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PAPER • OPEN ACCESS

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To cite this article: A Shodikin *et al* 2019 *J. Phys.: Conf. Ser.* **1188** 012004View the [article online](#) for updates and enhancements.

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Thinking errors of pre-service mathematics teachers in solving mathematical modelling task

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Abstract. Mathematical modelling is important. However, mathematics pre-service teachers often experience thinking errors in solving mathematical modelling task. This article aims to explore the thinking errors experienced by pre-service mathematics teachers in passing the mathematical modelling process. The explorative research was conducted by involving 21 mathematics pre-service teachers in East Java province, Indonesia. Data were obtained through a modelling task and interview. We have identified two of the most common thinking errors, that making inappropriate assumptions and failure in constructing a mathematical model that describes the real-world situation. The obstacles that cause these errors are they have not fully understood the problems faced and caught up in the routine problems they often get. Also, the verification process that should be used as a turning point for confirming the answers or improving answers has not been done optimally. We recommend providing sufficient scaffolding to mathematics pre-service teachers in understanding the problem, introducing non-routine problems in classroom learning, and optimizing verification of answers obtained to improve mathematical modelling abilities.

1. Introduction

Many people have experienced the advantages of mathematical modelling. Over the last two decades, there is rising attention to its importance in science, technology, engineering, and even the career of mathematics [1]. Therefore, the concept and skill of mathematical modelling are taught to students from elementary to higher education level [2–7]. Mathematical modelling is one of the important steps in solving mathematics problems. It can encourage students to explore the meaning of mathematics concept and procedure about the real-world situation [8]. The model which was made is the simplification of real-world phenomena in mathematics form.

In solving the problem of mathematical modelling, mathematics pre-service teachers often make thinking errors. The thinking error is a pattern of thinking and reasoning that causes errors in decision making. The thinking error is formed due to a person's understanding of a concept and is believed to be true. If these errors are not identified and straightened out, they will result from successive errors. These successive errors occurred because if later they become teachers, then the misinterpretation that they have will be transferred to their students. It can be said as a manifestation of thinking the error in applying problem-solving strategies [9].

The thinking error is caused by thinking obstacle, in addition to other factors such as ignorance, uncertainty, and chance [10]. Thinking obstacles are mistakes that occur as a result of the fascinating and successful prior knowledge but currently revealed as being wrong or just not realizing [11]. This



type of errors are regular (not erratic) and suddenly (unexpected). It often occurs due to the partial understanding of concept gained from during the learning. This obstacle is naturally found in any situation of mathematics task working, especially on the type of problem-solving task [12].

Until recently, there has been little research which explores the thinking errors of mathematics pre-service teachers, particularly in mathematical modelling. Previous studies related to mathematical modelling usually talk about teaching the concepts and skills of mathematical modelling [2–7], conceptualizing mathematical modelling [8,13–15], as well as the use of mathematical modelling [16–19]. Whereas, knowing the thinking obstacle is very necessary to discover the effects of learners' thinking activities [12] deeply. Also, by knowing the thinking errors, researchers and educators can use as consideration in making assignments, giving scaffolding, lesson planning, and an alternative solution if these obstacles arise. For that reason, this study aims to explore the thinking errors experienced by pre-service mathematics teachers in solving mathematical modelling task.

2. Method

This study utilized explorative research. Researchers developed a modelling task in Systems of Linear Equations in Two Variables (SLETV) with the inversely proportional topic. We selected this topic because after comparing SLETV with directly proportional, the SLETV is the more challenging and non-routine problem in classroom learning for most of the pre-service teachers. The modelling task created is based on the stages of the mathematical modelling cycle presented as follows.

A pond will be full of water in 4 hours if it is filled with two large pumps and one small pump at the same time. The pond will also be full of water in 4 hours if one large pump and three small pumps are used together. How many times is needed to make the pond full of water if four large pumps and four small pumps are used at the same time? To solve this case, make mathematical modelling in the first place.

This task tried to join the use of two types of pumps with the amount of time it takes to fill up a pond full. Students were given two prior conditions to calculate the time required under the conditions require. In this modelling tasks, SLETV with inversely proportional exists, where the more pumps were used, the time needed to fill the pond will be faster. This task is a kind of non-routine modelling problem for most of the pre-service mathematics teachers. This non-routine task is used to explore the thinking errors that arise in their mathematical modelling.

This modelling assignment was given to 21 pre-service mathematics teachers at one private university in East Java province, Indonesia. The pre-service mathematics teachers are taking a course in School Mathematics, where SLETV is one of the materials studied. Furthermore, in-depth interviews were conducted on eight selected pre-service mathematics teachers based on the similarity of answers and the types of mistakes made in the execution of tasks. Interviews are used to identify the thinking errors that they are experiencing and the obstacles that cause these errors to occur.

3. Results and Discussion

By reading the task and look at the situation of the problem, a model situation can be arranged as shown in Figure 1.

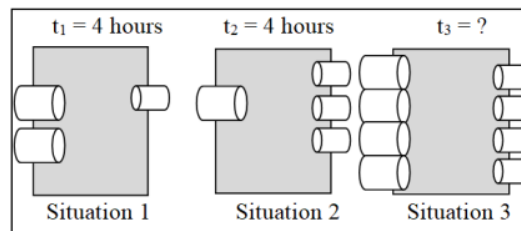


Figure 1. The model situation of the problem

Figure 1 shows that how the problem can be represented in the model situation. Supposing that a is part of the pond that will be filled by the big pumps for each hour, while b is part of the pond that will

be filled by the small pumps for each hour, and t is the time needed to fill up the pond. The problem can be modeled as a system of equation

$$\begin{cases} 2(4)a + 1(4)b = 1 \\ 1(4)a + 3(4)b = 1 \\ 4at + 4bt = 1 \end{cases}$$

while the question is finding the value of t .

By using algebraic manipulation which is in the stage of working mathematically, we could get relationships $a = 2b$ that lead to the value of $a = \frac{1}{10}$, $b = \frac{1}{20}$, and $t = \frac{5}{3}$. These results were further interpreted to the real-world as a tangible result, i.e. the amount of time required to fill up the pond. If four large pumps and four small pumps are used, it was approximately $\frac{5}{3}$ hours or 100 minutes needed to make the pond full of water. The next important step is to validate the results. Whether the results obtained reasonable or not? First, we know that the number of pumps used in Situation 3 is more than is used in Situation 1 and Situation 2, so the time required to fill the pond should be faster. Second, in this problem, we could assume that the amount of discharge water released for each pump is constant. Thus, we could form a mathematical model as a relationship of linear equations. 16

Errors that occur in the work of mathematical modelling tasks by the pre-service mathematics teachers are presented in Table 1.

Table 1. Types of error that appeared in the modelling task

Type	Error form	Stage	Frequency
Type-1	Making inappropriate assumptions	Simplifying	90%
Type-2	Failure in constructing a mathematical model that describes the real-world situation	Mathematizing	76%
Type-3	Mixing mathematical models with real-world problems	Mathematizing	19%
Type-4	Conducting an unreasonable process	Working mathematically	24%
Type-5	Mixing algebra operations with real-world problems	Working mathematically	28%
Type-6	Inadequate verification of results	Validating	57%

Table 1 shows that there are six identifiable forms of error from the modelling task, i.e. making inappropriate assumptions, failure to construct mathematical models appropriate to real-world problems, mixing mathematical models with real-world problems, mixing algebraic operations with real-world problems, performs unreasonable processes, and inadequate verification of results. These errors occur in simplifying/structuring activities, mathematizing, and validating. The absence of any other form of error is possible because the mathematics teacher does not experience any obstacles that cause errors in other forms.

From these results, two forms of error are making inappropriate assumptions (Type-1) in the simplifying activity and failure in constructing a mathematical model that describes the real-world situation (Type-2) in the mathematizing activity into errors that have the greatest percentage compared to other forms of error. Type-1 error occurs due to pre-service mathematics teachers not fully understand the problems encountered. There is some evidence from other studies that is reading texts and understanding situations, and problems are the cognitive impediment to learners [20,21]. In the matter of pool filling tasks, this is the most difficult part of the task. Conversation 1 illustrates an example of this difficulty.

Conversation 1

O : What do you understand from this matter?

P : (*Reading the text*). I suppose the large pump as x and the small pump as y . So, the first condition will be $2x + 1y = 4$. Then the second condition will be $1x + 3y = 4$. Then I do the elimination process to obtain the value of y first, Sir.

O : Did you get the value of y ?

P : Yes. $\frac{4}{5}$.

O : The value of x ?

P : $\frac{8}{5}$.

O : What does it mean by $y = \frac{4}{5}$?

P : The pump.

O : What do you mean? Is it the number of pump or time needed?

P : Eee (*thinking*) the time.

Most of them have realized that the problem they are facing is a matter of SLETV. They believe that they can do this task well and do the tasks as routine problems they often get. They feel they do not have to re-understand the problem situation to solve it. Next, they compile a mathematical model, make plans and work according to their original assumptions. At this time there was an understanding obstacle. Understanding obstacle occurs because they lacked variations in the types of problems and tendencies to use certain types of problems. In this case, the lack of variations in the type of problems in SLETV is distinguishing the inversely proportional and directly proportional problems, and the tendency to use directly proportional problems. Furthermore, it emerged the inappropriate assumption that "if it is a SLETV problem, then it corresponds to the directly proportional problem".

Furthermore, Type-2 error, we identify the existence of two forms of resistance. First, the result of making inappropriate assumptions causes them to construct the wrong mathematical model as their original understanding. On this modelling issue, they use SLETV comparative worth settlement, which should be a reversed ratio of value. As a result, they failed to make mathematical models that fit the real problem. This phenomenon is an obstacle that leads them to error. Second, they realize that this problem is not similar to the usual problem, but fails to make the corresponding relationship. They know that in Situation 3 it will take a shorter time to fill the pool compared to Situation 1 and Situation 2. However, many of them chose to remain using the old procedure for failing to construct appropriate mathematical models. They realized that their results were incorrect, though. They argued that the limitation of time to be the cause that they did not fix their answers. Some of them dared to explore to make an unusual model to gain an answer for Situation 3. This decision then drove him to develop correct mathematical modelling.

The lack of verification of the findings is also the cause of the error. This stage is important to re-check if the answer is correct and reasonable as it exists in the real world. Re-checking the answer is needed because when students are engaged in cognitive activities, they can understand the essence and content of knowledge gained, and they learn how to apply the knowledge to find solutions for the upcoming problem [22]. If the result of verification is reasonable, it will increase their confidence in the answer. However, if the verification obtained did not make sense, it would be a consideration for improvement. This consideration is in spite of the time limitation in the execution of tasks.

4. Conclusion

To conclude, the authors have predicted the types of errors in mathematical modelling and highlighted the two most common errors and obstacles that caused them. About the first error, making inappropriate assumptions, the pre-service mathematics teachers have been aware of the type of problem they are facing is SLETV problem and even believe that they can solve the problem well. They assume that the type of the problem is a type of routine problem they often get and leads to a loss of sensitivity to a new problem. Their belief in the misconception of previous experiences is the cause of this kind of error. This misconception is what we mean as the understanding obstacle.

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About the second error, failure in constructing a mathematical model that describes the real-world situation, as well as caused by making inappropriate assumptions, this error is also caused by the constructing obstacle. When experiencing these obstacles, mathematics pre-service teachers realize that the problem they face is a non-routine problem, but they fail to construct mathematical models that match in the real-world situation. To address both types of obstacles, we recommend to teachers providing sufficient scaffolding in understanding the problem, introducing non-routine issues in classroom learning, and emphasizing verification of answers obtained mathematics pre-service teachers.

5. Acknowledgement 13

We want to express our gratitude to all the pre-service mathematics teachers who participated in the research.

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