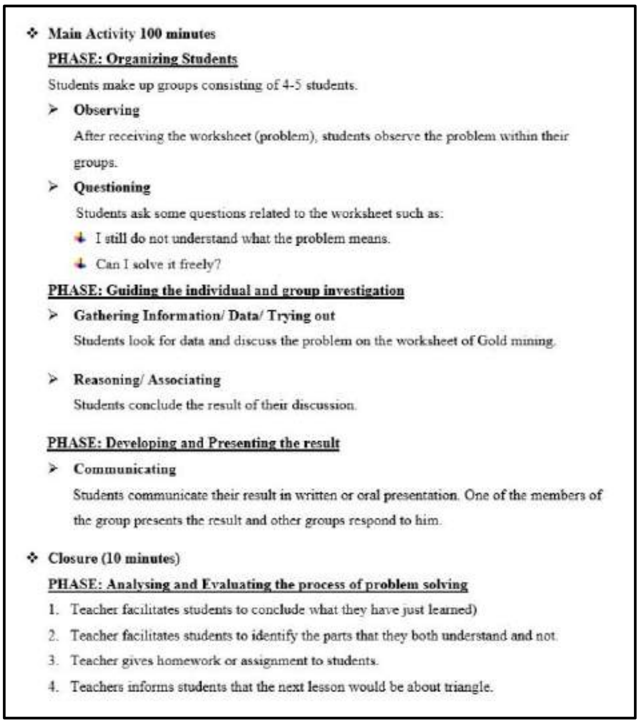
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	**23 (100%)**	**19 (100%)**

Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2's SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	**Introduction**		**Main**		**Closure**	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	**12**	**3**	**7**	**11**	**4**	**5**

T1 put more details of what students would ask to her on her lesson plan. For instance: 'Can I solve it freely?' has been put on her lesson plan. This is a proof of PCK in the lesson plan, but not specific to KCS.



- **Main Activity 100 minutes**
  PHASE: Organizing Students
  Students make up groups consisting of 4-5 students.
  - **Observing**
    After receiving the worksheet (problem), students observe the problem within their groups.
  - **Questioning**
    Students ask some questions related to the worksheet such as:
    - I still do not understand what the problem means.
    - Can I solve it freely?

  PHASE: Guiding the individual and group investigation
  - **Gathering Information/ Data/ Trying out**
    Students look for data and discuss the problem on the worksheet of Gold mining.
  - **Reasoning/ Associating**
    Students conclude the result of their discussion.

  PHASE: Developing and Presenting the result
  - **Communicating**
    Students communicate their result in written or oral presentation. One of the members of the group presents the result and other groups respond to him.
- **Closure (10 minutes)**
  PHASE: Analysing and Evaluating the process of problem solving
  1. Teacher facilitates students to conclude what they have just learned)
  2. Teacher facilitates students to identify the parts that they both understand and not.
  3. Teacher gives homework or assignment to students.
  4. Teachers informs students that the next lesson would be about triangle.

Figure 4. Teacher 1 Lesson Plan

In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4). On the phase of guiding the individual and group investigation, which should be rich of KCS. In this lesson plan, detail ways of students might solve it or make mistakes and how to facilitate it have not been depicted.

The T2 lesson plan of rectangle using Gold Rush task depicted detailed information about a possible student strategy. Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

Main Activity

- ☑ Teacher divide students into groups
- ☑ Teacher delivers the worksheet to be discussed
- ☑ Teacher facilitates the learning processes

- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
  for instance, p=10, l=… m
  p = 15 m, l = ...m, then the area = …. p=15 m, l= … m etc
  Students determine the largest area by themselves
- For the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
  Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
  How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan of Gold Rush

After finding the largest area of the rectangle, students had to find the largest area by joining two miners' ropes and how would they join it. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

- Students evaluate and make generalisation into questioning.
  - ☑ Teacher asks students to present in front of the class
  - ☑ Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Teachers T1 and T2 have more than five-years teaching experience each. Based on the questionnaire and interview, their schools are different in terms of location and students' background. These teachers themselves employed different abilities in solving the Gold Mining problem (

Figure ***3***). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : Are there other ways T1?
T1 : Yesterday, I just did that one.
T1 : …just let students find the ways to solve it …. Then, I will let them know that there are some ways to solve it. I give that opportunity to students

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

*T2 : To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*
R : So, you (T2), induce them by using the table?
T2 : Yes, by the table.
R : What do you think, how many ways to solve it?
*T2 : To me, I did one way I know it directly it would be a square**. I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and.., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

*T2 : I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*
T2 : However, I have thought only one strategy, which is global to solve a task. … I, I... know at least I understand my students' characteristic so that it will be difficult for my students. … It is not possible to come up if I let them to do it freely. … I am so careful to give it the various strategies because students would get confuse

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

T2 : By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.

T2 : To me, I did one way I know it directly it would be a square. I knew it already
T2 : …instead of table, we can make the variable x, then I will be a quadratic function,
R : Are there other ways to solve it?
T2 : For the time being, not yet, making rectangles and to the square
R : Do you think there are still other ways to solve that problem?
T2 : I could use the graph …

To some extent, from the lesson plan, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task (figure 5). However, by giving the students the strategy, he inadvertently is making the students dependent on him. Whereas, from the lesson plan, T1 is helping the students to make decisions themselves (figure 4). From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

*T1 : The task has missing information, it should be more, and some students would think that. So that they **have not thought** yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
*T1 : 100. Maybe **they thought that** that's the only think they know.*
R : … So, they would answer it 100, possibly
T1 : Yeah, possibly
*T2 : … for those who did not understand, **they would not know what 100** m rope is to with the perimeter. So that the concept of perimeter, for those who understood, they already make it but later **they would not think** the rectangles can be varied.*
*T2 : Students **would confuse** the meaning of maximum, which is the largest, **they have not thought about it.** So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7). The researcher proposed a possible mistake by a student of which the shape looks like a rectangle 25 x 26,5.

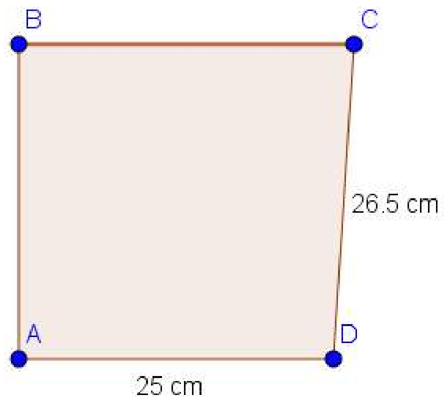


Figure 7. A student's possible mistake proposed by the researcher

If faced with a student mistake that they not have thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R : *If it happens if you see this (showing)*
T1 : *I would ask students back to try it then you calculate it as what being asked to you*
R : *They have not yet known the result!*
T1 *Try, try it, by trialing they would know that it is different, this one is more, and that one is like that, ...*
R : *T2, what if your students did this? what would you do?*
T2 : *I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...*

In this study, the lesson plans facilitated an insight into teachers' knowledge. In this case, it showed teacher's pedagogical knowledge as well as PCK. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid, and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Kow & Yeo, 2008). In Japanese lesson plans, they contain more detailed instructions (Nakahara & Koyama, 2000) which shows more information about teachers knowledge. In contrast, the two case of teachers in this study, have not yet shown detailed instructions but more in general instruction.

Teachers have different ways of supporting students to solve tasks (Yei, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task (possessing SMK and PCK). However, these results are not generalizable. The limited sample was not chosen and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in

Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS of a specific topic through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter of rectangle topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared in facing the situations during teaching. Developing problem solving skills and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment. Additionally, this study explored a rectangle topic, the result might vary in different topics Therefore, further investigation on different topic could be conducted. This study is not generalizable as it used limited research subjects.

ACKNOWLEDGMENTS

The authors wish to thank to Ministry of Education of Republic of Indonesia; Planning and Cooperation of Foreign Affairs for the scholarship. The authors also would like to thank SEAMEO QITEP in Mathematics for its endless support.

REFERENCES

An, S., Kulm, G., & Wu, Z. (2004). The Pedagogical Content Knowledge of Middle School, Mathematics Teachers in China and the U.S. *Journal of Mathematics Teacher Education*. https://doi.org/10.1023/b:jmte.0000021943.35739.1c

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*. https://doi.org/10.1007/BF00376322

Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project*. https://doi.org/10.1007/978-1-4614-5149-5_2

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., … Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*. https://doi.org/10.3102/0002831209345157

Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' Thinking About Instruction. *Remedial and Special Education*. https://doi.org/10.1177/074193259001100609

Burns, R. B., & Lash, A. A. (1988). Nine Seventh-Grade Teachers' Knowledge and Planning of Problem-Solving Instruction. *The Elementary School Journal*. https://doi.org/10.1086/461545

Butt, G. (2008). *Lesson Planning 3rd Edition*. Bloomsbury Publishing.

Carle, S. M. (1993). *Student held misconceptions regarding area and perimeter of rectangles*.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. In *Handbook of resaerch on teaching* . https://doi.org/10.1258/ijsa.2008.008228

Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from Lessons: Studying the Construction of Teacher Knowledge Catalysed by Purposefully-Designed Experimental Mathematics Lessons. *Mathematics Education Research Group of Australasia*.

Denscombe, M. (2010). The Good Research Guide For Small Scale Research Projects. In *Open University Press*. https://doi.org/10.1371/journal.pone.0017540

Dwi Kristanto, Y., Hariwangsa Panuluh, A., & Dian Atmajati, E. (2020). Development and validation of a test instrument to measure pre-service mathematics teachers' content knowledge and pedagogical content knowledge. *Journal of Physics: Conference Series*, *1470*(1). https://doi.org/10.1088/1742-6596/1470/1/012008

Fauskanger, J. (2015). Challenges in measuring teachers' knowledge. *Educational Studies in Mathematics*, *90*(1). https://doi.org/10.1007/s10649-015-9612-4

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*. https://doi.org/10.3102/00028312042002371

John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies*. https://doi.org/10.1080/00220270500363620

Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience: 3rd Edition* (pp. 79–100). Routledge Taylor & Francis Group. https://doi.org/10.4324/9780203844120

Korkmaz, H. I., & Şahin, Ö. (2019). Preservice Preschool Teachers' Pedagogical Content Knowledge on Geometric Shapes in Terms of Children's Mistakes. *Journal of Research in Childhood Education*, 1–21.

Kow, K., & Yeo, J. (2008). Teaching Area and Perimeter : Mathematics-Pedagogical-Content Knowledge-in-Action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*.

Lee, Y., Capraro, R. M., & Capraro, M. M. (2018). Mathematics Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge in Problem Posing. *International Electronic Journal of Mathematics Education*, *13*(2). https://doi.org/10.12973/iejme/2698

Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*. https://doi.org/10.1177/0022487108324554

Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1*.

Özerem, A. (2012). Misconceptions In Geometry And Suggested Solutions For Seventh Grade Students. *Procedia - Social and Behavioral Sciences*. https://doi.org/10.1016/j.sbspro.2012.09.557

Planning for Mathematics Instruction: A Model of Experienced Teachers' Planning Processes in the Context of a Reform Mathematics Curriculum. (2008). *Mathematics Educator*.

Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical Content Knowledge of Mathematics Pre-service Teachers: Do they know their students? *Journal of Physics: Conference Series*. https://doi.org/10.1088/1742-6596/1097/1/012098

Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*. https://doi.org/10.3102/0013189X015002004

Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education*. https://doi.org/10.2307/749205

Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., … Rowley, G. (2012). Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). In *International Association for the Evaluation of Educational Achievement*.

Turnuklu, E., & Yesildere, S. (2007). The Pedagogical Content Knowledge in Mathematics: Pre-Service Primary Mathematics Teachers' Perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*.

Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating Preservice Mathematics Teachers' Pedagogical Content Knowledge through Microteaching. *REDIMAT-Journal of Research in Mathematics Education*, *9*(1), 62–87.

Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.

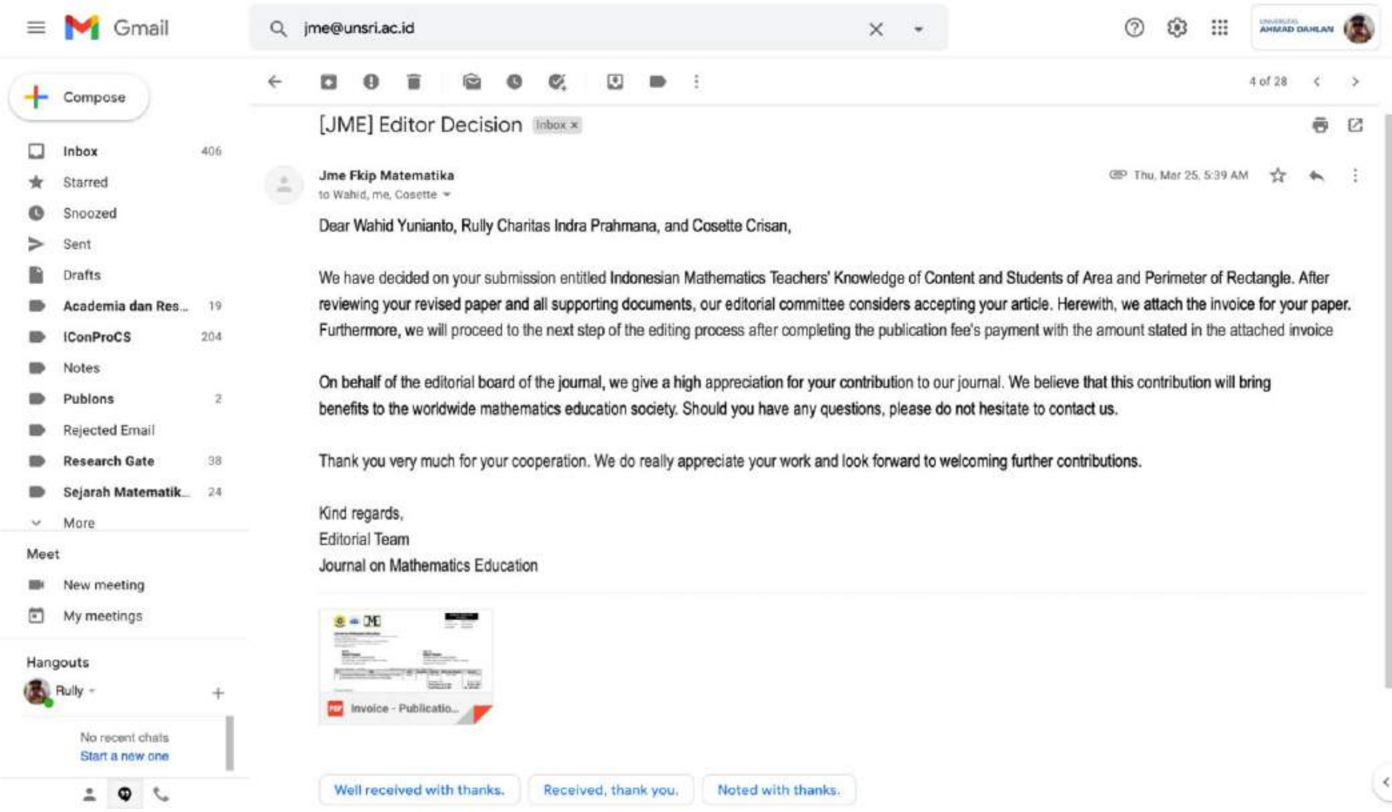
Widodo, & Tamimudin H, M. (2014). Three Training Strategies for Improving Mathematics Teacher Competences in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Yogyakarta: Mathematics and Technology, LLC. Retrieved from http://atcm.mathandtech.org/EP2014/index.html

Yin, R. K. (2014). Case study research: Design and methods (5th ed.). In *Thousand Oaks, CA: SAGE Publications*.

Yunianto, W. (2015). Supporting Students' Understanding of Area Measurement Through Verknippen Applet. *Southeast Asian Mathematics Education Journal*, *5*(1). https://doi.org/10.46517/seamej.v5i1.34

Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, *6*(2), 41–62.

Keputusan diterima pasca revisi pada tanggal 25 Maret 2021.



[JME] Editor Decision

Jme Fkip Matematika
to Wahid, me, Cosette
Thu, Mar 25, 5:39 AM

Dear Wahid Yunianto, Rully Charitas Indra Prahmana, and Cosette Crisan,

We have decided on your submission entitled Indonesian Mathematics Teachers' Knowledge of Content and Students of Area and Perimeter of Rectangle. After reviewing your revised paper and all supporting documents, our editorial committee considers accepting your article. Herewith, we attach the invoice for your paper. Furthermore, we will proceed to the next step of the editing process after completing the publication fee's payment with the amount stated in the attached invoice

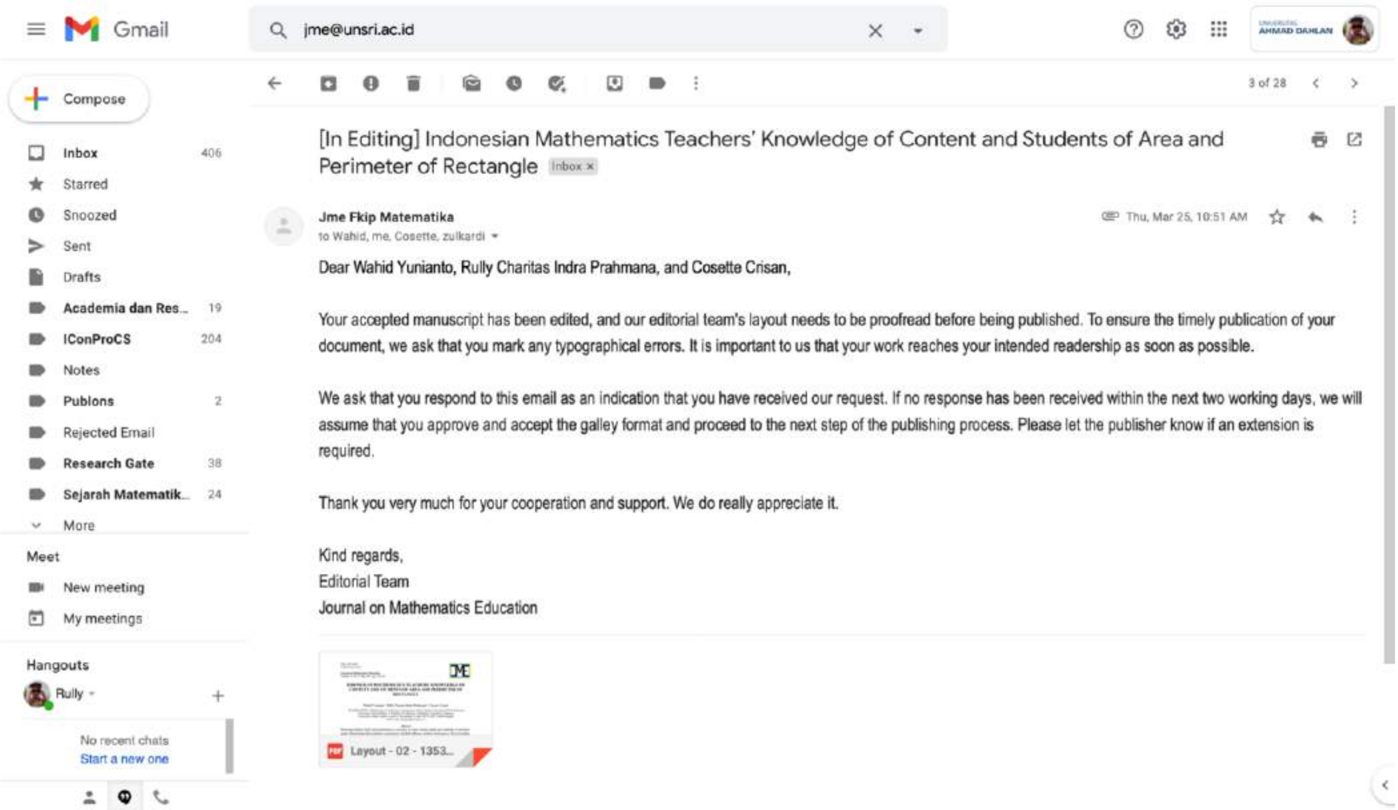
On behalf of the editorial board of the journal, we give a high appreciation for your contribution to our journal. We believe that this contribution will bring benefits to the worldwide mathematics education society. Should you have any questions, please do not hesitate to contact us.

Thank you very much for your cooperation. We do really appreciate your work and look forward to welcoming further contributions.

Kind regards,
Editorial Team
Journal on Mathematics Education

Invoice - Publicatio...

Konfirmasi hasil editing dan proofread pasca diterima pada tanggal 25 Maret 2021.



[In Editing] Indonesian Mathematics Teachers' Knowledge of Content and Students of Area and Perimeter of Rectangle

Jme Fkip Matematika
to Wahid, me, Cosette, zulkardi
Thu, Mar 25, 10:51 AM

Dear Wahid Yunianto, Rully Charitas Indra Prahmana, and Cosette Crisan,

Your accepted manuscript has been edited, and our editorial team's layout needs to be proofread before being published. To ensure the timely publication of your document, we ask that you mark any typographical errors. It is important to us that your work reaches your intended readership as soon as possible.

We ask that you respond to this email as an indication that you have received our request. If no response has been received within the next two working days, we will assume that you approve and accept the galley format and proceed to the next step of the publishing process. Please let the publisher know if an extension is required.

Thank you very much for your cooperation and support. We do really appreciate it.

Kind regards,
Editorial Team
Journal on Mathematics Education

Layout - 02 - 1353...

Paper in Editing Version

[Paper ID: 13537]

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 12, No. 2, May 2021, pp. 223-238

INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS OF AREA AND PERIMETER OF RECTANGLE

Wahid Yunianto[1], Rully Charitas Indra Prahmana[2], Cosette Crisan[3]

[1]SEAMEO QITEP in Mathematics, Jl. Kaliurang, Condongcatur, Depok, Sleman, Yogyakarta 55281, Indonesia
[2]Universitas Ahmad Dahlan, Jl. Pramuka 42, Pandeyan, Umbulharjo, Yogyakarta, Indonesia
[3]University College London, Gower St, Bloomsbury, London WC1E 6BT, United Kingdom
Email: rully.indra@mpmat.uad.ac.id

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. Research has shown teachers competencies and skills influence students' performances. Previous studies explored teachers' knowledge through testing. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items or tasks to assess mathematics teacher knowledge in the previous studies were dominated by subject matter knowledge problems. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' Knowledge of Content and Students (KCS) through lesson plans developed by the teachers. To accommodate the gap in the previous studies, this study focuses on KCS on the topic of area-perimeter through their designed lesson plans. Twenty-nine mathematics teachers attended a professional development activity voluntarily participated in this study. Two teachers were selected to be the focus of this case study. Content analysis of the lesson plan and semi-structured interviews were conducted, and then data were analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible strategies and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Penelitian ini menunjukkan bahwa komptensi dan keterampilan guru mempengaruhi performa siswa. Penelitian sebelumnya telah mengkaji pengetahuan gru melalui tes. Pengetahuan guru pada topik keliling-luas dan strategi pembelajaran juga telah dikaji melalui tes. Pada umumnya, banyaknya soal pada tes didominasi oleh soal-soal tentang pengetahuan subjek yang diajarkan. Oleh karena itu, asesmen seperti ini belum mencakup kesuluruhan pengetahuan dan kompetensi guru. Pada studi ini, peneliti menginvestigasi pengetahuan guru matematika tentang KCS pada rencana pelaksanaan pembelajaran yang mereka kembangkan. Untuk mengakomodasi kesenjangan pada penelitian sebelumnya, penelitian kali ini berfokus pada pengetahuan tentang konten dan siswa (KCS) pada topik keliling-luas pada rencana pelaksanaan pembelajaran. Dua puluh Sembilan guru matematika yang sedang mengikuti pelatihan peningkatan kompetensi secara suka rela mengikuti penelitian ini. Dua guru matematika menjadi fokus penelitian studi kasus ini. Konten analisis dan interview semi terstruktur dilakukan dan datanya dianalisis. Terungkap bahwa peserta ini mengalami tantangan dalam memprediksi kemungkinan respon yang diberikan siswa. Mereka belum menyadari strategi siswa yang biasanya digunakan untuk menyelesaikan persoalan memaksimalkan luas dari keliling yang ditentukan. Dengan pengetahuan yang terbatas pada kemungkinan strategi siswa dan kesalahan siswa, guru ini kurang siap dalam mendukung siswanya

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

How to Cite: Yunianto, W., Prahmana, R.C.I., & Crisan, C. (2021). Indonesian Mathematics Teachers' Knowledge of Content and Students of Area and Perimeter of Rectangle. *Journal on Mathematics Education, 12*(2), 223-238. http://doi.org/10.22342/jme.12.2.13537.223-238

Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research have shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

> *Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2012, p.394).*

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

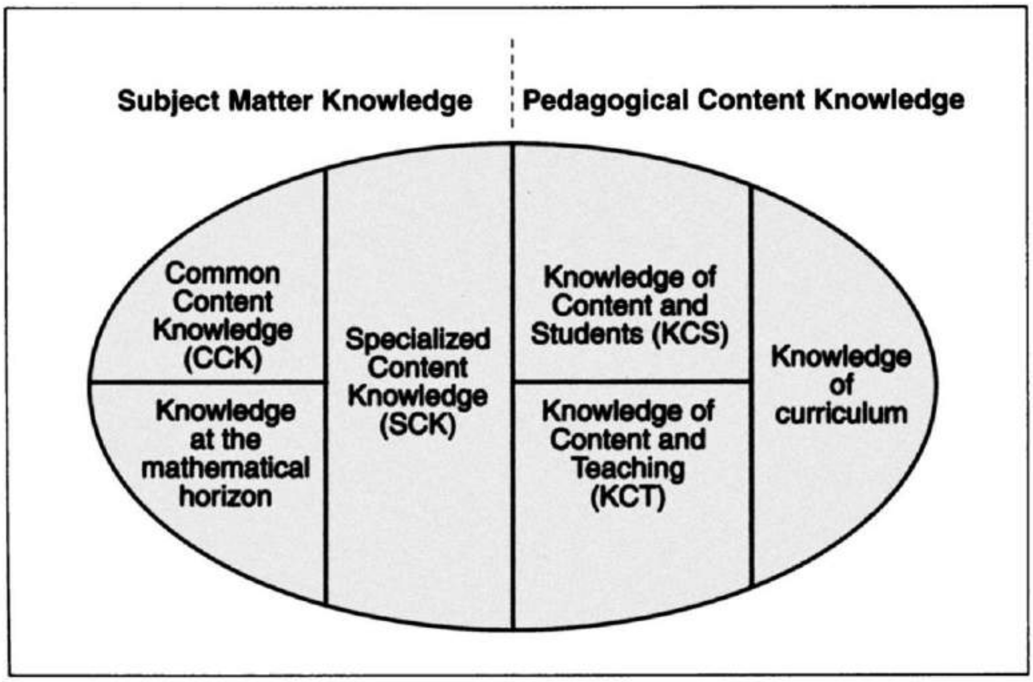


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). In the case of pre-service teachers, they have challenges with student's thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). It makes sense as they have limited teaching experiences or even have not taught yet. For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COACTIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

As one of the ways, testing is used to assess teachers. The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin, 2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Another study using testing; faced challenges in measuring teachers' knowledge (Fauskanger, 2015). An interesting finding of a study of pre-service teachers is that they possessed higher PCK scores than SMK from limited test items (Kristanto, Panuluh, & Atmajati, 2020). A case study in South Korea revealed that teachers with sufficient SMK of a certain competence/ topic faced challenges in incorporating KCS and KCT of that topic (Lee, Capraro, & Capraro, 2018). Therefore, testing to measure teachers' knowledge still face challenges.

Lesson plans are considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers can easily understand it when reading it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called 'Bansho', which anticipates and tries to elicit student mathematical thinking and student thinking schema for solving the given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed that the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). The students' engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom practices are influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

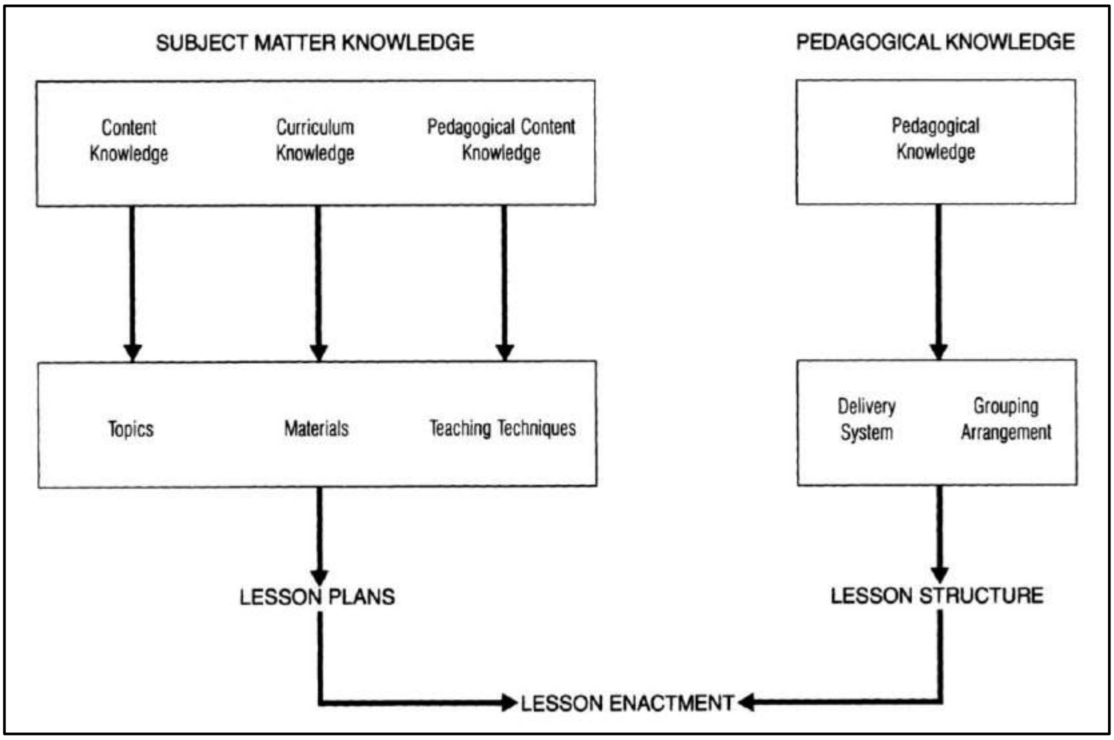


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zacahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013). It seems to make sense as they could manipulate and observe what changes happen by reshaping a figure (Yunianto, 2015).

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Thus, we should not expect teachers to deliver mathematics well if they do not have mastered it and do not understand how to teach it. Kow and Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason, (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Kow & Yeo, 2008).

This study intends to focus on a part of PCK, the KCS within lesson plans on the topic of area-perimeter of a rectangle. It is necessary to obtain a fuller insight into teacher knowledge as it influence students' performance. Beside testing, there might be alternative way such as lesson plans to investigate teachers' knowledge. How mathematics teachers prepare their lesson plans and how is PCK integrated in their lesson plans? How are the KCS integrated in the lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved humans and has been approved by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk) as this is a part of completion of the first author's dissertation. This study administrated a case study approach. This approach suits this study as it does not seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). The research subjects were the mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan as part of the whole PD. It was done somewhere in the middle of all complete sessions. As it is a case study, the researchers examined two selected lesson plans of two mathematics teachers. The remaining lesson plans have not been analyzed due to time limitation. The sample was chosen from twenty-nine teachers who attended a professional development (PD) session, and two teachers were selected for the lesson plan analysis and interview. Additionally. these teachers were selected based on their teaching experience; at least five years. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The topic that would be taught was area and perimeter for grade 7. The "Gold Rush/Mining" task was selected. This task was chosen because it is a problem-solving task and has several ways to be solved on area-perimeter of a rectangle (see Figure 3). Additionally, the complete Gold Rush activity showed the mistakes that students might do. Thus, it is considered as a good activity to be explored to understand how teachers prepare this activity.

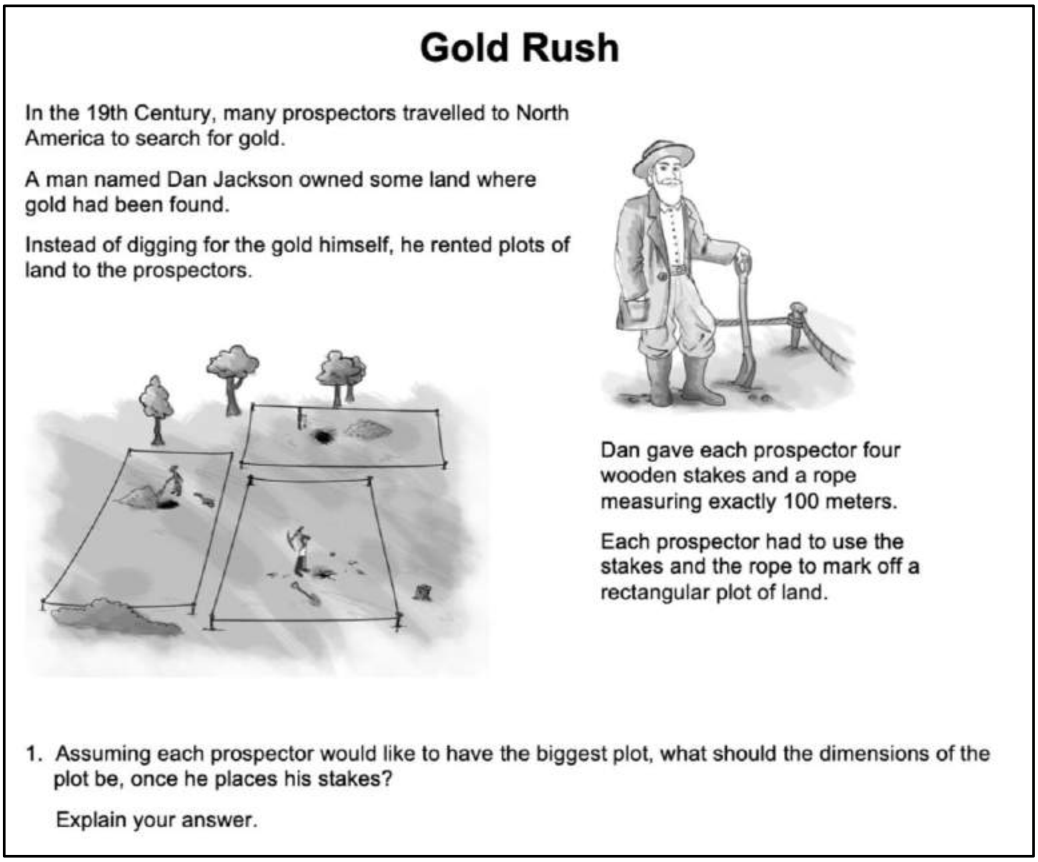


Figure 3. The Gold Rush problem (https://www.map.mathshell.org/download.php?fileid=1637)

To analyze the lesson plans, the researchers used content analysis. This method has the 'potential to disclose many hidden aspects of what is being communicated through the written text' (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers' knowledge of students' conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find themes by classifying instructions and KCS integrated in the lesson plans.

Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students' emerging and incomplete thinking

By using Table 2, it is easy to differentiate instructions' categories. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers

included information about what students would do to the task (KCS). The data were presented descriptively.

The two lesson plans were coded and analyzed. There were three types of instructions to refer to with the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3). The percentage is from type of instruction per total instructions written on the lesson plans.

Indonesian teachers follow the prescribed template of a lesson plan by MoEC. The template consists of three main parts namely; introduction, main and closure. It also consists learning goals and how teachers and students would do in the classroom.

Table 3. Proportions of the instructions

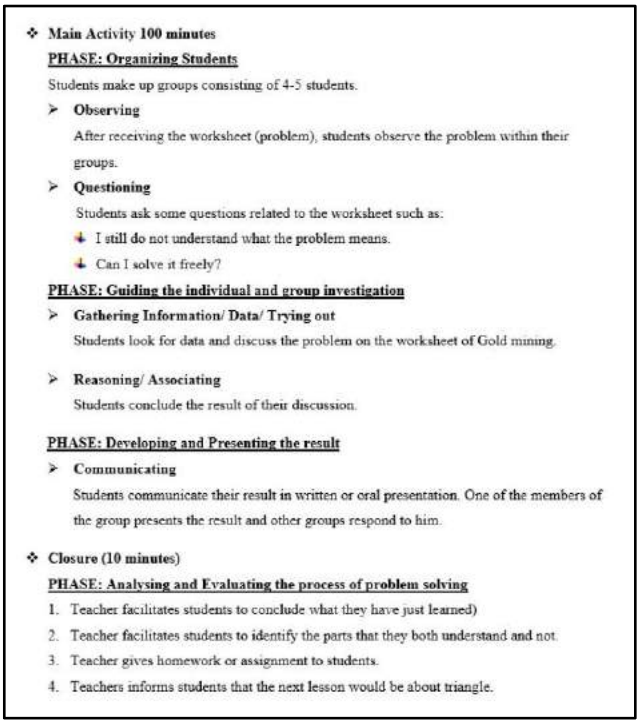
Instruction	T1	T2
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	**23 (100%)**	**19 (100%)**

Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2's SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	Introduction		Main		Closure	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	**12**	**3**	**7**	**11**	**4**	**5**

T1 put more details of what students would ask to her on her lesson plan. For instance: 'Can I solve it freely?' has been put on her lesson plan. This is a proof of PCK in the lesson plan, but not specific to KCS.



❖ **Main Activity 100 minutes**

PHASE: Organizing Students

Students make up groups consisting of 4-5 students.

➢ **Observing**

After receiving the worksheet (problem), students observe the problem within their groups.

➢ **Questioning**

Students ask some questions related to the worksheet such as:

- I still do not understand what the problem means.
- Can I solve it freely?

PHASE: Guiding the individual and group investigation

➢ **Gathering Information/ Data/ Trying out**

Students look for data and discuss the problem on the worksheet of Gold mining.

➢ **Reasoning/ Associating**

Students conclude the result of their discussion.

PHASE: Developing and Presenting the result

➢ **Communicating**

Students communicate their result in written or oral presentation. One of the members of the group presents the result and other groups respond to him.

❖ **Closure (10 minutes)**

PHASE: Analysing and Evaluating the process of problem solving

1. Teacher facilitates students to conclude what they have just learned)
2. Teacher facilitates students to identify the parts that they both understand and not.
3. Teacher gives homework or assignment to students.
4. Teachers informs students that the next lesson would be about triangle.

Figure 4. Teacher 1 Lesson Plan

In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4). On the phase of guiding the individual and group investigation, which should be rich of KCS. In this lesson plan, detail ways of students might solve it or make mistakes and how to facilitate it have not been depicted.

The T2 lesson plan of rectangle using Gold Rush task depicted detailed information about a possible student strategy (KCS). Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

Main Activity
- ☑ Teacher divide students into groups
- ☑ Teacher delivers the worksheet to be discussed
- ☑ Teacher facilitates the learning processes

- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
  for instance, p=10, l=… m
  p = 15 m, l = ...m, then the area = …. p=15 m, l= … m etc
  Students determine the largest area by themselves
- For the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
  Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
  How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan of Gold Rush

After finding the largest area of the rectangle, students had to find the largest area by joining two miners' ropes and how would they join it. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

- Students evaluate and make generalisation into questioning.
  - ☑ Teacher asks students to present in front of the class
  - ☑ Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Teachers T1 and T2 have more than five-years teaching experience each. Based on the questionnaire and interview, their schools are different in terms of location and students' background. These teachers themselves employed different abilities in solving the Gold Mining problem (Figure 3). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : Are there other ways T1?
T1 : Yesterday, I just did that one.
T1 : …just let students find the ways to solve it …. Then, I will let them know that there are some ways to solve it. I give that opportunity to students

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

*T2 : To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*
R : So, you (T2), induce them by using the table?
T2 : Yes, by the table.
R : What do you think, how many ways to solve it?
*T2 : To me, I did one way I know it directly it would be a square**. I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and.., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

*T2 : I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*
T2 : However, I have thought only one strategy, which is global to solve a task. … I, I... know at least I understand my students' characteristic so that it will be difficult for my students. … It is not possible to come up if I let them to do it freely. … I am so careful to give it the various strategies because students would get confuse

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

T2 : *By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.*
T2 : *To me, I did one way I know it directly it would be a square. I knew it already*
T2 : *…instead of table, we can make the variable x, then I will be a quadratic function,*
R : *Are there other ways to solve it?*
T2 : *For the time being, not yet, making rectangles and to the square*
R : *Do you think there are still other ways to solve that problem?*
T2 : *I could use the graph …*

To some extent, from the lesson plan, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task (Figure 5). However, by giving the students the strategy, he inadvertently is making the students dependent on him. Whereas, from the lesson plan, T1 is helping the students to make decisions themselves (Figure 4). From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

T1 : *The task has missing information, it should be more, and some students would think that. So that they* ***have not thought*** *yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
T1 : *100. Maybe* ***they thought that*** *that's the only think they know.*
R : *… So, they would answer it 100, possibly*
T1 : *Yeah, possibly*
T2 : *… for those who did not understand,* ***they would not know what 100*** *m rope is to with the perimeter. So that the concept of perimeter, for those who understood, they already make it but later* ***they would not think*** *the rectangles can be varied.*
T2 : *Students* ***would confuse*** *the meaning of maximum, which is the largest,* ***they have not thought about it.*** *So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7). The researcher proposed a possible mistake by a student of which the shape looks like a rectangle 25 x 26,5.

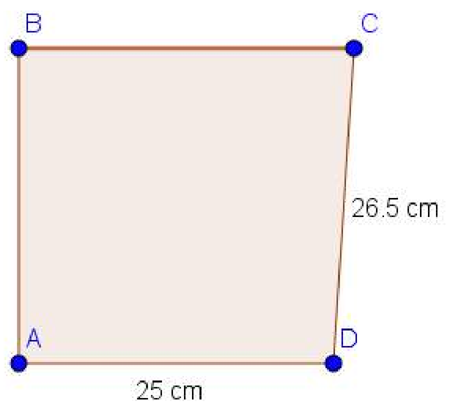


Figure 7. A student's possible mistake proposed by the researcher

If faced with a student mistake that they have not thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R	*:*	*If it happens if you see this (showing)*
T1	*:*	*I would ask students back to try it then you calculate it as what being asked to you*
R	*:*	*They have not yet known the result!*
T1		*Try, try it, by trialing they would know that it is different, this one is more, and that one is like that, ...*
R	*:*	*T2, what if your students did this? what would you do?*
T2	*:*	*I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...*

In this study, the lesson plans facilitated an insight into teachers' knowledge. In this case, it showed teacher's pedagogical knowledge as well as PCK. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid, and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Kow & Yeo, 2008). In Japanese lesson plans, they contain more detailed instructions (Nakahara & Koyama, 2000) which shows more information about teachers knowledge. In contrast, the two case of teachers in this study, have not yet shown detailed instructions but more in general instruction.

Teachers have different ways of supporting students to solve tasks (Yeo, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task (possessing SMK and PCK). However, these results are not generalizable. The limited sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully been aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS of a specific topic through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter of rectangle topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared in facing the situations during teaching. Developing problem solving skills and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment. Additionally, this study explored a rectangle topic, the result might vary in different topics Therefore, further investigation on different topic could be conducted. This study is not generalizable as it used limited research subjects.

ACKNOWLEDGMENTS

The authors wish to thank to Ministry of Education of Republic of Indonesia; Planning and Cooperation of Foreign Affairs for the scholarship. The authors also would like to thank SEAMEO QITEP in Mathematics for its endless support.

REFERENCES

An, S., Kulm, G., & Wu, Z. (2004). The Pedagogical Content Knowledge of Middle School, Mathematics Teachers in China and the U.S. *Journal of Mathematics Teacher Education*, *7*, 145-172. https://doi.org/10.1023/b:jmte.0000021943.35739.1c

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, *31*, 235-268. https://doi.org/10.1007/BF00376322

Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers:*

Results from the COACTIV Project (pp. 25-48). Boston: Springer. https://doi.org/10.1007/978-1-4614-5149-5_2

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., … Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*(1), 133-180. https://doi.org/10.3102/0002831209345157

Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' Thinking About Instruction. *Remedial and Special Education, 11*(6), 40-49. https://doi.org/10.1177/074193259001100609

Burns, R. B., & Lash, A. A. (1988). Nine Seventh-Grade Teachers' Knowledge and Planning of Problem-Solving Instruction. *The Elementary School Journal, 88*(4), 369-386. https://doi.org/10.1086/461545

Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.

Carle, S. M. (1993). Student held misconceptions regarding area and perimeter of rectangles. *Critical and Creative Thinking Capstones Collection, 46*. http://scholarworks.umb.edu/cct_capstone/46

Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136–143). Adelaide: Australian Association of Mathematics Teachers.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. *Handbook of research on teaching*. New York: MacMillan

Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from Lessons: Studying the Construction of Teacher Knowledge Catalysed by Purposefully-Designed Experimental Mathematics Lessons. *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165-172). Sunshine Coast: MERGA

Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.

Fauskanger, J. (2015). Challenges in measuring teachers' knowledge. *Educational Studies in Mathematics, 90*, 57-73. https://doi.org/10.1007/s10649-015-9612-4

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372-400. https://www.jstor.org/stable/pdf/40539304

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406. https://doi.org/10.3102/00028312042002371

John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies, 38*(4), 483-498. https://doi.org/10.1080/00220270500363620

Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience: 3rd Edition* (pp. 79–100). London: Routledge Taylor & Francis Group. https://doi.org/10.4324/9780203844120

Korkmaz, H. I., & Şahin, Ö. (2019). Preservice Preschool Teachers' Pedagogical Content Knowledge on Geometric Shapes in Terms of Children's Mistakes. *Journal of Research in Childhood Education, 34*(3), 385-405. https://doi.org/10.1080/02568543.2019.1701150

Kow, K., & Yeo, J. (2008). Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action. *Proceedings of the 31st Annual Conference of the Mathematics Education*

Research Group of Australasia.

Kristanto, Y. D., Panuluh, A. H., & Atmajati, E. D. (2020). Development and validation of a test instrument to measure pre-service mathematics teachers' content knowledge and pedagogical content knowledge. *Journal of Physics: Conference Series*, *1470*(1), 012008. https://doi.org/10.1088/1742-6596/1470/1/012008

Lee, Y., Capraro, R. M., & Capraro, M. M. (2018). Mathematics Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge in Problem Posing. *International Electronic Journal of Mathematics Education*, *13*(2), 75-90. https://doi.org/10.12973/iejme/2698

Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1*.

Özerem, A. (2012). Misconceptions In Geometry And Suggested Solutions For Seventh Grade Students. *Procedia - Social and Behavioral Sciences*, *55*, 720-729. https://doi.org/10.1016/j.sbspro.2012.09.557

Superfine, A. M. C. (2008). Planning for Mathematics Instruction: A Model of Experienced Teachers' Planning Processes in the Context of a Reform Mathematics Curriculum. *The Mathematics Educator, 18*(2), 11-22. https://ojs01.galib.uga.edu/tme/article/view/1925/1830

Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical Content Knowledge of Mathematics Pre-service Teachers: Do they know their students? *Journal of Physics: Conference Series*, *1097*(1), 012098. https://doi.org/10.1088/1742-6596/1097/1/012098

Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*, *15*(2), 4-14. https://doi.org/10.3102/0013189X015002004

Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education*, *26*(2), 114-145. https://doi.org/10.2307/749205

Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., … Rowley, G. (2012). *Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M)*. Amsterdam: International Association for the Evaluation of Educational Achievement.

Turnuklu, E., & Yesildere, S. (2007). The Pedagogical Content Knowledge in Mathematics: Pre-Service Primary Mathematics Teachers' Perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, *1*, 1-13.

Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating Preservice Mathematics Teachers' Pedagogical Content Knowledge through Microteaching. *REDIMAT-Journal of Research in Mathematics Education*, *9*(1), 62–87. http://dx.doi.org/10.17583/redimat.2020.3353

Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.

White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2012). Teachers learning from teachers. In *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13

Widodo, & Tamimudin, M. (2014). Three Training Strategies for Improving Mathematics Teacher Competences in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Yogyakarta: Mathematics and Technology, LLC. Retrieved from http://atcm.mathandtech.org/EP2014/index.html

Yeo, K. K. Y. (2008). Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 621-627). Brisbane: The University of Queensland. Retrieved from https://repository.nie.edu.sg/bitstream/10497/14397/1/MERGA-2008-621-YeoKK_a.pdf

Yin, R. K. (2014). *Case study research: Design and methods*. Thousand Oaks: SAGE Publications.

Yunianto, W. (2015). Supporting Students' Understanding of Area Measurement Through Verknippen Applet. *Southeast Asian Mathematics Education Journal*, *5*(1), 73-82. https://doi.org/10.46517/seamej.v5i1.34

Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, *6*(2), 41–62. https://doi.org/10.26220/rev.1627

Artikel diterbitkan pada tanggal 26 Maret 2021

Gmail | jme@unsri.ac.id | 2 of 28

Compose

Inbox 406
Starred
Snoozed
Sent
Drafts
Academia dan Res... 19
IConProCS 204
Notes
Publons 2
Rejected Email
Research Gate 38
Sejarah Matematik... 24
More

Meet
New meeting
My meetings

Hangouts
Rully
No recent chats
Start a new one

[JME] Your article has been published

Jme Fkip Matematika — Fri, Mar 26, 2:03 PM
to Wahid, me, Cosette, zulkardi

Dear Wahid Yunianto, Rully Charitas Indra Prahmana, and Cosette Crisan,

Congratulations!

After our editorial staff revised your paper based on your remark and finalization of your payment process, your article entitled **Indonesian Mathematics Teachers' Knowledge of Content and Students of Area and Perimeter of Rectangle** has just been published and is available at the following link:

https://ejournal.unsri.ac.id/index.php/jme/article/view/13537

Your manuscript will now form part of this open-access publication and your content will be licensed under the Creative Commons Attribution License. It is freely accessible to any user and will shortly be indexed in various international research repositories for further dissemination and reach in readership. There is no need for you to order print issues or copies.

Citation Information
As an online-first article, your paper is fully citable even before the journal's full issue has been compiled. Your article can be cited by its unique Digital Object Identifier (DOI), https://doi.org/10.22342/jme.12.2.13537.223-238. We encourage you to mention your article and its DOI on your website or your social media profiles. If you are interested to receive publication notifications from the journal, register for the RSS feed at https://ejournal.unsri.ac.id/index.php/jme/gateway/plugin/WebFeedGatewayPlugin/atom.

****

Please help us to improve your experience as an author by taking a few minutes to tell us about the service that you have received. We appreciate your participation and want to make sure we met your expectations, which will give us the opportunity to better serve the community.

Feedback:
https://forms.gle/BeTU3Y6hCiTSxoUX9

****

Thank you again for publishing with the Journal on Mathematics Education. We look forward to your future contributions.

Kind regards,
Editorial Office
Journal on Mathematics Education

Journal on Mathematics Education
https://ejournal.unsri.ac.id/index.php/jme

Crisan, Cosette <c.crisan@ucl.ac.uk> — Fri, Mar 26, 4:48 PM
to Wahid, me

Dear Wahid and Indra,

This is such good news!
Many thanks for persevering with the publication of this paper. I have added it to my research profile - very proud!

It would be great if we had a Zoom chat at some point? We are interested in encouraging MA students to publish their dissertation work - Wahid, if you have 40min or so, you could come and talk to our current cohort.

with best wishes, Cosette

Artikel terbit di Journal on Mathematics Education Vol. 12 No. 2, 223-238
[DOI: https://doi.org/10.22342/jme.12.2.13537.223-238]

ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 12, No. 2, May 2021, pp. 223-238

INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS OF AREA AND PERIMETER OF RECTANGLE

Wahid Yunianto[1], Rully Charitas Indra Prahmana[2], Cosette Crisan[3]

[1]SEAMEO QITEP in Mathematics, Jl. Kaliurang, Condongcatur, Depok, Sleman, Yogyakarta 55281, Indonesia
[2]Universitas Ahmad Dahlan, Jl. Pramuka 42, Pandeyan, Umbulharjo, Yogyakarta, Indonesia
[3]University College London, Gower St, Bloomsbury, London WC1E 6BT, United Kingdom
Email: rully.indra@mpmat.uad.ac.id

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. Research has shown teachers competencies and skills influence students' performances. Previous studies explored teachers' knowledge through testing. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items or tasks to assess mathematics teacher knowledge in the previous studies were dominated by subject matter knowledge problems. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' Knowledge of Content and Students (KCS) through lesson plans developed by the teachers. To accommodate the gap in the previous studies, this study focuses on KCS on the topic of area-perimeter through their designed lesson plans. Twenty-nine mathematics teachers attended a professional development activity voluntarily participated in this study. Two teachers were selected to be the focus of this case study. Content analysis of the lesson plan and semi-structured interviews were conducted, and then data were analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible strategies and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Penelitian ini menunjukkan bahwa komptensi dan keterampilan guru mempengaruhi performa siswa. Penelitian sebelumnya telah mengkaji pengetahuan gru melalui tes. Pengetahuan guru pada topik keliling-luas dan strategi pembelajaran juga telah dikaji melalui tes. Pada umumnya, banyaknya soal pada tes didominasi oleh soal-soal tentang pengetahuan subjek yang diajarkan. Oleh karena itu, asesmen seperti ini belum mencakup kesuluruhan pengetahuan dan kompetensi guru. Pada studi ini, peneliti menginvestigasi pengetahuan guru matematika tentang KCS pada rencana pelaksanaan pembelajaran yang mereka kembangkan. Untuk mengakomodasi kesenjangan pada penelitian sebelumnya, penelitian kali ini berfokus pada pengetahuan tentang konten dan siswa (KCS) pada topik keliling-luas pada rencana pelaksanaan pembelajaran. Dua puluh Sembilan guru matematika yang sedang mengikuti pelatihan peningkatan kompetensi secara suka rela mengikuti penelitian ini. Dua guru matematika menjadi fokus penelitian studi kasus ini. Konten analisis dan interview semi terstruktur dilakukan dan datanya dianalisis. Terungkap bahwa peserta ini mengalami tantangan dalam memprediksi kemungkinan respon yang diberikan siswa. Mereka belum menyadari strategi siswa yang biasanya digunakan untuk menyelesaikan persoalan memaksimalkan luas dari keliling yang ditentukan. Dengan pengetahuan yang terbatas pada kemungkinan strategi siswa dan kesalahan siswa, guru ini kurang siap dalam mendukung siswanya

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

How to Cite: Yunianto, W., Prahmana, R.C.I., & Crisan, C. (2021). Indonesian Mathematics Teachers' Knowledge of Content and Students of Area and Perimeter of Rectangle. *Journal on Mathematics Education, 12*(2), 223-238. http://doi.org/10.22342/jme.12.2.13537.223-238

Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research have shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

> *Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2012, p.394).*

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

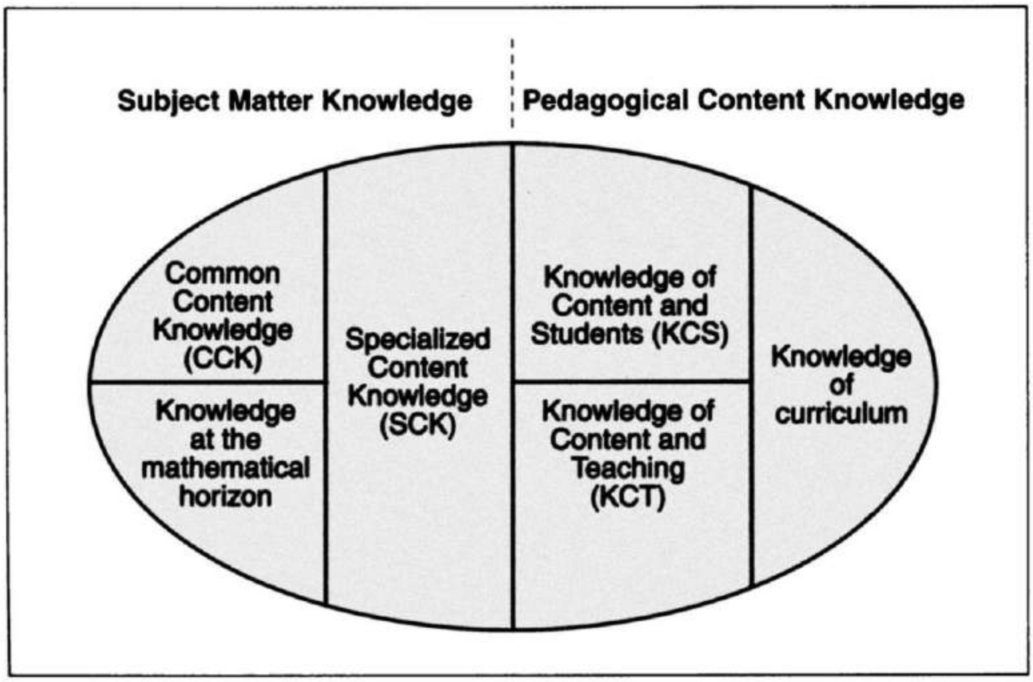


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). In the case of pre-service teachers, they have challenges with student's thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). It makes sense as they have limited teaching experiences or even have not taught yet. For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COACTIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

As one of the ways, testing is used to assess teachers. The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin, 2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Another study using testing faced challenges in measuring teachers' knowledge (Fauskanger, 2015). An interesting finding of a study of pre-service teachers is that they possessed higher PCK scores than SMK from limited test items (Kristanto, Panuluh, & Atmajati, 2020). A case study in South Korea revealed that teachers with sufficient SMK of a certain competence/ topic faced challenges in incorporating KCS and KCT of that topic (Lee, Capraro, & Capraro, 2018). Therefore, testing to measure teachers' knowledge still face challenges.

Lesson plans are considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers can easily understand it when reading it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called, 'Bansho', which anticipates and tries to elicit student mathematical thinking and student thinking schema for solving the given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed that the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). The students' engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom practices are influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

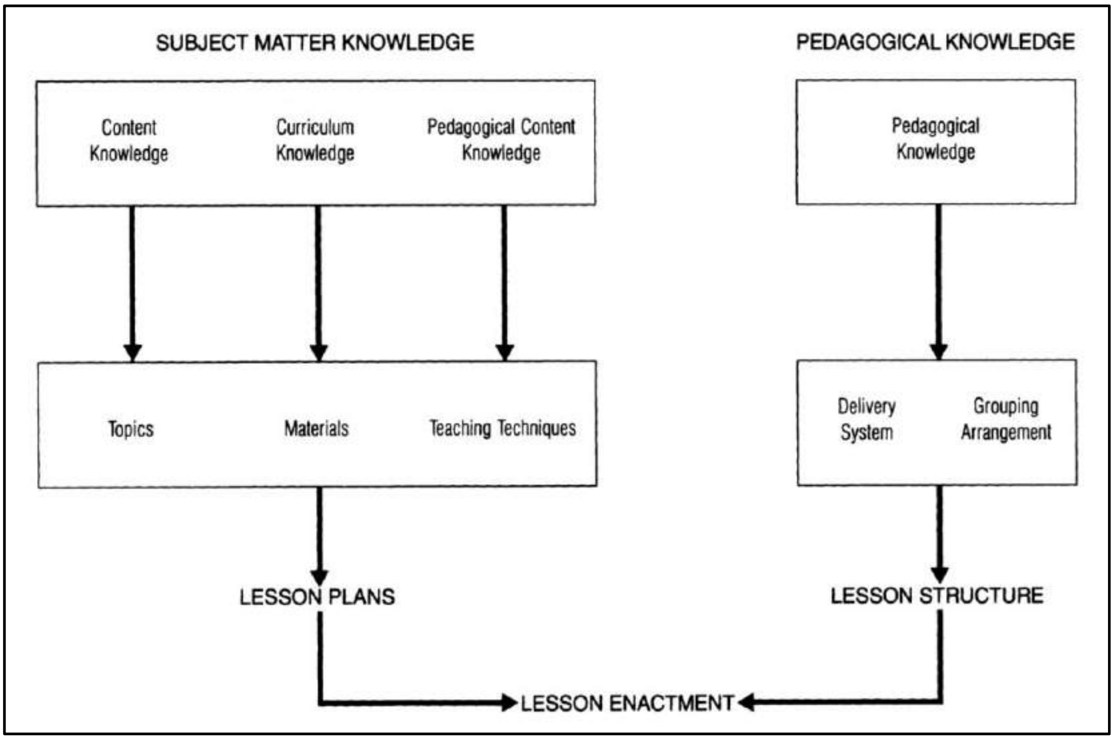


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zacahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013). It seems to make sense as they could manipulate and observe what changes happen by reshaping a figure (Yunianto, 2015).

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Thus, we should not expect teachers to deliver mathematics well if they do not have mastered it and do not understand how to teach it. Kow and Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Kow & Yeo, 2008).

This study intends to focus on a part of PCK, the KCS within lesson plans on the topic of area-perimeter of a rectangle. It is necessary to obtain a fuller insight into teacher knowledge as it influence students' performance. Beside testing, there might be alternative way such as lesson plans to investigate teachers' knowledge. How are mathematics teachers prepare their lesson plans and how is PCK integrated in their lesson plans? How are the KCS integrated in the lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved humans and has been approved by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk) as this is a part of completion of the first author's dissertation. This study administrated a case study approach. This approach suits this study as it does not seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). The research subjects were the mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan as part of the whole PD. It was done somewhere in the middle of all complete sessions. As it is a case study, the researchers examined two selected lesson plans of two mathematics teachers. The remaining lesson plans have not been analyzed due to time limitation. The sample was chosen from twenty-nine teachers who attended a professional development (PD) session, and two teachers were selected for the lesson plan analysis and interview. Additionally. these teachers were selected based on their teaching experience; at least five years. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The topic that would be taught was area and perimeter for grade 7. The "Gold Rush/Mining" task was selected. This task was chosen because it is a problem-solving task and has several ways to be solved on area-perimeter of a rectangle (see Figure 3). Additionally, the complete Gold Rush activity showed the mistakes that students might do. Thus, it is considered as a good activity to be explored to understand how teachers prepare this activity.

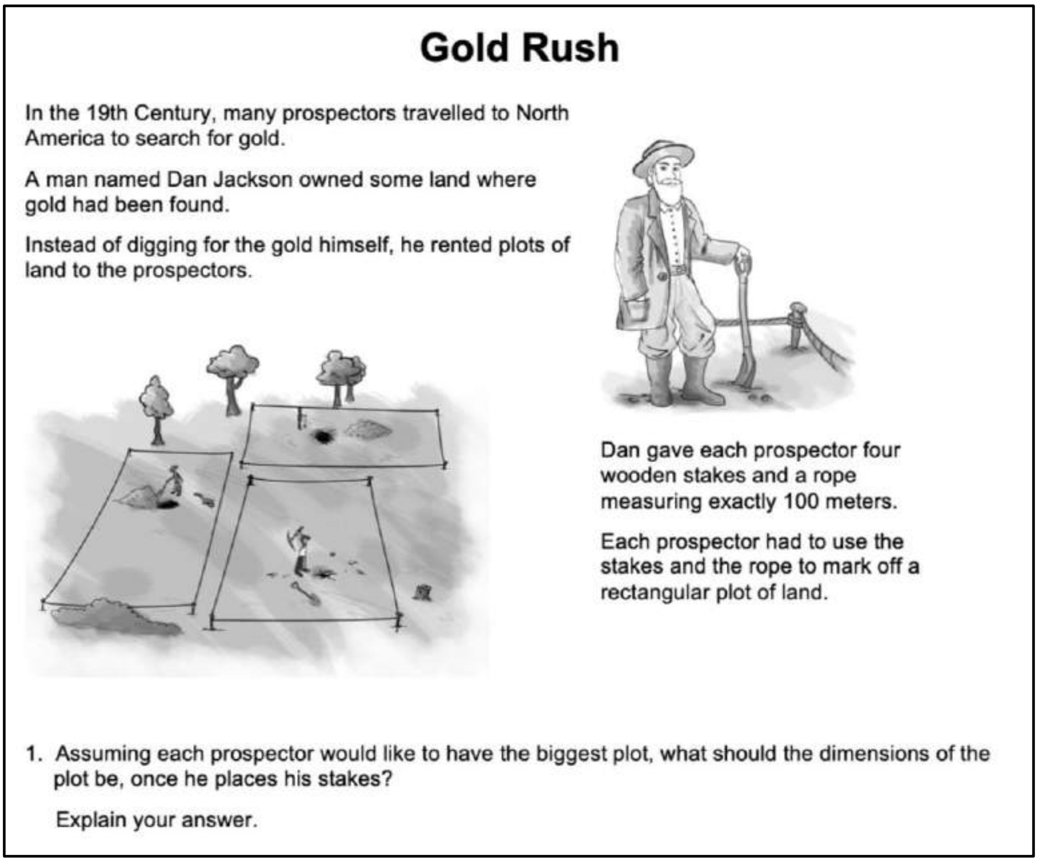


Figure 3. The Gold Rush problem (https://www.map.mathshell.org/download.php?fileid=1637)

To analyze the lesson plans, the researchers used content analysis. This method has the 'potential to disclose many hidden aspects of what is being communicated through the written text' (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers' knowledge of students' conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find themes by classifying instructions and KCS integrated in the lesson plans.

Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students' emerging and incomplete thinking

By using Table 2, it is easy to differentiate instructions' categories. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers

included information about what students would do to the task (KCS). The data were presented descriptively.

The two lesson plans were coded and analyzed. There were three types of instructions to refer to with the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3). The percentage is from type of instruction per total instructions written on the lesson plans.

Indonesian teachers follow the prescribed template of a lesson plan by MoEC. The template consists of three main parts namely; introduction, main and closure. It also consists learning goals and how teachers and students would do in the classroom.

Table 3. Proportions of the instructions

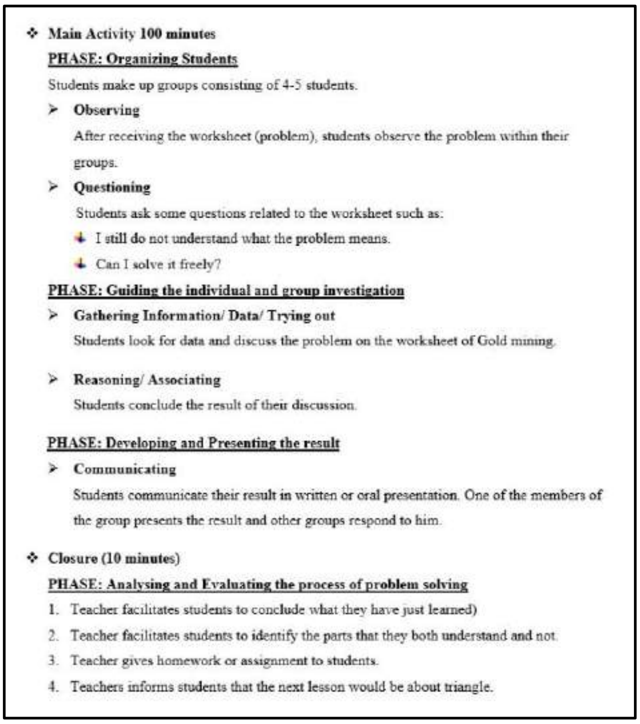
Instruction	**T1**	**T2**
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	**23 (100%)**	**19 (100%)**

Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2's SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	**Introduction**		**Main**		**Closure**	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	**12**	**3**	**7**	**11**	**4**	**5**

T1 put more details of what students would ask to her on her lesson plan. For instance: 'Can I solve it freely?' has been put on her lesson plan. This is a proof of PCK in the lesson plan, but not specific to KCS.



❖ **Main Activity 100 minutes**

PHASE: Organizing Students

Students make up groups consisting of 4-5 students.

➢ **Observing**

After receiving the worksheet (problem), students observe the problem within their groups.

➢ **Questioning**

Students ask some questions related to the worksheet such as:

- I still do not understand what the problem means.
- Can I solve it freely?

PHASE: Guiding the individual and group investigation

➢ **Gathering Information/ Data/ Trying out**

Students look for data and discuss the problem on the worksheet of Gold mining.

➢ **Reasoning/ Associating**

Students conclude the result of their discussion.

PHASE: Developing and Presenting the result

➢ **Communicating**

Students communicate their result in written or oral presentation. One of the members of the group presents the result and other groups respond to him.

❖ **Closure (10 minutes)**

PHASE: Analysing and Evaluating the process of problem solving

1. Teacher facilitates students to conclude what they have just learned)
2. Teacher facilitates students to identify the parts that they both understand and not.
3. Teacher gives homework or assignment to students.
4. Teachers informs students that the next lesson would be about triangle.

Figure 4. Teacher 1 Lesson Plan

In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4). On the phase of guiding the individual and group investigation which be rich of KCS. In this lesson plan, detail ways of students might solve it or make mistakes and how to facilitate it have not been depicted.

The T2 lesson plan of rectangle using Gold Rush task depicted detailed information about a possible student strategy (KCS). Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

Main Activity
- ☑ Teacher divide students into groups
- ☑ Teacher delivers the worksheet to be discussed
- ☑ Teacher facilitates the learning processes

- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
  for instance, p=10, l=… m
  p = 15 m, l = ...m, then the area = …. p=15 m, l= … m etc
  Students determine the largest area by themselves
- For the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
  Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
  How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan of Gold Rush

After finding the largest area of the rectangle, students had to find the largest area by joining two miners' ropes and how would they join it. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

- Students evaluate and make generalisation into questioning.
  - ☑ Teacher asks students to present in front of the class
  - ☑ Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Teachers T1 and T2 have more than five-years teaching experience each. Based on the questionnaire and interview, their schools are different in terms of location and students' background. These teachers themselves employed different abilities in solving the Gold Mining problem (Figure 3). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : Are there other ways T1?
T1 : Yesterday, I just did that one.
T1 : …just let students find the ways to solve it …. Then, I will let them know that there are some ways to solve it. I give that opportunity to students

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

*T2 : To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*
R : So, you (T2), induce them by using the table?
T2 : Yes, by the table.
R : What do you think, how many ways to solve it?
*T2 : To me, I did one way I know it directly it would be a square. **I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and.., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

*T2 : I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*
T2 : However, I have thought only one strategy, which is global to solve a task. … I, I... know at least I understand my students' characteristic so that it will be difficult for my students. … It is not possible to come up if I let them to do it freely. … I am so careful to give it the various strategies because students would get confuse

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

T2 : *By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.*
T2 : *To me, I did one way I know it directly it would be a square. I knew it already*
T2 : *...instead of table, we can make the variable x, then I will be a quadratic function,*
R : *Are there other ways to solve it?*
T2 : *For the time being, not yet, making rectangles and to the square*
R : *Do you think there are still other ways to solve that problem?*
T2 : *I could use the graph ...*

To some extent, from the lesson plan, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task (Figure 5). However, by giving the students the strategy, he inadvertently is making the students dependent on him. Whereas, from the lesson plan, T1 is helping the students to make decisions themselves (Figure 4). From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

T1 : *The task has missing information, it should be more, and some students would think that. So that they* ***have not thought*** *yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
T1 : *100. Maybe* ***they thought that*** *that's the only think they know.*
R : *... So, they would answer it 100, possibly*
T1 : *Yeah, possibly*
T2 : *... for those who did not understand,* ***they would not know what 100*** *m rope is to with the perimeter. So that the concept of perimeter, for those who understood, they already make it but later* ***they would not think*** *the rectangles can be varied.*
T2 : *Students* ***would confuse*** *the meaning of maximum, which is the largest,* ***they have not thought about it.*** *So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7). The researcher proposed a possible mistake by a student of which the shape looks like a rectangle 25 x 26,5.

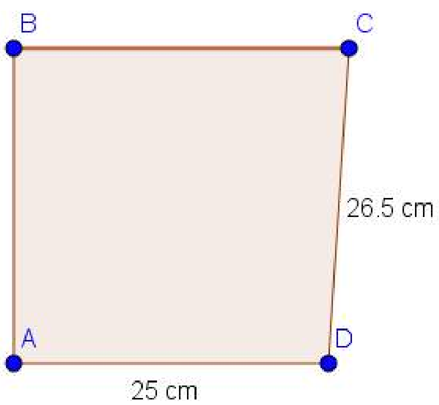


Figure 7. A student's possible mistake proposed by the researcher

If faced with a student mistake that they have not thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R : If it happens if you see this (showing)
T1 : I would ask students back to try it then you calculate it as what being asked to you
R : They have not yet known the result!
T1 Try, try it, by trialing they would know that it is different, this one is more, and that one is like that, ...
R : T2, what if your students did this? what would you do?
T2 : I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...

In this study, the lesson plans facilitated an insight into teachers' knowledge. In this case, it showed teacher's pedagogical knowledge as well as PCK. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid, and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Kow & Yeo, 2008). In Japanese lesson plans, they contain more detailed instructions (Nakahara & Koyama, 2000) which shows more information about teachers knowledge. In contrast, the two case of teachers in this study, have not yet shown detailed instructions but more in general instruction.

Teachers have different ways of supporting students to solve tasks (Yeo, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task (possessing SMK and PCK). However, these results are not generalizable. The limited sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully been aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS of a specific topic through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter of rectangle topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared in facing the situations during teaching. Developing problem solving skills and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment. Additionally, this study explored a rectangle topic, the result might vary in different topics. Therefore, further investigation on different topic could be conducted. This study is not generalizable as it used limited research subjects.

ACKNOWLEDGMENTS

The authors wish to thank to Ministry of Education of Republic of Indonesia; Planning and Cooperation of Foreign Affairs for the scholarship. The authors also would like to thank SEAMEO QITEP in Mathematics for its endless support.

REFERENCES

An, S., Kulm, G., & Wu, Z. (2004). The Pedagogical Content Knowledge of Middle School, Mathematics Teachers in China and the U.S. *Journal of Mathematics Teacher Education*, *7*, 145-172. https://doi.org/10.1023/b:jmte.0000021943.35739.1c

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, *31*, 235-268. https://doi.org/10.1007/BF00376322

Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers:*

Results from the COACTIV Project (pp. 25-48). Boston: Springer. https://doi.org/10.1007/978-1-4614-5149-5_2

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., … Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*(1), 133-180. https://doi.org/10.3102/0002831209345157

Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' Thinking About Instruction. *Remedial and Special Education, 11*(6), 40-49. https://doi.org/10.1177/074193259001100609

Burns, R. B., & Lash, A. A. (1988). Nine Seventh-Grade Teachers' Knowledge and Planning of Problem-Solving Instruction. *The Elementary School Journal, 88*(4), 369-386. https://doi.org/10.1086/461545

Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.

Carle, S. M. (1993). Student held misconceptions regarding area and perimeter of rectangles. *Critical and Creative Thinking Capstones Collection, 46*. http://scholarworks.umb.edu/cct_capstone/46

Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136–143). Adelaide: Australian Association of Mathematics Teachers.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. *Handbook of research on teaching*. New York: MacMillan

Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from Lessons: Studying the Construction of Teacher Knowledge Catalysed by Purposefully-Designed Experimental Mathematics Lessons. *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165-172). Sunshine Coast: MERGA

Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.

Fauskanger, J. (2015). Challenges in measuring teachers' knowledge. *Educational Studies in Mathematics, 90*, 57-73. https://doi.org/10.1007/s10649-015-9612-4

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372-400. https://www.jstor.org/stable/pdf/40539304

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406. https://doi.org/10.3102/00028312042002371

John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies, 38*(4), 483-498. https://doi.org/10.1080/00220270500363620

Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience: 3rd Edition* (pp. 79–100). London: Routledge Taylor & Francis Group. https://doi.org/10.4324/9780203844120

Korkmaz, H. I., & Şahin, Ö. (2019). Preservice Preschool Teachers' Pedagogical Content Knowledge on Geometric Shapes in Terms of Children's Mistakes. *Journal of Research in Childhood Education, 34*(3), 385-405. https://doi.org/10.1080/02568543.2019.1701150

Kow, K., & Yeo, J. (2008). Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action. *Proceedings of the 31st Annual Conference of the Mathematics Education*

Research Group of Australasia.

Kristanto, Y. D., Panuluh, A. H., & Atmajati, E. D. (2020). Development and validation of a test instrument to measure pre-service mathematics teachers' content knowledge and pedagogical content knowledge. *Journal of Physics: Conference Series*, *1470*(1), 012008. https://doi.org/10.1088/1742-6596/1470/1/012008

Lee, Y., Capraro, R. M., & Capraro, M. M. (2018). Mathematics Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge in Problem Posing. *International Electronic Journal of Mathematics Education*, *13*(2), 75-90. https://doi.org/10.12973/iejme/2698

Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1*.

Özerem, A. (2012). Misconceptions In Geometry And Suggested Solutions For Seventh Grade Students. *Procedia - Social and Behavioral Sciences*, *55*, 720-729. https://doi.org/10.1016/j.sbspro.2012.09.557

Superfine, A. M. C. (2008). Planning for Mathematics Instruction: A Model of Experienced Teachers' Planning Processes in the Context of a Reform Mathematics Curriculum. *The Mathematics Educator, 18*(2), 11-22. https://ojs01.galib.uga.edu/tme/article/view/1925/1830

Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical Content Knowledge of Mathematics Pre-service Teachers: Do they know their students? *Journal of Physics: Conference Series*, *1097*(1), 012098. https://doi.org/10.1088/1742-6596/1097/1/012098

Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*, *15*(2), 4-14. https://doi.org/10.3102/0013189X015002004

Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education*, *26*(2), 114-145. https://doi.org/10.2307/749205

Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., … Rowley, G. (2012). *Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M)*. Amsterdam: International Association for the Evaluation of Educational Achievement.

Turnuklu, E., & Yesildere, S. (2007). The Pedagogical Content Knowledge in Mathematics: Pre-Service Primary Mathematics Teachers' Perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, *1*, 1-13.

Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating Preservice Mathematics Teachers' Pedagogical Content Knowledge through Microteaching. *REDIMAT-Journal of Research in Mathematics Education*, *9*(1), 62–87. http://dx.doi.org/10.17583/redimat.2020.3353

Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.

White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2012). Teachers learning from teachers. In *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13

Widodo, & Tamimudin, M. (2014). Three Training Strategies for Improving Mathematics Teacher Competences in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Yogyakarta: Mathematics and Technology, LLC. Retrieved from http://atcm.mathandtech.org/EP2014/index.html

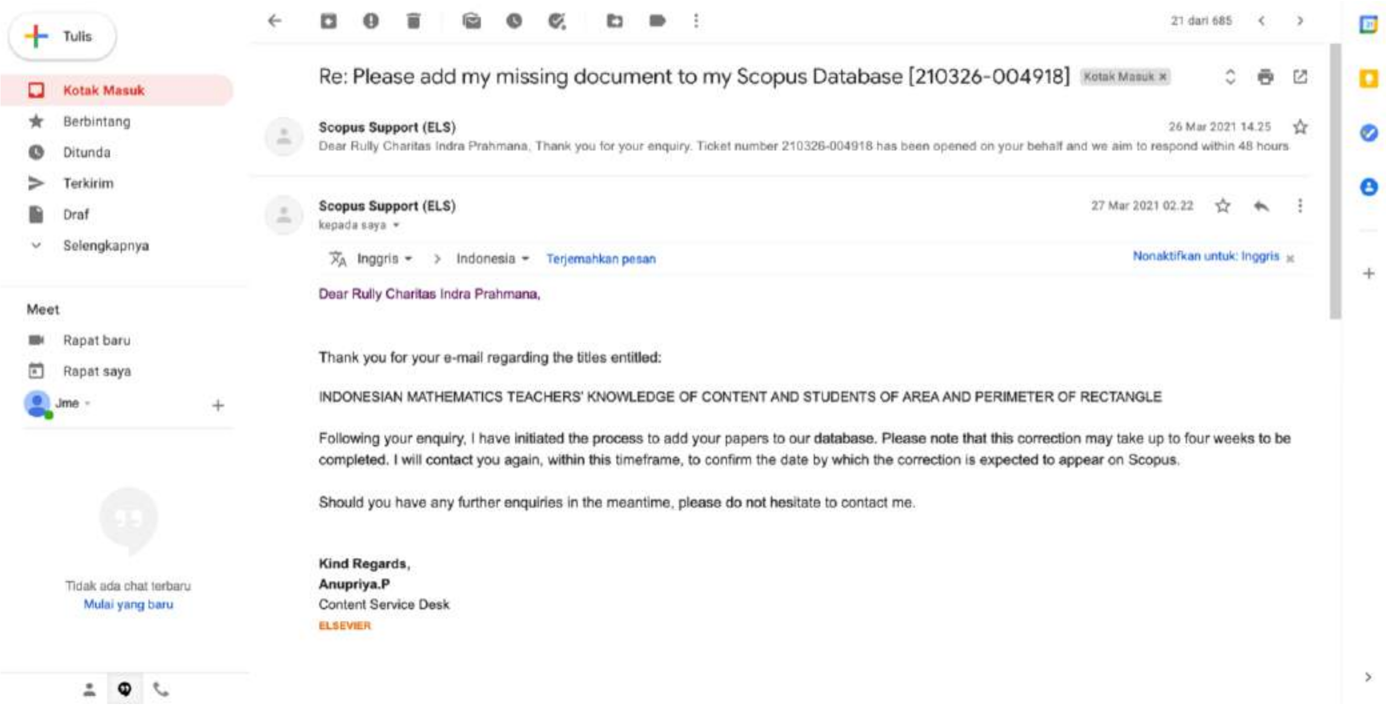
Yeo, K. K. Y. (2008). Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 621-627). Brisbane: The University of Queensland. Retrieved from https://repository.nie.edu.sg/bitstream/10497/14397/1/MERGA-2008-621-YeoKK_a.pdf

Yin, R. K. (2014). *Case study research: Design and methods*. Thousand Oaks: SAGE Publications.

Yunianto, W. (2015). Supporting Students' Understanding of Area Measurement Through Verknippen Applet. *Southeast Asian Mathematics Education Journal*, *5*(1), 73-82. https://doi.org/10.46517/seamej.v5i1.34

Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, *6*(2), 41–62. https://doi.org/10.26220/rev.1627

Artikel masih dalam proses untuk masuk ke dalam database Scopus.



Re: Please add my missing document to my Scopus Database [210326-004918]

Scopus Support (ELS) 26 Mar 2021 14.25

Dear Rully Charitas Indra Prahmana, Thank you for your enquiry. Ticket number 210326-004918 has been opened on your behalf and we aim to respond within 48 hours

Scopus Support (ELS) 27 Mar 2021 02.22

kepada saya

Dear Rully Charitas Indra Prahmana,

Thank you for your e-mail regarding the titles entitled:

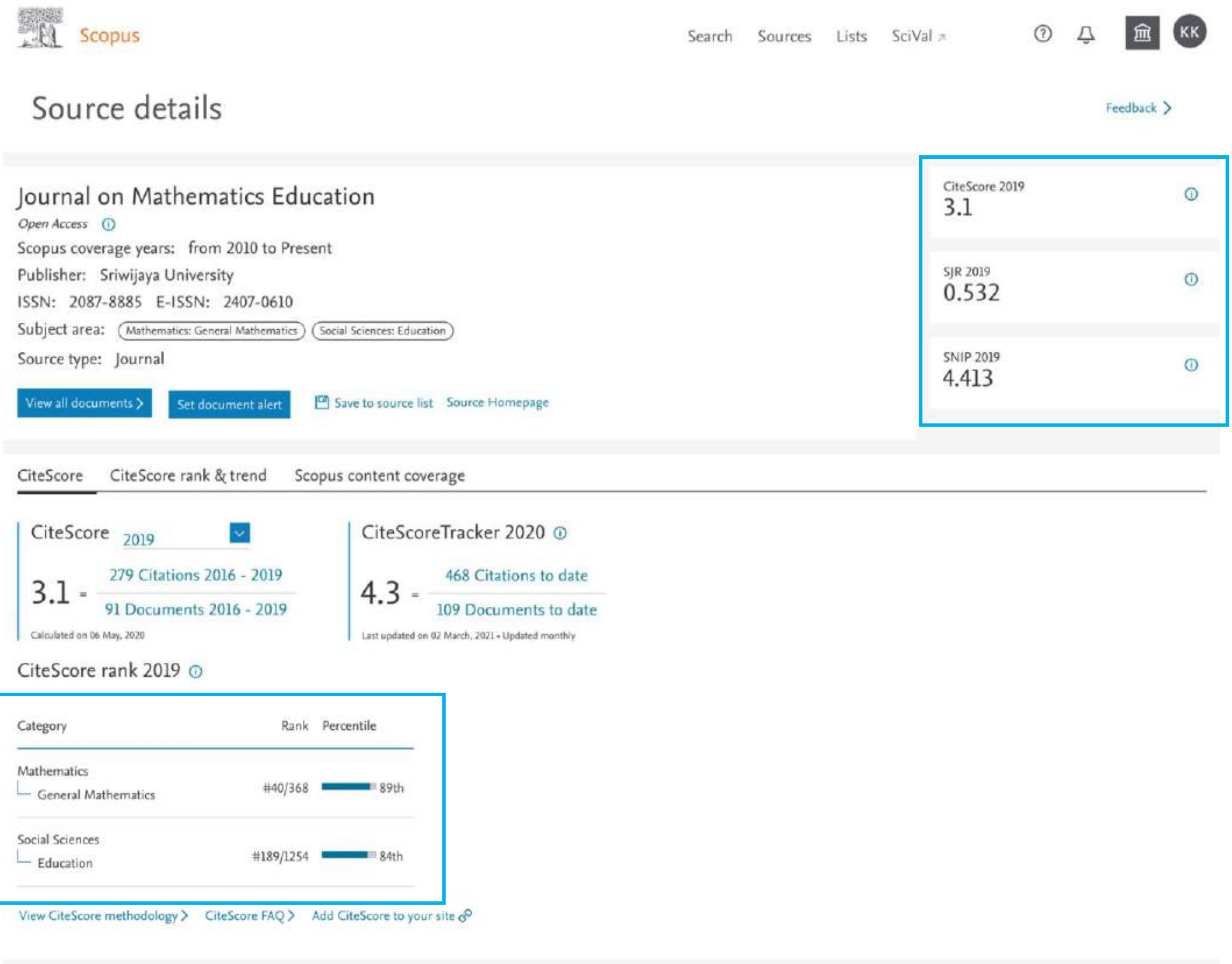
INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS OF AREA AND PERIMETER OF RECTANGLE

Following your enquiry, I have initiated the process to add your papers to our database. Please note that this correction may take up to four weeks to be completed. I will contact you again, within this timeframe, to confirm the date by which the correction is expected to appear on Scopus.

Should you have any further enquiries in the meantime, please do not hesitate to contact me.

Kind Regards,
Anupriya.P
Content Service Desk
ELSEVIER

Profile Jurnal di Website Scopus



Source details

Journal on Mathematics Education

Open Access

Scopus coverage years: from 2010 to Present

Publisher: Sriwijaya University

ISSN: 2087-8885 E-ISSN: 2407-0610

Subject area: Mathematics: General Mathematics; Social Sciences: Education

Source type: Journal

CiteScore 2019: 3.1

SJR 2019: 0.532

SNIP 2019: 4.413

CiteScore 2019: 3.1 = 279 Citations 2016 - 2019 / 91 Documents 2016 - 2019

Calculated on 06 May, 2020

CiteScoreTracker 2020: 4.3 = 468 Citations to date / 109 Documents to date

Last updated on 02 March, 2021 • Updated monthly

CiteScore rank 2019

Category	Rank	Percentile
Mathematics — General Mathematics	#40/368	89th
Social Sciences — Education	#189/1254	84th