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Title and Abstract

Title INDONESIA MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS OF AREA AND PERIMETER OF RECTANGLE

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. Research has shown teachers' competencies and skills influence students' performances. Previous studies explored teachers' knowledge through testing. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items or tasks to assess mathematics teacher knowledge in the previous studies were dominated by subject matter knowledge problems. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' Knowledge of Content and Students (KCS) through lesson plans developed by the teachers. To accommodate the gap in the previous studies, this study focuses on KCS on the topic of area-perimeter through their designed lesson plans. Twenty-nine mathematics teachers attended a professional development activity voluntarily participated in this study. Two teachers were selected to be the focus of this case study. Content analysis of the lesson plan and semi-structured interviews were conducted, and then data were analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible strategies and mistakes, these teachers were poorly prepared to support student learning.

Indexing

Keywords Knowledge of Content and Students; Mathematics Teacher; Area and Perimeter; Teachers' Skills and Competencies

Language en

Supporting Agencies

Agencies Ministry of Education of Republic of Indonesia, Planning and Cooperation of Foreign Affairs, and SEAMEO QITEP in Mathematics

References

- References
- An, S., Kulm, G., & Wu, Z. (2004). The Pedagogical Content Knowledge of Middle School Mathematics Teachers in China and the U.S. *Journal of Mathematics Teacher Education*, 7, 145-172. <https://doi.org/10.1023/b:jmt.0000021943.35739.1c>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>
- Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, 31, 235-268. <https://doi.org/10.1007/BF00376322>
- Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project* (pp. 25-48). Boston: Springer. https://doi.org/10.1007/978-1-4614-5149-5_2
- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., ... Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 133-180. <https://doi.org/10.3102/0002831209345157>
- Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' Thinking About Instruction. *Remedial and Special Education*, 11(6), 40-49. <https://doi.org/10.1177/074193259001100609>
- Burns, R. B., & Lash, A. A. (1988). Nine Seventh-Grade Teachers' Knowledge and Planning of Problem-Solving Instruction. *The Elementary School Journal*, 88(4), 369-386. <https://doi.org/10.1086/461545>
- Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.
- Carle, S. M. (1993). Student held misconceptions regarding area and perimeter of rectangles. *Critical and Creative Thinking Capstones Collection*, 46. http://scholarworks.umb.edu/ccct_capstone/46
- Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136-143). Adelaide: Australian Association of Mathematics Teachers.
- Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. *Handbook of research on teaching*. New York: MacMillan
- Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from Lessons: Studying the Construction of Teacher Knowledge Catalysed by Purposefully-Designed Experimental Mathematics Lessons. *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165-172). Sunshine Coast: MERGA
- Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.
- Fauskanger, J. (2015). Challenges in measuring teachers' knowledge. *Educational Studies in Mathematics*, 90, 57-73. <https://doi.org/10.1007/s10649-015-9612-4>
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372-400. <https://www.jstor.org/stable/pdf/40539304>
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406. <https://doi.org/10.3102/00028312042002371>
- John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies*, 38(4), 483-498. <https://doi.org/10.1080/00220270500363620>
- Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience: 3rd Edition* (pp. 79-100). London: Routledge Taylor & Francis Group. <https://doi.org/10.4324/9780203844120>
- Korkmaz, H. I., & Tahin, D. (2019). Preservice Preschool Teachers' Pedagogical Content Knowledge on Geometric Shapes in Terms of Children's Mistakes. *Journal of Research in Childhood Education*, 34(3), 385-405. <https://doi.org/10.1080/02568543.2019.1701150>
- Kow, K., & Yeo, J. (2008). Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*.
- Kristanto, Y. D., Panuluh, A. H., & Atmajati, E. D. (2020). Development and validation of a test instrument to measure pre-service mathematics teachers' content knowledge and pedagogical content knowledge. *Journal of Physics: Conference Series*, 1470(1), 012008. <https://doi.org/10.1088/1742-6596/1470/1/012008>
- Lee, Y., Capraro, R. M., & Capraro, M. M. (2018). Mathematics Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge in Problem Posing. *International Electronic Journal of Mathematics Education*, 13(2), 75-90. <https://doi.org/10.12973/iejme/2698>



KEYWORDS

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PMRI Realistic
Mathematics Education
Subtraction design research
learning trajectory mathematical proof
multiplication

- Nakahara, T., & Koyama, M. (2000). Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME) 24th, Hiroshima, Japan, July 23-27, 2000, Volume 1.
- Özerem, A. (2012). Misconceptions In Geometry And Suggested Solutions For Seventh Grade Students. *Procedia - Social and Behavioral Sciences*, 55, 720-729. <https://doi.org/10.1016/j.sbspro.2012.09.557>
- Superfine, A. M. C. (2008). Planning for Mathematics Instruction: A Model of Experienced Teachers' Planning Processes in the Context of a Reform Mathematics Curriculum. *The Mathematics Educator*, 18(2), 11-22. <https://ojs01.galib.uga.edu/tme/article/view/1925/1830>
- Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical Content Knowledge of Mathematics Pre-service Teachers: Do they know their students? *Journal of Physics: Conference Series*, 1097(1), 012098. <https://doi.org/10.1088/1742-6596/1097/1/012098>
- Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*, 15(2), 4-14. <https://doi.org/10.3102/0013189X015002004>
- Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education*, 26(2), 114-145. <https://doi.org/10.2307/749205>
- Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., ... Rowley, G. (2012). Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). Amsterdam: International Association for the Evaluation of Educational Achievement.
- Turnuklu, E., & Yesildere, S. (2007). The Pedagogical Content Knowledge in Mathematics: Pre-Service Primary Mathematics Teachers' Perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, 1, 1-13.
- Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating Preservice Mathematics Teachers' Pedagogical Content Knowledge through Microteaching. *REDIMAT-Journal of Research in Mathematics Education*, 9(1), 62-87. <http://dx.doi.org/10.17583/redimat.2020.3353>
- Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.
- White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2012). Teachers learning from teachers. In *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13
- Widodo, & Taminudin, M. (2014). Three Training Strategies for Improving Mathematics Teacher Competences in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Yogyakarta: Mathematics and Technology, LLC. Retrieved from <http://atcm.mathandtech.org/EP2014/index.html>
- Yeo, K. K. Y. (2008). Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action. Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia (pp. 621-627). Brisbane: The University of Queensland. Retrieved from https://repository.nie.edu.sg/bitstream/10497/14397/1/MERGA-2008-621-YeoKK_a.pdf
- Yin, R. K. (2014). *Case study research: Design and methods*. Thousand Oaks: SAGE Publications.
- Yunianto, W. (2015). Supporting Students' Understanding of Area Measurement Through Verknippen Applet. *Southeast Asian Mathematics Education Journal*, 5(1), 73-82. <https://doi.org/10.46517/seamej.v5i1.34>
- Zachros, K., & Chassaplis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, 6(2), 41-62. <https://doi.org/10.26220/rev.1627>

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Blind Review Artikel yang di submit pada tanggal 17 Januari 2021 dengan judul awal
“Indonesian Mathematics Teachers’ Knowledge of Content
and Students: Predicting and Responding to Students’
Responses to the Topic of Area and Perimeter”



INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS: PREDICTING AND RESPONDING TO STUDENTS' RESPONSES TO THE TOPIC OF AREA AND PERIMETER

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. However, to some extent of teacher assessment has not yet completely covered the full range of teacher skills and competencies. This study investigates focuses on Knowledge of Content and Students (KCS) on the topic of area-perimeter through their designed lesson plans. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items to assess mathematics teacher knowledge are dominated by subject matter knowledge. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' KCS through lesson plans developed by the teachers. Mathematics teachers attended a professional development activity and voluntarily participated in this study. Content analysis of the lesson plan and semi-structured interviews were conducted, and the data analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible methods and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Namun, dalam beberapa hal, penilaian guru belum sepenuhnya mencakup seluruh keterampilan dan kompetensi guru. Fokus penelitian ini adalah menyelidiki *Knowledge of Content and Students* (KCS) pada topik luas dan keliling melalui rancangan rencana pembelajaran mereka. Pengetahuan guru tentang topik luas dan keliling dan strategi pengajaran telah dinilai melalui pengujian. Secara umum, materi untuk menilai pengetahuan guru matematika didominasi oleh materi pengetahuan. Dengan demikian, penilaian tersebut tampaknya belum sepenuhnya mencakup seluruh pengetahuan dan kompetensi guru. Dalam penelitian ini, peneliti menginvestigasi KCS guru matematika melalui RPP yang dikembangkan oleh guru. Guru matematika mengikuti kegiatan pengembangan profesional dan secara sukarela berpartisipasi dalam penelitian ini. Analisis isi RPP dan wawancara semi-terstruktur dilakukan, dan data dianalisis. Hasil penelitian ini mengungkapkan bahwa guru yang berpartisipasi ditantang ketika membuat prediksi kemungkinan tanggapan siswa. Mereka tampaknya tidak menyadari strategi siswa biasa, yang digunakan untuk menyelesaikan memaksimalkan luas dari keliling tertentu. Dengan pengetahuan yang terbatas tentang kemungkinan metode dan kesalahan siswa, para guru ini kurang siap untuk mendukung pembelajaran siswa.

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

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Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research has shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related

to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2013, p.394).

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

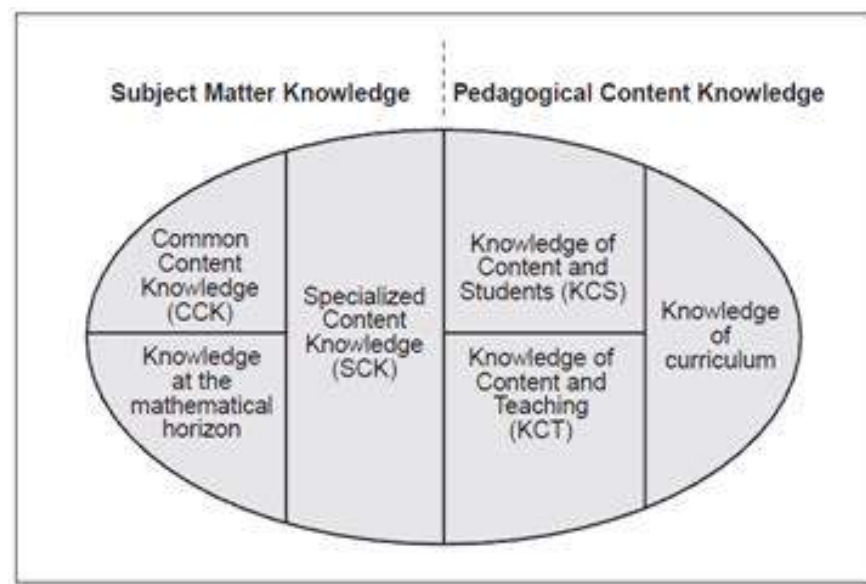


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). Pre-service teachers have challenges with student thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COACTIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin H,

2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Lesson planning is considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers who read it can easily understand it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called ‘Bansho’, it anticipates student mathematical thinking and student thinking schema for solving given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed the teacher, in planning a lesson, gave attention to students’ engagement (Clarke, Clarke, Roche, & Chan, 2015). Student engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom behaviour is influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

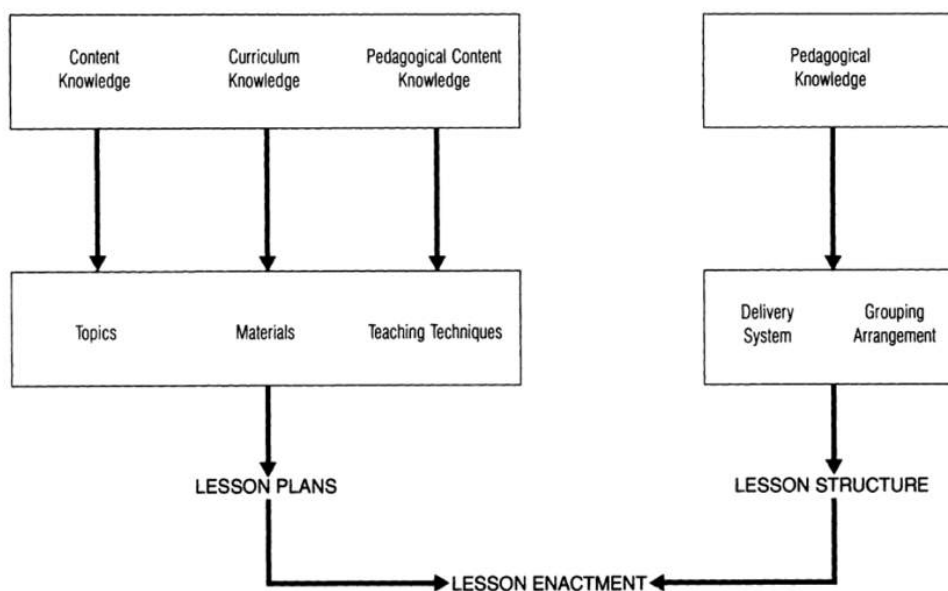


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zazahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013).

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason, (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Yeo, 2008).

This study intends to focus on a part of PCK pedagogical content knowledge, the KCS within lesson plans on the topic of area-perimeter. It is necessary to obtain a fuller insight into teacher knowledge. How mathematics teachers in Yogyakarta prepare their lesson plans and how is PCK integrated in their lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved human and had been through research ethics approval by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk). This study administrated a case study approach. This approach suits this study as it doesn't seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). Through this approach, the researchers examined two selected lesson plans of the mathematics teachers. The sample was chosen from twenty-nine teachers who attended a Professional Development (PD) session, and two teachers were selected for the lesson plan analysis and interview. The interview scenario was a semi-structured interview, and

the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan. The topic that would be taught was area and perimeter for grade 7. The “Gold Rush/Mining” task was selected. This task has several ways to be solved (see

Figure 3).

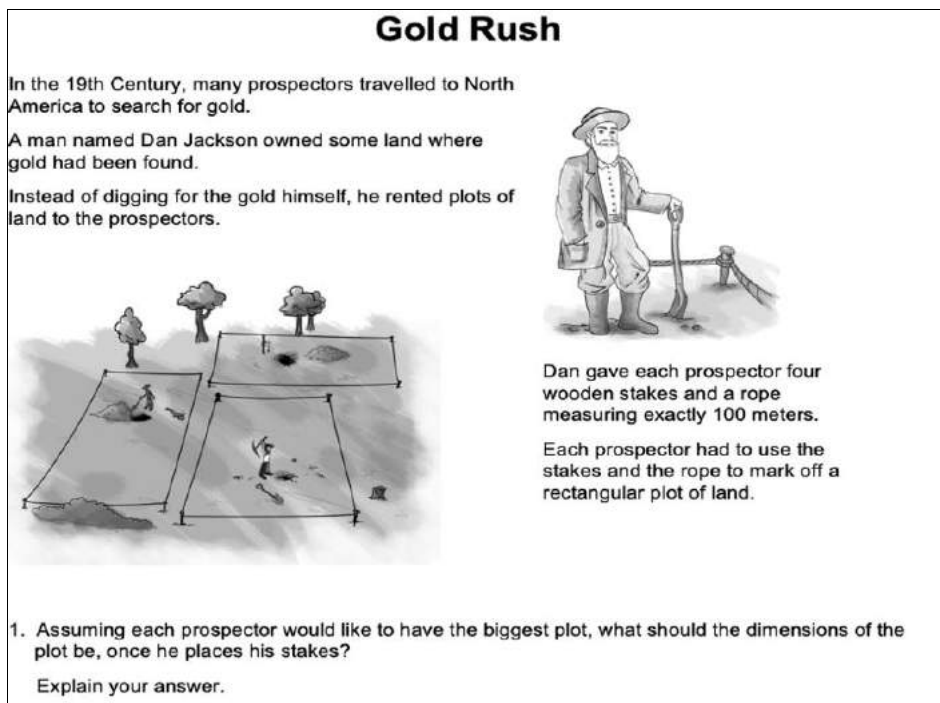


Figure 3. The Gold Rush problem (<https://www.map.mathshell.org/download.php?fileid=1637>)

To analyze the lesson plans, the researchers used content analysis. This method has the ‘potential to disclose many hidden aspects of what is being communicated through the written text’ (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers’ knowledge of students’ conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find the themes. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers included information about what students would do to the task. The data were presented descriptively.

Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing

2. The ability to predict what students will find interesting and motivating when choosing a task
3. The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4. The ability to hear and interpret students' emerging and incomplete thinking

The two lesson plans were coded and analyzed. There were three types of instructions to refer to the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?"	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3).

Table 3. Proportions of the instructions

Instruction	T1	T2
GI	8 (35%)	6 (31.6%)

SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	23 (100%)	19 (100%)

Indonesian teachers follow the prescribed template of a lesson plan. The template consists of three main parts namely; introduction, main and closure. Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2’s SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	Introduction		Main		Closure	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	12	3	7	11	4	5

T1 put more details of what students would ask to her on her lesson plan. For instance: ‘Can I solve it freely?’ has been put on her lesson plan. In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4).

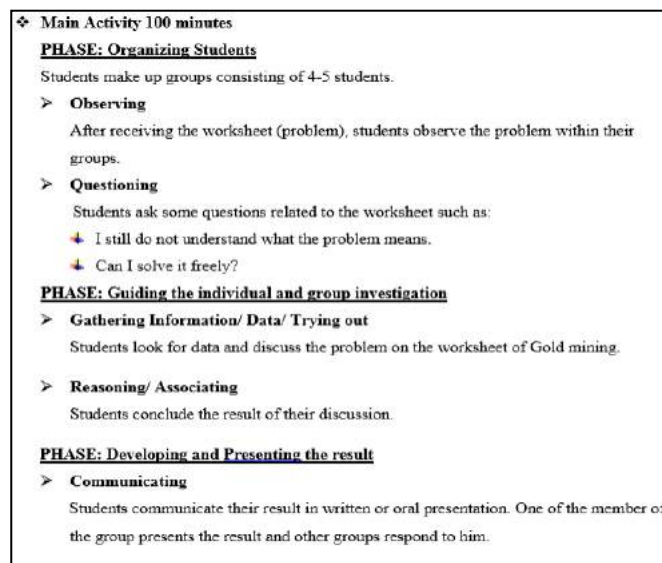


Figure 4. Teacher 1 Lesson Plan

The T2 lesson plan depicted detailed information about a possible student strategy. Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that

table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

<p>Main Activity</p> <ul style="list-style-type: none"> <input checked="" type="checkbox"/> Teacher divide students into groups <input checked="" type="checkbox"/> Teacher delivers the worksheet to be discussed <input checked="" type="checkbox"/> Teacher facilitates the learning processes ○ For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m. for instance $p=10, l=\dots m$ then the area = ... $p = 15 m, l = \dots m$, then the area = Students determine the largest area by themselves ○ for the second question, after students have solved the largest area for one miner, then how if it is for 2 miners? Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area? How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan

After finding the largest area, students had to find the largest area by joining two miners' ropes. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

<ul style="list-style-type: none"> ○ Students evaluate and make generalisation into questioning. <ul style="list-style-type: none"> <input checked="" type="checkbox"/> Teacher asks students to present in front of the class <input checked="" type="checkbox"/> Other students respond the presenter
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Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Teachers T1 and T2 have more than five-years teaching experience each. However, their schools are different in terms of location and students. These teachers themselves employed different abilities in solving the Gold Mining problem (

Figure 3). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : *Are there other ways T1?*

T1 : *Yesterday, I just did that one.*

T1 : *...just let students find the ways to solve it Then, I will let them know that there are some ways to solve it. I give that opportunity to students*

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

T2 : *To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*

R : *So, you (T2), induce them by using the table?*

T2 : *Yes, by the table.*

R : *What do you think, how many ways to solve it?*

T2 : *To me, I did one way I know it directly it would be a square. **I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and..., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

T2 : *I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*

T2 : *However, I have thought only one strategy, which is global to solve a task. ... I, I... know at least I understand my students' characteristic so that it will be difficult for my students. ... It is not possible to come up if I let them to do it freely. ... I am so careful to give it the various strategies because students would get confuse*

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

T2 : *By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.*

T2 : *To me, I did one way I know it directly it would be a square. I knew it already*

T2 : *...instead of table, we can make the variable x , then I will be a quadratic function,*

R : *Are there other ways to solve it?*

T2 : *For the time being, not yet, making rectangles and to the square*

- R : *Do you think there are still other ways to solve that problem?*
 T2 : *I could use the graph ...*

To some extent, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task. However, by giving the students the strategy, he inadvertently is making the student dependent on him. Whereas, T1 is helping the students to make decisions themselves. From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

- T1 : *The task has missing information, it should be more, and some students would think that. So that they **have not thought** yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
 T1 : *100. Maybe **they thought that** that's the only think they know.*
 R : *... So, they would answer it 100, possibly*
 T1 : *Yeah, possibly*
 T2 : *... for those who did not understand, **they would not know what 100 m rope is** to with the perimeter. So that the concept of perimeter, for those who understood, they already make it but later **they would not think** the rectangles can be varied.*
 T2 : *Students **would confuse** the meaning of maximum, which is the largest, **they have not thought about it**. So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7).

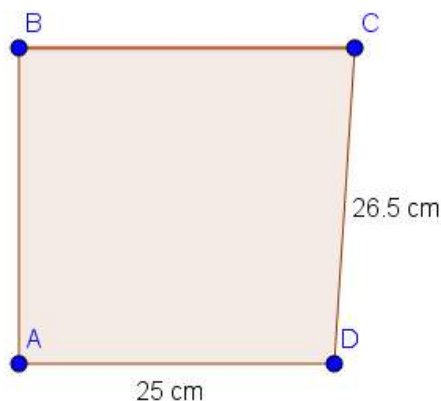


Figure 7. A student's possible mistake proposed by the researcher

If faced with a student mistake that they not have thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

- R : *If it happens if you see this (showing)*

- T1 : I would ask students back to try it then you calculate it as what being asked to you*
- R : They have not yet known the result!*
- T1 : Try, try it, by trialing they would know that it is different, this one is more, and that one is like that,*
- R : T2, what if your students did this? what would you do?*
- T2 : I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...*

In this study, the lesson plans facilitated an insight into teachers' knowledge. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Yeo, 2008).

Teachers have different ways of supporting students to solve tasks (Yei, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task. However, these results are not generalizable. The sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter topic, and that this knowledge was not

necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared. Developing higher order thinking and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment.

ACKNOWLEDGMENTS

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REFERENCES

- An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, 7, 145-172. <https://doi.org/10.1023/b:jmte.0000021943.35739.1c>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>
- Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, 31, 235-268. <https://doi.org/10.1007/BF00376322>
- Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project* (pp. 25-48). https://doi.org/10.1007/978-1-4614-5149-5_2
- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., ... Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 133-180. <https://doi.org/10.3102/0002831209345157>
- Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' thinking about instruction. *Remedial and Special Education*, 11(6), 40-49. <https://doi.org/10.1177/074193259001100609>
- Burns, R. B., & Lash, A. A. (1988). Nine seventh-grade teachers' knowledge and planning of problem-solving instruction. *The Elementary School Journal*, 8(4), 369-386. <https://doi.org/10.1086/461545>
- Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.
- Carle, S. M. (1993). *Student held misconceptions regarding area and perimeter of rectangles* (University of Massachusetts Boston). Retrieved from https://scholarworks.umb.edu/cgi/viewcontent.cgi?article=1045&context=cct_capstone
- Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves,

- & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136–143). Adelaide: Australian Association of Mathematics Teachers.
- Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching, 3rd Edition* (pp. 255-296). New York: Macmillan.
- Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from lessons: Studying the construction of teacher knowledge catalysed by purposefully-designed experimental mathematics lessons. In M. Marshman, V. Geiger, & A Bennison (Eds.), *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165–172). Sunshine Coast: MEGA.
- Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406. <https://doi.org/10.3102/00028312042002371>
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39, 372-400. Retrieved from <https://www.jstor.org/stable/40539304>
- John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies*, 38(4), 483-498. <https://doi.org/10.1080/00220270500363620>
- Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In S. Johnston-Wilder, C. Lee, & D. Pimm (Eds.), *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience, 3rd Edition* (pp. 79-100). London: Routledge. <https://doi.org/10.4324/9780203844120>
- Korkmaz, H. I., & Şahin, Ö. (2019). Preservice preschool teachers' pedagogical content knowledge on geometric shapes in terms of children's mistakes. *Journal of Research in Childhood Education*, 34(3), 385-405. <https://doi.org/10.1080/02568543.2019.1701150>
- Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1*. Retrieved from <https://files.eric.ed.gov/fulltext/ED452031.pdf>
- Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *Procedia - Social and Behavioral Sciences*, 55, 720-729. <https://doi.org/10.1016/j.sbspro.2012.09.557>
- Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical content knowledge of mathematics pre-service teachers: Do they know their students? *Journal of Physics: Conference Series*, 1097(1), 012098. <https://doi.org/10.1088/1742-6596/1097/1/012098>
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 2(15), 4-14. <https://doi.org/10.3102/0013189X015002004>
- Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26(2), 114-145. <https://doi.org/10.2307/749205>
- Superfine, A. C. (2008). Planning for mathematics instruction: A model of experienced teachers' planning processes in the context of a reform mathematics curriculum. *Mathematics Educator*, 18(2), 11-22. Retrieved from <http://tme.journals.libs.uga.edu/index.php/tme/article/view/198>

- Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., ... Rowley, G. (2012). Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). In *International Association for the Evaluation of Educational Achievement*. Retrieved from <https://files.eric.ed.gov/fulltext/ED542380.pdf>
- Turnuklu, E., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers, 1*, 1-13. Retrieved from <http://www.k-12prep.math.ttu.edu/journal/1.contentknowledge/yesildere01/article.pdf>
- Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating preservice mathematics teachers' pedagogical content knowledge through microteaching. *REDIMAT-Journal of Research in Mathematics Education*, 9(1), 62-87. <https://doi.org/http://dx.doi.org/10.17583/redimat.2020.3353>
- Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.
- White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2013). Teachers learning from teachers. In M.A. Clements, A.J. Bishop, C. Keitel, J. Kilpatrick, & F.K.S. Leung (Eds.), *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13
- Widodo, & Tamimudin, H. M. (2014). Three training strategies for improving mathematics teacher competence in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Retrieved from <http://atcm.mathandtech.org/EP2014/index.html>
- Yeo, K. K. Y. (2008). Teaching area and perimeter : Mathematics-pedagogical-content knowledge-in-action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*. Retrieved from <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.471.9965&rep=rep1&type=pdf>
- Yin, R. K. (2014). *Case Study Research: Design and Methods* (5th ed.). Thousand Oaks: SAGE Publications.
- Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, 6(2), 41-62. <https://doi.org/10.26220/rev.1627>

[JME] Editor Decision Inbox

Jme Fkip Matematika <jme@unsri.ac.id>
to Wahid, me, Cosette

Mon, Mar 8, 8:45 PM

Dear Wahid Yudianto, Rully Charitas Indra Prahmana, and Cosette Crisan,

It is my great pleasure to inform you that your paper entitled "Indonesian Mathematics Teachers' Knowledge of Content and Students: Predicting and Responding to Students' Responses to the Topic of Area and Perimeter" has been Accepted with Major Revision and will be published in the Journal on Mathematics Education (JME). Your paper will be published for forthcoming issues after suitable revision and fulfil the JME's standard.

Authors are encouraged to carefully consider the reviewers' comments and suggestions for improvement of your manuscript, such as:

Reviewer A:
The research question seems to address how teachers prepare lessons, while the question is whether lesson plans can be used to evaluate teachers' PCK. Improving the question is a minor detail. I have a bit more concerns with the method and the scope of the conclusions. The procedure of performing the task for the teachers is not clear to me. To what extent were these two teachers aware of the purpose of the task and how was that related to the PD course they were attending? This is relevant for interpreting the results. E.g. I missed the teachers' reflections on intentions for the lesson plan (learning about area? Eliciting misconceptions? Learning about problem solving? Etc.). Since the study is a case study with two teachers, the risk is to generalize too quickly without understanding the context. This context and a detailed account of the method are needed to describe limitations of the study and possible generalizations or theoretical reflections. Detailed information about my concerns can be found in the document.

Reviewer B:
The research topic is intriguing and important in mathematics education. Examining and developing math teachers' knowledge in general and KCS in specific is challenging. The current research attempted to address this. I must say that the attempt is already good but still insufficient to comprehensively address the teachers' KCS. I found some issues:
Firstly, the abstract and introduction need a rework if this manuscript will be accepted. An essential missing part of the introduction is what did the study offer compared to the prior relevant studies. Furthermore, they have not been logically and sequentially arranged.
Secondly, the first issue is a technical one; it can be revised. However, the second issue is principle one- the research procedures were not able to fully reveal the teachers' KCS. This review is based on my knowledge on the topic. Others might have different standpoints.
Here is my argumentation for the second point:

After reading the introduction with its limitations, I was wondering how the indicators of KCS were examined in this study. So, I decided to jump into Table 1 (The indicators of KCS). Well, that is great; four indicators. Then, the question popped up to my mind was how the research procedures facilitate the intended indicators.

Initially, the researchers asked the teachers to devise a lesson plan with the Gold Rush task. The teachers did it and two selected lesson plans (it has not been explained why they chose these two) were examined to reveal teachers' KCS (Table 1). I was thrilled to read the findings; how the content analysis revealed the teachers' KCS. I tried carefully and repeatedly reading the findings to make sure that I did not miss the details. What I found is that the findings do not comprehensively represent the indicators of KCS. The findings of the content analysis on the lessons plan are:
1. Types of instructions (Table 2-3)
2. The delivery of the lessons (Figure 4-6 along with its interpretations and interview scripts)
In my opinion which refer to the indicators of KCS, those findings are not KCS. Finding 2 tends to be teachers' pedagogical knowledge instead of KCS

Scrolling to the next pages of Findings, all of a sudden, I found that the researchers provided a possible students' answer on the task. This procedure has not been explained in the method. Technically, this can be revised. However, the data (interview transcripts) were yet able to reveal the teachers' KCS.

In short, KCS, which is clearly shown in the indicators, is about teachers' mathematical content knowledge and how they utilize this knowledge to facilitate students' understanding of the task or the construction of mathematical knowledge in general. In this case, a typical procedure of research on this topic is
1. How the teachers' knowledge to solve the task
2. Teachers' solution is confronted with various possible students' answers on the task
3. Using point 2, the teachers plan to facilitate the students' understanding on the tasks
In my perspective, the current research missed some of the points.

To conclude, the procedures of the current research were not able to fully reveal the teachers' KCS.
Referring to point 2, I am a bit sceptic whether the manuscript meets the expectation or standard of the journal or not. However, in my personal view, the research need to be extended to address the topic. That is to say, as a reviewer, I decline the manuscript.

Reviewer C:
The manuscript covers an interesting topic. However, there are many changes to be made to improve the readability and cohesiveness, especially for the excerpts of interview transcripts. In relation, the author should place a more in- depth discussion with support from the literature.

I want to suggest that the editor and author make sure the article's English writing level is at least par before sending it to any reviewers. By saying this, I recommend the author to use a proofreader service to get a better quality manuscript.

Reviewer D:
This paper is worthy to be published. However, several additional information is needed and the representation of the data analysis need more effort to make it more comprehensive relate those to the theory about knowledge

Reviewer E:
Ask English expert to consult the paper. Please see the reviewer's comments on the manuscript.

Meet

- New meeting
- My meetings

Hangouts

Rully +

No recent chats
Start a new one

- Inbox 406
- Starred
- Snoozed
- Sent
- Drafts
- Academia dan Res... 19
- IConProCS 204
- Notes
- Publons 2
- Rejected Email
- Research Gate 38
- Sejarah Matematik... 24
- More

Meet

- New meeting
- My meetings

Hangouts

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- Rejected Email
- Research Gate 38
- Sejarah Matematik... 24
- More

Meet

- New meeting
- My meetings

Hangouts

Rully +

No recent chats
Start a new one

Reviewer F:

I suggest accepting the article with minor changes. However, before accepting the editor should look at the following remark regarding plagiarism.

Many parts of the article are identical to a master thesis by Wahid Yuniarto. On "Research Gate," this thesis has the remarkable title "Dissertation Cover Sheet PART 1-TO BE COMPLETED BY THE STUDENT Student Name."

This article refers both to one author (researcher) and more authors (researchers). If there are more authors a reference to this thesis is needed at all sentences that come from this thesis. If Wahid was the only author, there is the question of self-plagiarism. What is the policy of the journal regarding self-plagiarism? I would think that a remark that this article is an adaptation/modification of that thesis is needed.

If Wahid is not one of the authors, then there is a serious issue with plagiarism and the article should be refused.

I would suggest the author (authors) to use a good spelling and grammar checker to have a better look at the text. It will improve the readability of the article. They could try Grammarly and trinka (or both).

Furthermore, the author must make sure all references have DOI and follow this guideline to ensure that your final file is complete including the blind references from authors and in the correct format (<https://bit.ly/33GvXT3>) for preparing their paper strictly. You can follow the paper that was already published in JME. The fluent, comprehensible, and correct use of English is the main criterion in publishing proven by proofread certificate from a reputable proof reader (attached in revision submission process). Lastly, the recapitulation of the contents of the revised article and similarity check result file must be attached as a supplementary file in the revision submission process.

To support the cost of wide-open access dissemination of research results, to manage the various costs associated with handling and editing of the submitted manuscripts, and the Journal management and publication in general, the authors or the author's institution is requested to pay the Article Processing Charge (APC).

- * Publication fee : USD 250
- * Layout & Editing fee : USD 35

* Total : USD 285

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Bank Account name (please be exact): Dr. Zulkardi, M.Ikom., M.Sc
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Please submit the final revised paper along with your payment receipt, Copyright Transfer Agreement, the recapitulation of the contents of the revised article, similarity check result file, and proofread certificate from a reputable proof reader to email: jme@unsri.ac.id or via your JME OJS Account as an Author Version. The revised manuscript should be submitted by March 17, 2021; if you anticipate that you will be unable to meet this deadline, please notify the Editorial Office. Should you have any questions, please do not hesitate to contact us.

Thank you very much for your cooperation. I do really appreciate it.

Kind regards,

Prof. Dr. Zulkardi, M.Ikom., M.Sc.
 Editor in Chief
 Journal on Mathematics Education

IMPORTANT:

"Please revise and give a comment in the attached file and it is not permitted to send a new article file, other than the revised results of the attached file"

7 Attachments

The attachments include: 13537-35584-1-RV..., 13537-35933-1-RV..., 13537-36158-1-RV..., 13537-35552-1-RV..., Recapitulation the..., Author_Guidelines..., and JME_Copyright Tra...

Received, thank you. Noted with thanks. Thank you for the feedback.

Reply Reply all Forward

Hasil review dari 6 reviewer dengan 4 diantaranya memberikan catatan pada artikel nya secara langsung, yaitu Reviewer A, B, D, dan E

[Paper ID: 13537]



INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS: PREDICTING AND RESPONDING TO STUDENTS' RESPONSES TO THE TOPIC OF AREA AND PERIMETER

Abstract

Measuring teachers' skills and competencies is necessary to ensure teachers' quality and contribute to education quality. However, to some extent of teacher assessment has not yet completely covered the full range of teacher skills and competencies. This study investigates focuses on Knowledge of Content and Students (KCS) on the topic of area-perimeter through their designed lesson plans. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items to assess mathematics teacher knowledge are dominated by subject matter knowledge. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' KCS through lesson plans developed by the teachers. Mathematics teachers attended a professional development activity and voluntarily participated in this study. Content analysis of the lesson plan and semi-structured interviews were conducted, and the data analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible methods and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Namun, dalam beberapa hal, penilaian guru belum sepenuhnya mencakup seluruh keterampilan dan kompetensi guru. Fokus penelitian ini adalah menyelidiki *Knowledge of Content and Students* (KCS) pada topik luas dan keliling melalui rancangan rencana pembelajaran mereka. Pengetahuan guru tentang topik luas dan keliling dan strategi pengajaran telah dinilai melalui pengujian. Secara umum, materi untuk menilai pengetahuan guru matematika didominasi oleh materi pengetahuan. Dengan demikian, penilaian tersebut tampaknya belum sepenuhnya mencakup seluruh pengetahuan dan kompetensi guru. Dalam penelitian ini, peneliti menginvestigasi KCS guru matematika melalui RPP yang dikembangkan oleh guru. Guru matematika mengikuti kegiatan pengembangan profesional dan secara sukarela berpartisipasi dalam penelitian ini. Analisis isi RPP dan wawancara semi-terstruktur dilakukan, dan data dianalisis. Hasil penelitian ini mengungkapkan bahwa guru yang berpartisipasi ditantang ketika membuat prediksi kemungkinan tanggapan siswa. Mereka tampaknya tidak menyadari strategi siswa biasa, yang digunakan untuk menyelesaikan memaksimalkan luas dari keliling tertentu. Dengan pengetahuan yang terbatas tentang kemungkinan metode dan kesalahan siswa, para guru ini kurang siap untuk mendukung pembelajaran siswa.

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

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Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research have shown that students' achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of

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The research examined (in-services) mathematics' teachers KCS, which is part of PCK using their lessons plan. The rationale for this research is that large assessments have not focused on KCS. To this point, it is great. However, I was wondering, within the prior studies, what this study offers? If it is only to understand teachers' knowledge on content and students, some prior studies have addressed this. They even offer complete pictures of the topic (e.g., Hill, Ball, & Schilling, 2008) since the studies involved planning and real actions in the classroom.

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instruction relates to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2013, p.394).

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

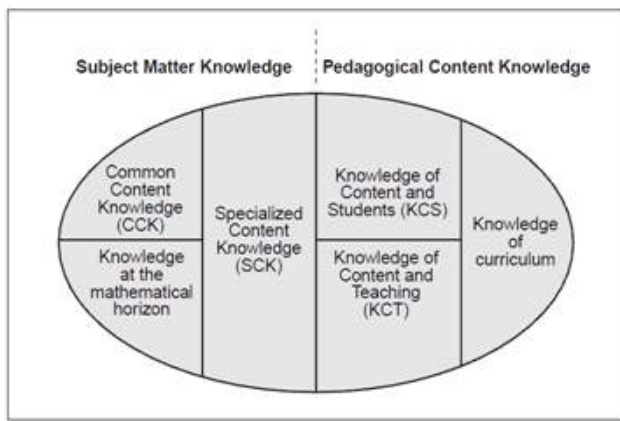


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). Pre-service teachers have challenges with student thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COCATIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin H,

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2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Lesson plans are considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instructions so that teachers can easily understand when reading it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned, called 'Bansho', which anticipates student mathematical thinking and student thinking schema for solving the given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed that the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). The students' engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom behaviour is influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions. Therefore, arguably it is possible to look at the teachers' lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

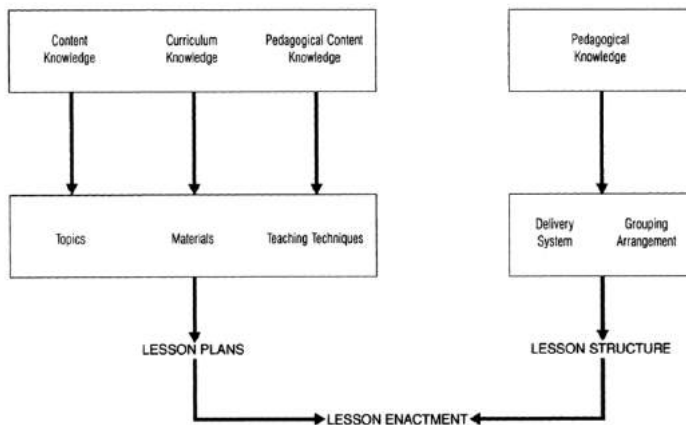


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

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Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zawahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013).

It has been shown that SMK and PCK of mathematics teachers influenced students' performance (Baumert et al., 2010). Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Yeo, 2008).

This study intends to focus on a part of PCK, the KCS within lesson plans on the topic of area-perimeter. It is necessary to obtain a fuller insight into teacher knowledge. How mathematics teachers in Yogyakarta prepare their lesson plans and how is PCK integrated in their lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved human and had been through research ethics approval by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk). This study administrated a case study approach. This approach suits this study as it doesn't seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). Through this approach, the researchers examined two selected lesson plans of the mathematics teachers. The sample was chosen from twenty-nine teachers who attended a Professional Development (PD) session, and two teachers were selected

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Initially, the researchers asked the teachers to devise a lesson plan with the Gold Rush task. The teachers did it and two selected lessons plan (it has not been explicated why chose these two) were examined to reveal teachers' KCS (Table 1). I was thrilled to read the findings; how the content analysis revealed the teachers' KCS. I tried carefully and repeatedly reading the findings to make sure that I did not miss the details. What I found is that the findings do not comprehensively represent the indicators of KCS. The findings of the content analysis on the lessons plan are;

1. Types of instructions (Table 2-3)
2. The delivery of the lessons (Figure 4-6 along with its interpretations and interview scripts)

In my opinion which refer to the indicators of KCS, those findings are not KCS. Finding 2 tends to be teachers' pedagogical knowledge instead of KCS

Scrolling to the next pages of Findings, all of a sudden, I found that the researchers provided a possible students' answer on the task. This procedure has not been explained in the method. Technically, this can be revised. However, the data (interview transcripts) were yet able to reveal the teachers' KCS.

In short, KCS, which is clearly shown in the indicators, is about teachers' mathematical content knowledge and how they utilize this knowledge to facilitate students' understanding of the task or the construction of mathematical knowledge in general. In this case, a typical procedure of research on this topic is

1. How the teachers' knowledge to solve the task
2. Teachers' solution is confronted with various possible students' answers on the task
3. Using point 2, the teachers plan to facilitate the students' understanding on the tasks

In my perspective, the current research missed some of the points.


To conclude, the procedures of the current research were not able to fully reveal the teachers' KCS.

for the lesson plan analysis and interview. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan. The topic that would be taught was area and perimeter for grade 7. The “Gold Rush/Mining” task was selected. This task has several ways to be solved (see

Figure 3).

Gold Rush

In the 19th Century, many prospectors travelled to North America to search for gold.
 A man named Dan Jackson owned some land where gold had been found.
 Instead of digging for the gold himself, he rented plots of land to the prospectors.



Dan gave each prospector four wooden stakes and a rope measuring exactly 100 meters.
 Each prospector had to use the stakes and the rope to mark off a rectangular plot of land.

1. Assuming each prospector would like to have the biggest plot, what should the dimensions of the plot be, once he places his stakes?
 Explain your answer.

Figure 3. The Gold Rush problem (<https://www.map.mathshell.org/download.php?fileid=1637>)

To analyze the lesson plans, the researchers used content analysis. This method has the ‘potential to disclose many hidden aspects of what is being communicated through the written text’ (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers’ knowledge of students’ conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find the themes. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers included information about what students would do to the task. The data were presented descriptively.

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Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students' emerging and incomplete thinking

The two lesson plans were coded and analyzed. There were three types of instructions to refer to the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?"	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3).

Table 3. Proportions of the instructions

Instruction	T1	T2
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	23 (100%)	19 (100%)

Indonesian teachers follow the prescribed template of a lesson plan. The template consists of three main parts namely; introduction, main and closure. Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2's SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	Introduction		Main		Closure	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	12	3	7	11	4	5

T1 put more details of what students would ask to her on her lesson plan. For instance: 'Can I solve it freely?' has been put on her lesson plan. In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4).

<p>Main Activity 100 minutes</p> <p>PHASE: Organizing Students</p> <p>Students make up groups consisting of 4-5 students.</p> <p>> Observing</p> <p>After receiving the worksheet (problem), students observe the problem within their groups.</p> <p>> Questioning</p> <p>Students ask some questions related to the worksheet such as:</p> <ul style="list-style-type: none"> ⬇ I still do not understand what the problem means. ⬇ Can I solve it freely? <p>PHASE: Guiding the individual and group investigation</p> <p>> Gathering Information/ Data/ Trying out</p> <p>Students look for data and discuss the problem on the worksheet of Gold mining.</p> <p>> Reasoning/ Associating</p> <p>Students conclude the result of their discussion.</p> <p>PHASE: Developing and Presenting the result</p> <p>> Communicating</p> <p>Students communicate their result in written or oral presentation. One of the member of the group presents the result and other groups respond to him.</p>

Figure 4. Teacher 1 Lesson Plan

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The T2 lesson plan depicted detailed information about a possible student strategy. [Figure 5](#) shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2’s instruction can be understood as providing a method to solve the task, with much support given to students.

Main Activity

- Teacher divide students into groups
- Teacher delivers the worksheet to be discussed
- Teacher facilitates the learning processes
- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
for instance $p=10, l=... m$ then the area = ...
 $p = 15 m, l = ...m$, then the area =
Students determine the largest area by themselves
- for the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan

After finding the largest area, students had to find the largest area by joining two miners’ ropes. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in [Figure 6](#).

- Students evaluate and make generalisation into questioning.
 - Teacher asks students to present in front of the class
 - Other students respond the presenter

Figure 6. T2’s lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that ‘the largest area would always be a square’. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

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Teachers T1 and T2 have more than five-years teaching experience each. However, their schools are different in terms of location and students. These teachers themselves employed different abilities in solving the Gold Mining problem (

Figure 3). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : *Are there other ways T1?*

T1 : *Yesterday, I just did that one.*

T1 : *...just let students find the ways to solve it Then, I will let them know that there are some ways to solve it. I give that opportunity to students*

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

T2 : *To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*

R : *So, you (T2), induce them by using the table?*

T2 : *Yes, by the table.*

R : *What do you think, how many ways to solve it?*

T2 : *To me, I did one way I know it directly it would be a square. **I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and..., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

T2 : *I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*

T2 : *However, I have thought only one strategy, which is global to solve a task. ... I, I... know at least I understand my students' characteristic so that it will be difficult for my students. ... It is not possible to come up if I let them to do it freely. ... I am so careful to give it the various strategies because students would get confuse*

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

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- T2 : *By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.*
 T2 : *To me, I did one way I know it directly it would be a square. I knew it already*
 T2 : *...instead of table, we can make the variable x, then I will be a quadratic function,*
 R : *Are there other ways to solve it?*
 T2 : *For the time being, not yet, making rectangles and to the square*
 R : *Do you think there are still other ways to solve that problem?*
 T2 : *I could use the graph ...*

To some extent, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task. However, by giving the students the strategy, he inadvertently is making the student dependent on him. Whereas, T1 is helping the students to make decisions themselves. From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

- T1 : *The task has missing information, it should be more, and some students would think that. So that they **have not thought** yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
 T1 : *100. Maybe **they thought that** that's the only think they know.*
 R : *... So, they would answer it 100, possibly*
 T1 : *Yeah, possibly*
 T2 : *... for those who did not understand, **they would not know what 100 m rope is to with the perimeter.** So that the concept of perimeter, for those who understood, they already make it but later **they would not think** the rectangles can be varied.*
 T2 : *Students **would confuse** the meaning of maximum, which is the largest, **they have not thought about it.** So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7).

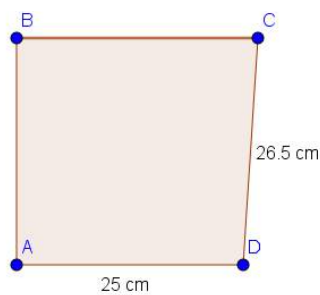


Figure 7. A student's possible mistake proposed by the researcher

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If faced with a student mistake that they not have thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R : If it happens if you see this (showing)

T1 : I would ask students back to try it then you calculate it as what being asked to you

R : They have not yet known the result!

T1 : Try, try it, by trialing they would know that it is different, this one is more, and that one is like that,

R : T2, what if your students did this? what would you do?

T2 : I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...

In this study, the lesson plans facilitated an insight into teachers' knowledge. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Yeo, 2008).

Teachers have different ways of supporting students to solve tasks (Yei, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task. However, these results are not generalizable. The sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared. Developing higher order thinking and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment.

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REFERENCES

- An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, 7, 145-172. <https://doi.org/10.1023/b:jmte.0000021943.35739.1c>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>
- Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, 31, 235-268. <https://doi.org/10.1007/BF00376322>
- Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project* (pp. 25-48). https://doi.org/10.1007/978-1-4614-5149-5_2
- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., ... Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 133-180. <https://doi.org/10.3102/0002831209345157>
- Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' thinking about instruction. *Remedial and Special Education*, 11(6), 40-49. <https://doi.org/10.1177/074193259001100609>
- Burns, R. B., & Lash, A. A. (1988). Nine seventh-grade teachers' knowledge and planning of problem-

- solving instruction. *The Elementary School Journal*, 8(4), 369-386. <https://doi.org/10.1086/461545>
- Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.
- Carle, S. M. (1993). *Student held misconceptions regarding area and perimeter of rectangles* (University of Massachusetts Boston). Retrieved from https://scholarworks.umb.edu/cgi/viewcontent.cgi?article=1045&context=cct_capstone
- Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136-143). Adelaide: Australian Association of Mathematics Teachers.
- Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching, 3rd Edition* (pp. 255-296). New York: Macmillan.
- Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from lessons: Studying the construction of teacher knowledge catalysed by purposefully-designed experimental mathematics lessons. In M. Marshman, V. Geiger, & A Bennison (Eds.), *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165-172). Sunshine Coast: MEGA.
- Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406. <https://doi.org/10.3102/00028312042002371>
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39, 372-400. Retrieved from <https://www.jstor.org/stable/40539304>
- John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies*, 38(4), 483-498. <https://doi.org/10.1080/00220270500363620>
- Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In S. Johnston-Wilder, C. Lee, & D. Pimm (Eds.), *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience, 3rd Edition* (pp. 79-100). London: Routledge. <https://doi.org/10.4324/9780203844120>
- Korkmaz, H. I., & Şahin, Ö. (2019). Preservice preschool teachers' pedagogical content knowledge on geometric shapes in terms of children's mistakes. *Journal of Research in Childhood Education*, 34(3), 385-405. <https://doi.org/10.1080/02568543.2019.1701150>
- Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1*. Retrieved from <https://files.eric.ed.gov/fulltext/ED452031.pdf>
- Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *Procedia - Social and Behavioral Sciences*, 55, 720-729. <https://doi.org/10.1016/j.sbspro.2012.09.557>
- Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical content knowledge of mathematics pre-service teachers: Do they know their students? *Journal of Physics: Conference Series*, 1097(1), 012098. <https://doi.org/10.1088/1742-6596/1097/1/012098>

- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 2(15), 4-14. <https://doi.org/10.3102/0013189X015002004>
- Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26(2), 114-145. <https://doi.org/10.2307/749205>
- Superfine, A. C. (2008). Planning for mathematics instruction: A model of experienced teachers' planning processes in the context of a reform mathematics curriculum. *Mathematics Educator*, 18(2), 11-22. Retrieved from <http://tme.journals.libs.uga.edu/index.php/tme/article/view/198>
- Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., ... Rowley, G. (2012). Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). In *International Association for the Evaluation of Educational Achievement*. Retrieved from <https://files.eric.ed.gov/fulltext/ED542380.pdf>
- Turnuklu, E., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, 1, 1-13. Retrieved from <http://www.k-12prep.math.ttu.edu/journal/1.contentknowledge/yesildere01/article.pdf>
- Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating preservice mathematics teachers' pedagogical content knowledge through microteaching. *REDIMAT-Journal of Research in Mathematics Education*, 9(1), 62-87. <https://doi.org/http://dx.doi.org/10.17583/redimat.2020.3353>
- Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.
- White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2013). Teachers learning from teachers. In M.A. Clements, A.J. Bishop, C. Keitel, J. Kilpatrick, & F.K.S. Leung (Eds.), *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13
- Widodo, & Tamimudin, H. M. (2014). Three training strategies for improving mathematics teacher competence in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Retrieved from <http://atcm.mathandtech.org/EP2014/index.html>
- Yeo, K. K. Y. (2008). Teaching area and perimeter : Mathematics-pedagogical-content knowledge-in-action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*. Retrieved from <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.471.9965&rep=rep1&type=pdf>
- Yin, R. K. (2014). *Case Study Research: Design and Methods* (5th ed.). Thousand Oaks: SAGE Publications.
- Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, 6(2), 41-62. <https://doi.org/10.26220/rev.1627>



INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS: PREDICTING AND RESPONDING TO STUDENTS' RESPONSES TO THE TOPIC OF AREA AND PERIMETER

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. However, to some extent of teacher assessment has not yet completely covered the full range of teacher skills and competencies. This study investigates focuses on Knowledge of Content and Students (KCS) on the topic of area-perimeter through their designed lesson plans. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items to assess mathematics teacher knowledge are dominated by subject matter knowledge. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' KCS through lesson plans developed by the teachers. Mathematics teachers attended a professional development activity and voluntarily participated in this study. Content analysis of the lesson plan and semi-structured interviews were conducted, and the data analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible methods and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Namun, dalam beberapa hal, penilaian guru belum sepenuhnya mencakup seluruh keterampilan dan kompetensi guru. Fokus penelitian ini adalah menyelidiki *Knowledge of Content and Students* (KCS) pada topik luas dan keliling melalui rancangan rencana pembelajaran mereka. Pengetahuan guru tentang topik luas dan keliling dan strategi pengajaran telah dinilai melalui pengujian. Secara umum, materi untuk menilai pengetahuan guru matematika didominasi oleh materi pengetahuan. Dengan demikian, penilaian tersebut tampaknya belum sepenuhnya mencakup seluruh pengetahuan dan kompetensi guru. Dalam penelitian ini, peneliti menginvestigasi KCS guru matematika melalui RPP yang dikembangkan oleh guru. Guru matematika mengikuti kegiatan pengembangan profesional dan secara sukarela berpartisipasi dalam penelitian ini. Analisis isi RPP dan wawancara semi-terstruktur dilakukan, dan data dianalisis. Hasil penelitian ini mengungkapkan bahwa guru yang berpartisipasi ditantang ketika membuat prediksi kemungkinan tanggapan siswa. Mereka tampaknya tidak menyadari strategi siswa biasa, yang digunakan untuk menyelesaikan memaksimalkan luas dari keliling tertentu. Dengan pengetahuan yang terbatas tentang kemungkinan metode dan kesalahan siswa, para guru ini kurang siap untuk mendukung pembelajaran siswa.

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

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Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research has shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related

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to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see [Figure 1](#)).

Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2013, p.394).

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

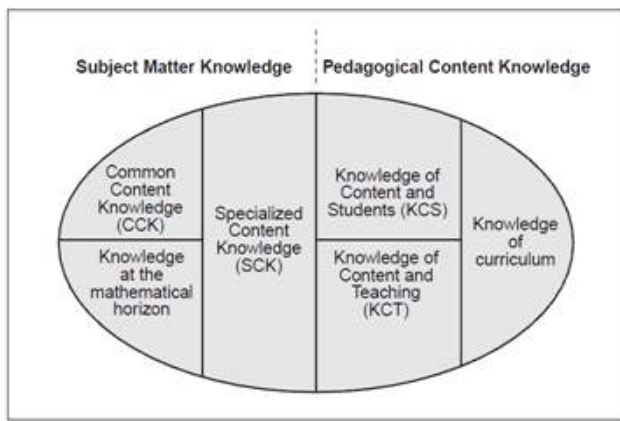


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). Pre-service teachers have challenges with student thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COCATIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin H,

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2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Lesson planning is considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers who read it can easily understand it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called ‘Bansho’, it anticipates student mathematical thinking and student thinking schema for solving given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed the teacher, in planning a lesson, gave attention to students’ engagement (Clarke, Clarke, Roche, & Chan, 2015). Student engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom behaviour is influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in [Figure 2](#).

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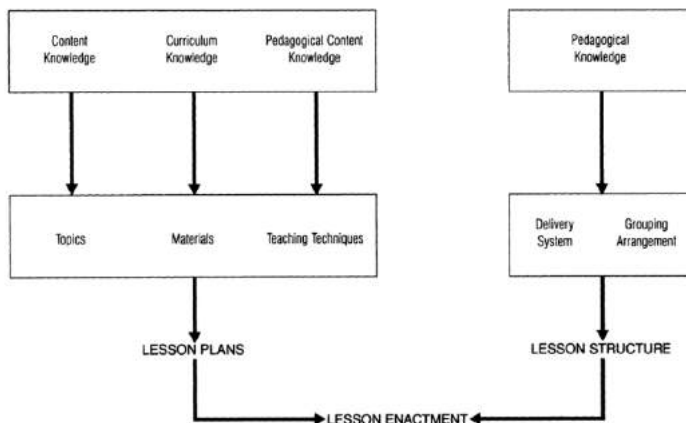


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zacahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013).

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason, (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Yeo, 2008).

This study intends to focus on a part of PCK pedagogical content knowledge, the KCS within lesson plans on the topic of area-perimeter. It is necessary to obtain a fuller insight into teacher knowledge. How mathematics teachers in Yogyakarta prepare their lesson plans and how is PCK integrated in their lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved humans and has been approved by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk). This study administrated a case study approach. This approach suits this study as it doesn't seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). Through this approach, the researchers examined two selected lesson plans of two mathematics teachers. The sample was chosen from twenty-nine teachers who attended a Professional Development (PD) session, and two teachers were selected for the lesson plan

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analysis and interview. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teaches in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan. The topic that would be taught was area and perimeter for grade 7. The “Gold Rush/Mining” task was selected. This task has several ways to be solved (see

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Figure 3).

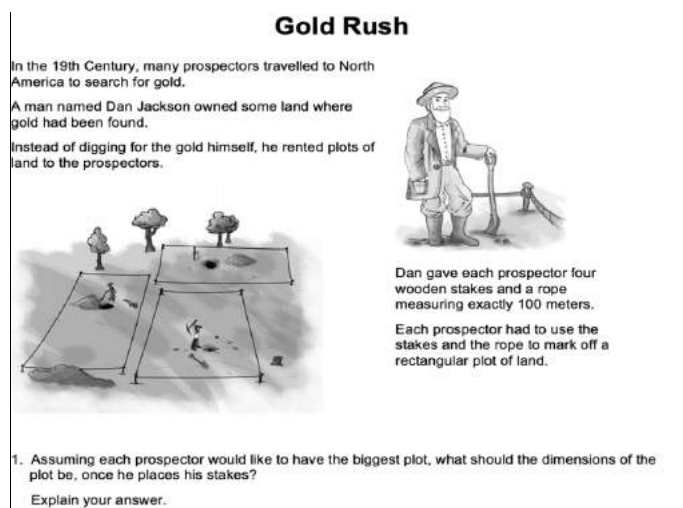


Figure 3. The Gold Rush problem (<https://www.map.mathshell.org/download.php?fileid=1637>)

To analyze the lesson plans, the researchers used content analysis. This method has the ‘potential to disclose many hidden aspects of what is being communicated through the written text’ (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers’ knowledge of students’ conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find the themes. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers included information about what students would do to the task. The data were presented descriptively.

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Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students' emerging and incomplete thinking

The two lesson plans were coded and analyzed. There were three types of instructions to refer to with the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?"	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

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RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3).

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Table 3. Proportions of the instructions

Instruction	T1	T2
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	23 (100%)	19 (100%)

Indonesian teachers follow the prescribed template of a lesson plan. The template consists of three main parts namely; introduction, main and closure. Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2's SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	Introduction		Main		Closure	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	12	3	7	11	4	5

T1 put more details of what students would ask to her on her lesson plan. For instance: 'Can I solve it freely?' has been put on her lesson plan. In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4).

<p>Main Activity 100 minutes</p> <p>PHASE: Organizing Students</p> <p>Students make up groups consisting of 4-5 students.</p> <p>> Observing</p> <p>After receiving the worksheet (problem), students observe the problem within their groups.</p> <p>> Questioning</p> <p>Students ask some questions related to the worksheet such as:</p> <ul style="list-style-type: none"> ⬇ I still do not understand what the problem means. ⬇ Can I solve it freely? <p>PHASE: Guiding the individual and group investigation</p> <p>> Gathering Information/ Data/ Trying out</p> <p>Students look for data and discuss the problem on the worksheet of Gold mining.</p> <p>> Reasoning/ Associating</p> <p>Students conclude the result of their discussion.</p> <p>PHASE: Developing and Presenting the result</p> <p>> Communicating</p> <p>Students communicate their result in written or oral presentation. One of the member of the group presents the result and other groups respond to him.</p>

Figure 4. Teacher 1 Lesson Plan

Commented [A16]: All? Part of instruction in teacher education?

Commented [A17]: Did the template require them to formulate learning goals? (or misconceptions being elicited and solved?) What is the teacher's purpose for organizing a lesson with the problem?

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The T2 lesson plan depicted detailed information about a possible student strategy. [Figure 5](#) shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

Main Activity

- Teacher divide students into groups
- Teacher delivers the worksheet to be discussed
- Teacher facilitates the learning processes
- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
 for instance $p=10, l=... m$ then the area = ...
 $p = 15 m, l = ...m$, then the area =
 Students determine the largest area by themselves
- for the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
 Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
 How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan

After finding the largest area, students had to find the largest area by joining two miners' ropes. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in [Figure 6](#).

- Students evaluate and make generalisation into questioning.
 - Teacher asks students to present in front of the class
 - Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

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Teachers T1 and T2 have more than five-years teaching experience each. However, their schools are different in terms of location and students. These teachers themselves employed different abilities in solving the Gold Mining problem (

Figure 3). From the conversation below, it seems that they have three correct strategies or less to solve it: T1- Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : *Are there other ways T1?*

T1 : *Yesterday, I just did that one.*

T1 : *...just let students find the ways to solve it Then, I will let them know that there are some ways to solve it. I give that opportunity to students*

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

T2 : *To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*

R : *So, you (T2), induce them by using the table?*

T2 : *Yes, by the table.*

R : *What do you think, how many ways to solve it?*

T2 : *To me, I did one way I know it directly it would be a square. **I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and..., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

T2 : *I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*

T2 : *However, I have thought only one strategy, which is global to solve a task. ... I, I... know at least I understand my students' characteristic so that it will be difficult for my students. ... It is not possible to come up if I let them to do it freely. ... I am so careful to give it the various strategies because students would get confuse*

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

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- T2 : *By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.*
 T2 : *To me, I did one way I know it directly it would be a square. I knew it already*
 T2 : *...instead of table, we can make the variable x, then I will be a quadratic function,*
 R : *Are there other ways to solve it?*
 T2 : *For the time being, not yet, making rectangles and to the square*
 R : *Do you think there are still other ways to solve that problem?*
 T2 : *I could use the graph ...*

To some extent, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task. However, by giving the students the strategy, he inadvertently is making the students dependent on him. Whereas, T1 is helping the students to make decisions themselves. From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

- T1 : *The task has missing information, it should be more, and some students would think that. So that they **have not thought** yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
 T1 : *100. Maybe **they thought that** that's the only think they know.*
 R : *... So, they would answer it 100, possibly*
 T1 : *Yeah, possibly*
 T2 : *... for those who did not understand, **they would not know what 100 m rope is to with the perimeter.** So that the concept of perimeter, for those who understood, they already make it but later **they would not think** the rectangles can be varied.*
 T2 : *Students **would confuse** the meaning of maximum, which is the largest, **they have not thought about it.** So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7).

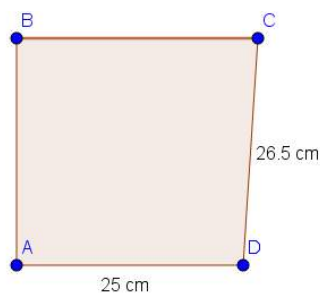


Figure 7. A student's possible mistake proposed by the researcher

Commented [A18]: Well, is that visible in the lesson plan? How to guide the students from West to East? This problem is also designed to create opportunities for discussing and developing inquiry skills in mathematics: the importance of first trying some cases, try to see what happens, organize calculations, and organize them systematically to find a pattern. This only works when you address these issues explicitly. A teacher modelling a solution strategy (e.g. by providing a table) can have a role in this learning process, as long as (s)he is aware of this process (next step might be to give students the responsibility in organizing calculations).

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If faced with a student mistake that they not have thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R : If it happens if you see this (showing)

T1 : I would ask students back to try it then you calculate it as what being asked to you

R : They have not yet known the result!

T1 : Try, try it, by trialing they would know that it is different, this one is more, and that one is like that,

R : T2, what if your students did this? what would you do?

T2 : I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...

In this study, the lesson plans facilitated an insight into teachers' knowledge. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Yeo, 2008).

Teachers have different ways of supporting students to solve tasks (Yei, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task. However, these results are not generalizable. The sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

Commented [A19]: The section below seems to be a discussion of the results. New heading?

Commented [A20]: Well, for me the interview was more informative than the only the lesson plan. And still not fully clear to me what the student teachers' intentions were with their lesson plans for this particular problem.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared. Developing higher order thinking and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment.

Commented [A21]: Please be more modest (e.g. aspects of KCS in the domain of area during a PD course... etc.).

Commented [A22]: Yes!

Commented [A23]: For designing lesson plans? Or for teaching? These conclusions seem to generalize (too) quick...

Commented [A24]: What is this? Not part of the paper...

Commented [A25]: Somewhere in a learning trajectory it can be helpful to model as a teacher a particular way. Don't suggest that this is always wrong... (or provide a more clear argument).

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REFERENCES

- An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, 7, 145-172. <https://doi.org/10.1023/b:jimte.0000021943.35739.1c>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>
- Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, 31, 235-268. <https://doi.org/10.1007/BF00376322>
- Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project* (pp. 25-48). https://doi.org/10.1007/978-1-4614-5149-5_2
- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., ... Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 133-180. <https://doi.org/10.3102/0002831209345157>
- Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' thinking about instruction. *Remedial and Special Education*, 11(6), 40-49. <https://doi.org/10.1177/074193259001100609>
- Burns, R. B., & Lash, A. A. (1988). Nine seventh-grade teachers' knowledge and planning of problem-

- solving instruction. *The Elementary School Journal*, 8(4), 369-386. <https://doi.org/10.1086/461545>
- Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.
- Carle, S. M. (1993). *Student held misconceptions regarding area and perimeter of rectangles* (University of Massachusetts Boston). Retrieved from https://scholarworks.umb.edu/cgi/viewcontent.cgi?article=1045&context=cct_capstone
- Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136-143). Adelaide: Australian Association of Mathematics Teachers.
- Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching, 3rd Edition* (pp. 255-296). New York: Macmillan.
- Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from lessons: Studying the construction of teacher knowledge catalysed by purposefully-designed experimental mathematics lessons. In M. Marshman, V. Geiger, & A Bennison (Eds.), *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165-172). Sunshine Coast: MEGA.
- Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406. <https://doi.org/10.3102/00028312042002371>
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39, 372-400. Retrieved from <https://www.jstor.org/stable/40539304>
- John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies*, 38(4), 483-498. <https://doi.org/10.1080/00220270500363620>
- Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In S. Johnston-Wilder, C. Lee, & D. Pimm (Eds.), *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience, 3rd Edition* (pp. 79-100). London: Routledge. <https://doi.org/10.4324/9780203844120>
- Korkmaz, H. I., & Şahin, Ö. (2019). Preservice preschool teachers' pedagogical content knowledge on geometric shapes in terms of children's mistakes. *Journal of Research in Childhood Education*, 34(3), 385-405. <https://doi.org/10.1080/02568543.2019.1701150>
- Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1*. Retrieved from <https://files.eric.ed.gov/fulltext/ED452031.pdf>
- Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *Procedia - Social and Behavioral Sciences*, 55, 720-729. <https://doi.org/10.1016/j.sbspro.2012.09.557>
- Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical content knowledge of mathematics pre-service teachers: Do they know their students? *Journal of Physics: Conference Series*, 1097(1), 012098. <https://doi.org/10.1088/1742-6596/1097/1/012098>

- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 2(15), 4-14. <https://doi.org/10.3102/0013189X015002004>
- Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26(2), 114-145. <https://doi.org/10.2307/749205>
- Superfine, A. C. (2008). Planning for mathematics instruction: A model of experienced teachers' planning processes in the context of a reform mathematics curriculum. *Mathematics Educator*, 18(2), 11-22. Retrieved from <http://tme.journals.libs.uga.edu/index.php/tme/article/view/198>
- Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., ... Rowley, G. (2012). Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). In *International Association for the Evaluation of Educational Achievement*. Retrieved from <https://files.eric.ed.gov/fulltext/ED542380.pdf>
- Turnuklu, E., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, 1, 1-13. Retrieved from <http://www.k-12prep.math.ttu.edu/journal/1.contentknowledge/yesildere01/article.pdf>
- Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating preservice mathematics teachers' pedagogical content knowledge through microteaching. *REDIMAT-Journal of Research in Mathematics Education*, 9(1), 62-87. <https://doi.org/http://dx.doi.org/10.17583/redimat.2020.3353>
- Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.
- White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2013). Teachers learning from teachers. In M.A. Clements, A.J. Bishop, C. Keitel, J. Kilpatrick, & F.K.S. Leung (Eds.), *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13
- Widodo, & Tamimudin, H. M. (2014). Three training strategies for improving mathematics teacher competence in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Retrieved from <http://atcm.mathandtech.org/EP2014/index.html>
- Yeo, K. K. Y. (2008). Teaching area and perimeter : Mathematics-pedagogical-content knowledge-in-action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*. Retrieved from <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.471.9965&rep=rep1&type=pdf>
- Yin, R. K. (2014). *Case Study Research: Design and Methods* (5th ed.). Thousand Oaks: SAGE Publications.
- Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, 6(2), 41-62. <https://doi.org/10.26220/rev.1627>



INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS: PREDICTING AND RESPONDING TO STUDENTS' RESPONSES TO THE TOPIC OF AREA AND PERIMETER

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. However, to some extent of teacher assessment has not yet completely covered the full range of teacher skills and competencies. This study investigates focuses on Knowledge of Content and Students (KCS) on the topic of area-perimeter through their designed lesson plans. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items to assess mathematics teacher knowledge are dominated by subject matter knowledge. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' KCS through lesson plans developed by the teachers. Mathematics teachers attended a professional development activity and voluntarily participated in this study. Content analysis of the lesson plan and semi-structured interviews were conducted, and the data analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible methods and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Namun, dalam beberapa hal, penilaian guru belum sepenuhnya mencakup seluruh keterampilan dan kompetensi guru. Fokus penelitian ini adalah menyelidiki *Knowledge of Content and Students* (KCS) pada topik luas dan keliling melalui rancangan rencana pembelajaran mereka. Pengetahuan guru tentang topik luas dan keliling dan strategi pengajaran telah dinilai melalui pengujian. Secara umum, materi untuk menilai pengetahuan guru matematika didominasi oleh materi pengetahuan. Dengan demikian, penilaian tersebut tampaknya belum sepenuhnya mencakup seluruh pengetahuan dan kompetensi guru. Dalam penelitian ini, peneliti menginvestigasi KCS guru matematika melalui RPP yang dikembangkan oleh guru. Guru matematika mengikuti kegiatan pengembangan profesional dan secara sukarela berpartisipasi dalam penelitian ini. Analisis isi RPP dan wawancara semi-terstruktur dilakukan, dan data dianalisis. Hasil penelitian ini mengungkapkan bahwa guru yang berpartisipasi ditantang ketika membuat prediksi kemungkinan tanggapan siswa. Mereka tampaknya tidak menyadari strategi siswa biasa, yang digunakan untuk menyelesaikan memaksimalkan luas dari keliling tertentu. Dengan pengetahuan yang terbatas tentang kemungkinan metode dan kesalahan siswa, para guru ini kurang siap untuk mendukung pembelajaran siswa.

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

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Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research has shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related

to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2013, p.394).

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

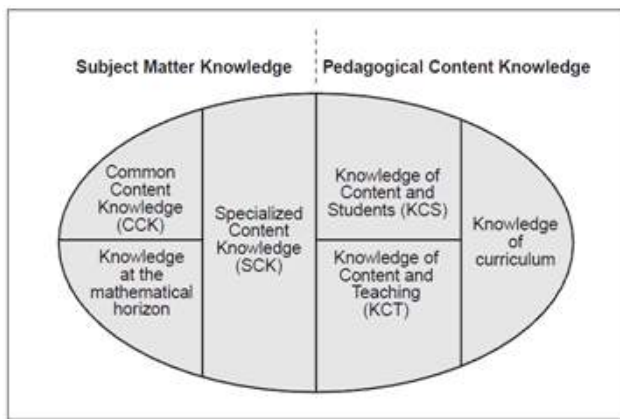


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). Pre-service teachers have challenges with student thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COCATIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin H,

2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Lesson planning is considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers who read it can easily understand it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called 'Bansho', it anticipates student mathematical thinking and student thinking schema for solving given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). Student engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom behaviour is influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

Commented [Ma1]: Author needs to explain current research in PCK, there are several research in relation to PCK and describe the gap

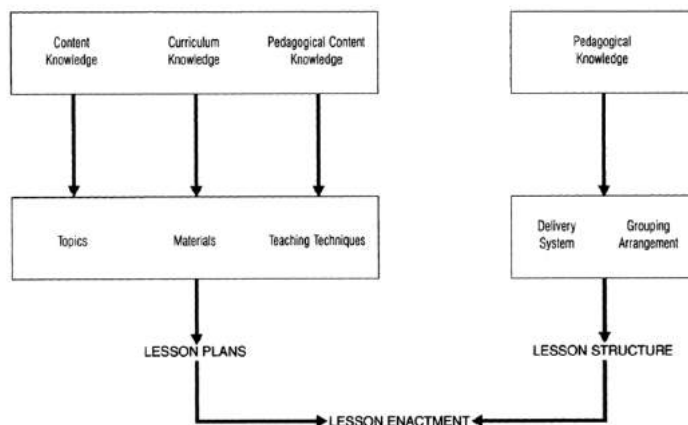


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zawahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013).

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason, (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Yeo, 2008).

This study intends to focus on a part of PCK pedagogical content knowledge, the KCS within lesson plans on the topic of area-perimeter. It is necessary to obtain a fuller insight into teacher knowledge. How mathematics teachers in Yogyakarta prepare their lesson plans and how is PCK integrated in their lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD


This research involved human and had been through research ethics approval by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk). This study administrated a case study approach. This approach suits this study as it doesn't seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). Through this approach, the researchers examined two selected lesson plans of the mathematics teachers. The sample was chosen from twenty-nine teachers who attended a Professional Development (PD) session, and two teachers were selected

for the lesson plan analysis and interview. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan. The topic that would be taught was area and perimeter for grade 7. The “Gold Rush/Mining” task was selected. This task has several ways to be solved (see


Figure 3).

Gold Rush

In the 19th Century, many prospectors travelled to North America to search for gold.
 A man named Dan Jackson owned some land where gold had been found.
 Instead of digging for the gold himself, he rented plots of land to the prospectors.



Dan gave each prospector four wooden stakes and a rope measuring exactly 100 meters.
 Each prospector had to use the stakes and the rope to mark off a rectangular plot of land.



1. Assuming each prospector would like to have the biggest plot, what should the dimensions of the plot be, once he places his stakes?
 Explain your answer.

Figure 3. The Gold Rush problem (<https://www.map.mathshell.org/download.php?fileid=1637>)

To analyze the lesson plans, the researchers used content analysis. This method has the ‘potential to disclose many hidden aspects of what is being communicated through the written text’ (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers’ knowledge of students’ conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find the themes. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers included information about what students would do to the task. The data were presented descriptively.

Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students' emerging and incomplete thinking

Commented [Ma2]: How this part is coded in the coding for instruction

The two lesson plans were coded and analyzed. There were three types of instructions to refer to the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?"	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3).

Table 3. Proportions of the instructions

Instruction	T1	T2
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	23 (100%)	19 (100%)

Commented [Ma3]: How the percentage come out?

Indonesian teachers follow the prescribed template of a lesson plan. The template consists of three main parts namely; introduction, main and closure. Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2’s SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	Introduction		Main		Closure	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	12	3	7	11	4	5

T1 put more details of what students would ask to her on her lesson plan. For instance: ‘Can I solve it freely?’ has been put on her lesson plan. In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4).

<p>☛ Main Activity 100 minutes</p> <p>PHASE: Organizing Students</p> <p>Students make up groups consisting of 4-5 students.</p> <p>> Observing</p> <p>After receiving the worksheet (problem), students observe the problem within their groups.</p> <p>> Questioning</p> <p>Students ask some questions related to the worksheet such as:</p> <ul style="list-style-type: none"> ⚡ I still do not understand what the problem means. ⚡ Can I solve it freely? <p>PHASE: Guiding the individual and group investigation</p> <p>> Gathering Information/ Data/ Trying out</p> <p>Students look for data and discuss the problem on the worksheet of Gold mining.</p> <p>> Reasoning/ Associating</p> <p>Students conclude the result of their discussion.</p> <p>PHASE: Developing and Presenting the result</p> <p>> Communicating</p> <p>Students communicate their result in written or oral presentation. One of the member of the group presents the result and other groups respond to him.</p>

Figure 4. Teacher 1 Lesson Plan

The T2 lesson plan depicted detailed information about a possible student strategy. Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

<p>Main Activity</p> <ul style="list-style-type: none"> <input checked="" type="checkbox"/> Teacher divide students into groups <input checked="" type="checkbox"/> Teacher delivers the worksheet to be discussed <input checked="" type="checkbox"/> Teacher facilitates the learning processes ○ For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m. for instance $p=10, l=... m$ then the area = ... $p = 15 m, l = ...m$, then the area = Students determine the largest area by themselves ○ for the second question, after students have solved the largest area for one miner, then how if it is for 2 miners? Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area? How if you continue doing this for 3 miners and 4 miners until n miners?
--

Figure 5. Teacher 2 Lesson Plan

After finding the largest area, students had to find the largest area by joining two miners' ropes. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

<ul style="list-style-type: none"> ○ Students evaluate and make generalisation into questioning. <ul style="list-style-type: none"> <input checked="" type="checkbox"/> Teacher asks students to present in front of the class <input checked="" type="checkbox"/> Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Teachers T1 and T2 have more than five-years teaching experience each. However, their schools are different in terms of location and students. These teachers themselves employed different abilities in solving the Gold Mining problem (

Figure 3). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : Are there other ways T1?

T1 : Yesterday, I just did that one.

T1 : ...just let students find the ways to solve it Then, I will let them know that there are some ways to solve it. I give that opportunity to students

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

*T2 : To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*

R : So, you (T2), induce them by using the table?

T2 : Yes, by the table.

R : What do you think, how many ways to solve it?

*T2 : To me, I did one way I know it directly it would be a square. **I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and..., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

*T2 : I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*

T2 : However, I have thought only one strategy, which is global to solve a task. ... I, I... know at least I understand my students' characteristic so that it will be difficult for my students. ... It is not possible to come up if I let them to do it freely. ... I am so careful to give it the various strategies because students would get confuse

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

- T2 : *By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.*
 T2 : *To me, I did one way I know it directly it would be a square. I knew it already*
 T2 : *...instead of table, we can make the variable x, then I will be a quadratic function,*
 R : *Are there other ways to solve it?*
 T2 : *For the time being, not yet, making rectangles and to the square*
 R : *Do you think there are still other ways to solve that problem?*
 T2 : *I could use the graph ...*

To some extent, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task. However, by giving the students the strategy, he inadvertently is making the student dependent on him. Whereas, T1 is helping the students to make decisions themselves. From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

- T1 : *The task has missing information, it should be more, and some students would think that. So that they **have not thought** yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
 T1 : *100. Maybe **they thought that** that's the only think they know.*
 R : *... So, they would answer it 100, possibly*
 T1 : *Yeah, possibly*
 T2 : *... for those who did not understand, **they would not know what 100 m rope** is to with the perimeter. So that the concept of perimeter, for those who understood, they already make it but later **they would not think** the rectangles can be varied.*
 T2 : *Students **would confuse** the meaning of maximum, which is the largest, **they have not thought about it**. So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7).

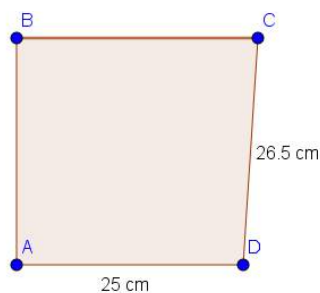


Figure 7. A student's possible mistake proposed by the researcher

If faced with a student mistake that they not have thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R : If it happens if you see this (showing)

T1 : I would ask students back to try it then you calculate it as what being asked to you

R : They have not yet known the result!

T1 : Try, try it, by trialing they would know that it is different, this one is more, and that one is like that,

R : T2, what if your students did this? what would you do?

T2 : I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...

In this study, the lesson plans facilitated an insight into teachers' knowledge. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Yeo, 2008).

Teachers have different ways of supporting students to solve tasks (Yei, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). **The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task. However, these results are not generalizable.** The sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

Commented [Ma4]: This result is important and can be related to the framework PCK

CONCLUSION

This study indicates that it is possible to assess teachers' KCS through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared. Developing higher order thinking and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment.

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REFERENCES

- An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, 7, 145-172. <https://doi.org/10.1023/b:jmte.0000021943.35739.1c>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>
- Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, 31, 235-268. <https://doi.org/10.1007/BF00376322>
- Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project* (pp. 25-48). https://doi.org/10.1007/978-1-4614-5149-5_2
- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., ... Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 133-180. <https://doi.org/10.3102/0002831209345157>
- Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' thinking about instruction. *Remedial and Special Education*, 11(6), 40-49. <https://doi.org/10.1177/074193259001100609>
- Burns, R. B., & Lash, A. A. (1988). Nine seventh-grade teachers' knowledge and planning of problem-

- solving instruction. *The Elementary School Journal*, 8(4), 369-386. <https://doi.org/10.1086/461545>
- Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.
- Carle, S. M. (1993). *Student held misconceptions regarding area and perimeter of rectangles* (University of Massachusetts Boston). Retrieved from https://scholarworks.umb.edu/cgi/viewcontent.cgi?article=1045&context=cct_capstone
- Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136-143). Adelaide: Australian Association of Mathematics Teachers.
- Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching, 3rd Edition* (pp. 255-296). New York: Macmillan.
- Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from lessons: Studying the construction of teacher knowledge catalysed by purposefully-designed experimental mathematics lessons. In M. Marshman, V. Geiger, & A Bennison (Eds.), *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165-172). Sunshine Coast: MEGA.
- Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406. <https://doi.org/10.3102/00028312042002371>
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39, 372-400. Retrieved from <https://www.jstor.org/stable/40539304>
- John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies*, 38(4), 483-498. <https://doi.org/10.1080/00220270500363620>
- Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In S. Johnston-Wilder, C. Lee, & D. Pimm (Eds.), *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience, 3rd Edition* (pp. 79-100). London: Routledge. <https://doi.org/10.4324/9780203844120>
- Korkmaz, H. I., & Şahin, Ö. (2019). Preservice preschool teachers' pedagogical content knowledge on geometric shapes in terms of children's mistakes. *Journal of Research in Childhood Education*, 34(3), 385-405. <https://doi.org/10.1080/02568543.2019.1701150>
- Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1*. Retrieved from <https://files.eric.ed.gov/fulltext/ED452031.pdf>
- Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *Procedia - Social and Behavioral Sciences*, 55, 720-729. <https://doi.org/10.1016/j.sbspro.2012.09.557>
- Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical content knowledge of mathematics pre-service teachers: Do they know their students? *Journal of Physics: Conference Series*, 1097(1), 012098. <https://doi.org/10.1088/1742-6596/1097/1/012098>

- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 2(15), 4-14. <https://doi.org/10.3102/0013189X015002004>
- Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26(2), 114-145. <https://doi.org/10.2307/749205>
- Superfine, A. C. (2008). Planning for mathematics instruction: A model of experienced teachers' planning processes in the context of a reform mathematics curriculum. *Mathematics Educator*, 18(2), 11-22. Retrieved from <http://tme.journals.libs.uga.edu/index.php/tme/article/view/198>
- Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., ... Rowley, G. (2012). Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). In *International Association for the Evaluation of Educational Achievement*. Retrieved from <https://files.eric.ed.gov/fulltext/ED542380.pdf>
- Turnuklu, E., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, 1, 1-13. Retrieved from <http://www.k-12prep.math.ttu.edu/journal/1.contentknowledge/yesildere01/article.pdf>
- Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating preservice mathematics teachers' pedagogical content knowledge through microteaching. *REDIMAT-Journal of Research in Mathematics Education*, 9(1), 62-87. <https://doi.org/http://dx.doi.org/10.17583/redimat.2020.3353>
- Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.
- White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2013). Teachers learning from teachers. In M.A. Clements, A.J. Bishop, C. Keitel, J. Kilpatrick, & F.K.S. Leung (Eds.), *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13
- Widodo, & Tamimudin, H. M. (2014). Three training strategies for improving mathematics teacher competence in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Retrieved from <http://atcm.mathandtech.org/EP2014/index.html>
- Yeo, K. K. Y. (2008). Teaching area and perimeter : Mathematics-pedagogical-content knowledge-in-action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*. Retrieved from <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.471.9965&rep=rep1&type=pdf>
- Yin, R. K. (2014). *Case Study Research: Design and Methods* (5th ed.). Thousand Oaks: SAGE Publications.
- Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, 6(2), 41-62. <https://doi.org/10.26220/rev.1627>



INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS: PREDICTING AND RESPONDING TO STUDENTS' RESPONSES TO THE TOPIC OF AREA AND PERIMETER

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. However, to some extent of teacher assessment has not yet completely covered the full range of teacher skills and competencies. This study investigates focuses on Knowledge of Content and Students (KCS) on the topic of area-perimeter through their designed lesson plans. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items to assess mathematics teacher knowledge are dominated by subject matter knowledge. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' KCS through lesson plans developed by the teachers. Mathematics teachers attended a professional development activity and voluntarily participated in this study. Content analysis of the lesson plan and semi-structured interviews were conducted, and the data analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible methods and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Namun, dalam beberapa hal, penilaian guru belum sepenuhnya mencakup seluruh keterampilan dan kompetensi guru. Fokus penelitian ini adalah menyelidiki *Knowledge of Content and Students* (KCS) pada topik luas dan keliling melalui rancangan rencana pembelajaran mereka. Pengetahuan guru tentang topik luas dan keliling dan strategi pengajaran telah dinilai melalui pengujian. Secara umum, materi untuk menilai pengetahuan guru matematika didominasi oleh materi pengetahuan. Dengan demikian, penilaian tersebut tampaknya belum sepenuhnya mencakup seluruh pengetahuan dan kompetensi guru. Dalam penelitian ini, peneliti menginvestigasi KCS guru matematika melalui RPP yang dikembangkan oleh guru. Guru matematika mengikuti kegiatan pengembangan profesional dan secara sukarela berpartisipasi dalam penelitian ini. Analisis isi RPP dan wawancara semi-terstruktur dilakukan, dan data dianalisis. Hasil penelitian ini mengungkapkan bahwa guru yang berpartisipasi ditantang ketika membuat prediksi kemungkinan tanggapan siswa. Mereka tampaknya tidak menyadari strategi siswa biasa, yang digunakan untuk menyelesaikan memaksimalkan luas dari keliling tertentu. Dengan pengetahuan yang terbatas tentang kemungkinan metode dan kesalahan siswa, para guru ini kurang siap untuk mendukung pembelajaran siswa.

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

How to Cite: Authors. (2021). Indonesian Mathematics Teachers' Knowledge of Content and Students: Predicting and Responding to Students' Responses to the Topic of Area and Perimeter. *Journal on Mathematics Education*, x (x), xx-xx.

Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research has shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related

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to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2013, p.394).

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

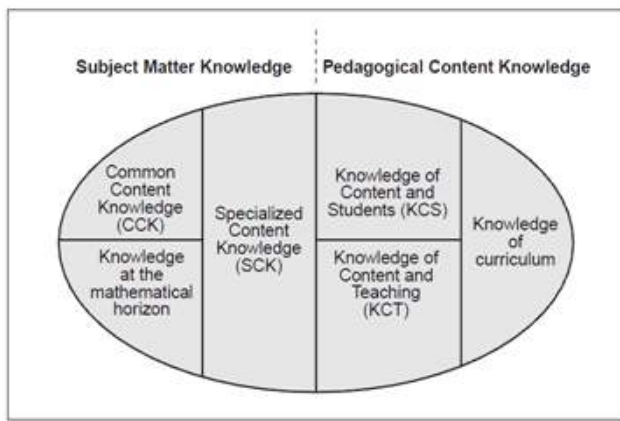


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). Pre-service teachers have challenges with student thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COCATIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin H,

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2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Lesson planning is considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers who read it can easily understand it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called ‘Bansho’, it anticipates student mathematical thinking and student thinking schema for solving given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed the teacher, in planning a lesson, gave attention to students’ engagement (Clarke, Clarke, Roche, & Chan, 2015). Student engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom behaviour is influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

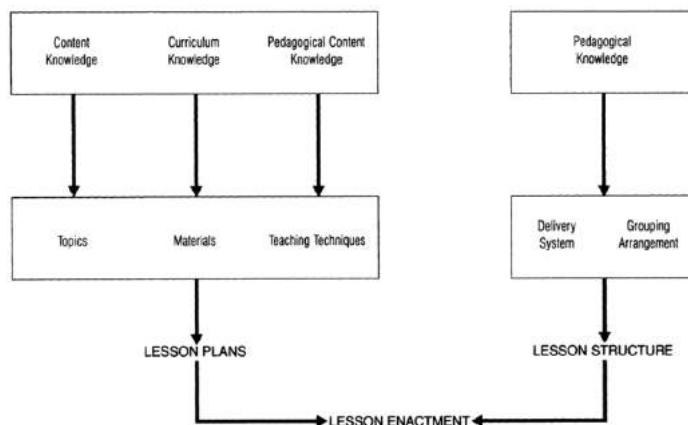


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zawahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013).

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason, (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Yeo, 2008).

This study intends to focus on a part of PCK pedagogical content knowledge, the KCS within lesson plans on the topic of area-perimeter. It is necessary to obtain a fuller insight into teacher knowledge. How mathematics teachers in Yogyakarta prepare their lesson plans and how is PCK integrated in their lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved human and had been through research ethics approval by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk). This study administrated a case study approach. This approach suits this study as it doesn't seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). Through this approach, the researchers examined two selected lesson plans of the mathematics teachers. The sample was chosen from twenty-nine teachers who attended a Professional Development (PD) session, and two teachers were selected

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- Please indicate clearly the research problem(s)/research gap/urgency for conducting this research;
- Rewrite this section so that it clearly presents: from research background, previous relevant studies, research gap, aim of the study, and its corresponding theoretical framework.
- Please reduce the use of abbreviations to improve readability of this section.

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- Research approach and its corresponding reasons
- Participants
- Instruments
- Data collection
- Data analysis

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for the lesson plan analysis and interview. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan. The topic that would be taught was area and perimeter for grade 7. The “Gold Rush/Mining” task was selected. This task has several ways to be solved (see

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Figure 3).

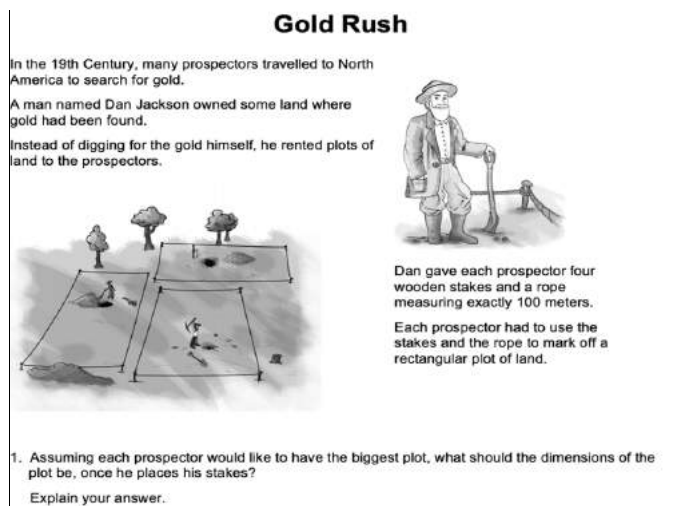


Figure 3. The Gold Rush problem (<https://www.map.mathshell.org/download.php?fileid=1637>)

To analyze the lesson plans, the researchers used content analysis. This method has the ‘potential to disclose many hidden aspects of what is being communicated through the written text’ (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers’ knowledge of students’ conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find the themes. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers included information about what students would do to the task. The data were presented descriptively.

Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students' emerging and incomplete thinking

The two lesson plans were coded and analyzed. There were three types of instructions to refer to the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?"	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3).

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Table 3. Proportions of the instructions

Instruction	T1	T2
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	23 (100%)	19 (100%)

Indonesian teachers follow the prescribed template of a lesson plan. The template consists of three main parts namely; introduction, main and closure. Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2’s SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	Introduction		Main		Closure	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	12	3	7	11	4	5

T1 put more details of what students would ask to her on her lesson plan. For instance: ‘Can I solve it freely?’ has been put on her lesson plan. In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4).

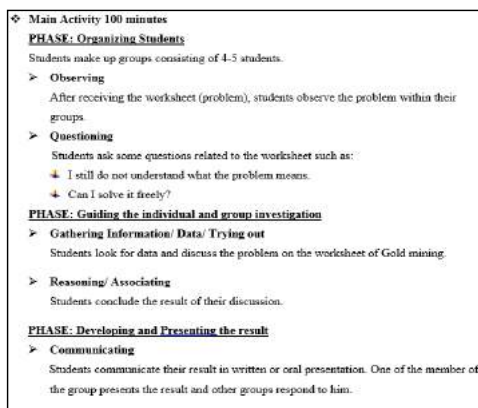


Figure 4. Teacher 1 Lesson Plan

The T2 lesson plan depicted detailed information about a possible student strategy. Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

Main Activity

- Teacher divide students into groups
- Teacher delivers the worksheet to be discussed
- Teacher facilitates the learning processes
- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
for instance $p=10, l=... m$ then the area = ...
 $p = 15 m, l = ...m$, then the area =
Students determine the largest area by themselves
- for the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan

After finding the largest area, students had to find the largest area by joining two miners' ropes. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

- Students evaluate and make generalisation into questioning.
 - Teacher asks students to present in front of the class
 - Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

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Teachers T1 and T2 have more than five-years teaching experience each. However, their schools are different in terms of location and students. These teachers themselves employed different abilities in solving the Gold Mining problem (

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Figure 3). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

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The interview with Teacher 1 showed that she has the ability to solve the problem.

R : Are there other ways T1?

T1 : Yesterday, I just did that one.

T1 : ...just let students find the ways to solve it Then, I will let them know that there are some ways to solve it. I give that opportunity to students

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

T2 : To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.

R : So, you (T2), induce them by using the table?

T2 : Yes, by the table.

R : What do you think, how many ways to solve it?

T2 : To me, I did one way I know it directly it would be a square. **I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and..., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

T2 : I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).

T2 : However, I have thought only one strategy, which is global to solve a task. ... I, I... know at least I understand my students' characteristic so that it will be difficult for my students. ... It is not possible to come up if I let them to do it freely. ... I am so careful to give it the various strategies because students would get confuse

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

- T2 : *By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.*
 T2 : *To me, I did one way I know it directly it would be a square. I knew it already*
 T2 : *...instead of table, we can make the variable x, then I will be a quadratic function,*
 R : *Are there other ways to solve it?*
 T2 : *For the time being, not yet, making rectangles and to the square*
 R : *Do you think there are still other ways to solve that problem?*
 T2 : *I could use the graph ...*

To some extent, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task. However, by giving the students the strategy, he inadvertently is making the student dependent on him. Whereas, T1 is helping the students to make decisions themselves. From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

- T1 : *The task has missing information, it should be more, and some students would think that. So that they **have not thought** yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
 T1 : *100. Maybe **they thought that** that's the only think they know.*
 R : *... So, they would answer it 100, possibly*
 T1 : *Yeah, possibly*
 T2 : *... for those who did not understand, **they would not know what 100 m rope** is to with the perimeter. So that the concept of perimeter, for those who understood, they already make it but later **they would not think** the rectangles can be varied.*
 T2 : *Students **would confuse** the meaning of maximum, which is the largest, **they have not thought about it**. So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7).

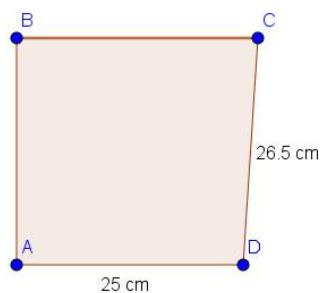


Figure 7. A student's possible mistake proposed by the researcher

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If faced with a student mistake that they not have thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R : If it happens if you see this (showing)

T1 : I would ask students back to try it then you calculate it as what being asked to you

R : They have not yet known the result!

T1 : Try, try it, by trialing they would know that it is different, this one is more, and that one is like that,

R : T2, what if your students did this? what would you do?

T2 : I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...

In this study, the lesson plans facilitated an insight into teachers' knowledge. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Yeo, 2008).

Teachers have different ways of supporting students to solve tasks (Yei, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task. However, these results are not generalizable. The sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

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CONCLUSION

This study indicates that it is possible to assess teachers' KCS through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared. Developing higher order thinking and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment.

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REFERENCES

- An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, 7, 145-172. <https://doi.org/10.1023/b:jimte.0000021943.35739.1c>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>
- Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, 31, 235-268. <https://doi.org/10.1007/BF00376322>
- Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project* (pp. 25-48). https://doi.org/10.1007/978-1-4614-5149-5_2
- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., ... Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 133-180. <https://doi.org/10.3102/0002831209345157>
- Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' thinking about instruction. *Remedial and Special Education*, 11(6), 40-49. <https://doi.org/10.1177/074193259001100609>
- Burns, R. B., & Lash, A. A. (1988). Nine seventh-grade teachers' knowledge and planning of problem-

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Please also add:



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- solving instruction. *The Elementary School Journal*, 8(4), 369-386. <https://doi.org/10.1086/461545>
- Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.
- Carle, S. M. (1993). *Student held misconceptions regarding area and perimeter of rectangles* (University of Massachusetts Boston). Retrieved from https://scholarworks.umb.edu/cgi/viewcontent.cgi?article=1045&context=cct_capstone
- Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136-143). Adelaide: Australian Association of Mathematics Teachers.
- Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching, 3rd Edition* (pp. 255-296). New York: Macmillan.
- Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from lessons: Studying the construction of teacher knowledge catalysed by purposefully-designed experimental mathematics lessons. In M. Marshman, V. Geiger, & A Bennison (Eds.), *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165-172). Sunshine Coast: MEGA.
- Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406. <https://doi.org/10.3102/00028312042002371>
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39, 372-400. Retrieved from <https://www.jstor.org/stable/40539304>
- John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies*, 38(4), 483-498. <https://doi.org/10.1080/00220270500363620>
- Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In S. Johnston-Wilder, C. Lee, & D. Pimm (Eds.), *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience, 3rd Edition* (pp. 79-100). London: Routledge. <https://doi.org/10.4324/9780203844120>
- Korkmaz, H. I., & Şahin, Ö. (2019). Preservice preschool teachers' pedagogical content knowledge on geometric shapes in terms of children's mistakes. *Journal of Research in Childhood Education*, 34(3), 385-405. <https://doi.org/10.1080/02568543.2019.1701150>
- Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1*. Retrieved from <https://files.eric.ed.gov/fulltext/ED452031.pdf>
- Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *Procedia - Social and Behavioral Sciences*, 55, 720-729. <https://doi.org/10.1016/j.sbspro.2012.09.557>
- Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical content knowledge of mathematics pre-service teachers: Do they know their students? *Journal of Physics: Conference Series*, 1097(1), 012098. <https://doi.org/10.1088/1742-6596/1097/1/012098>


- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 2(15), 4-14. <https://doi.org/10.3102/0013189X015002004>
- Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26(2), 114-145. <https://doi.org/10.2307/749205>
- Superfine, A. C. (2008). Planning for mathematics instruction: A model of experienced teachers' planning processes in the context of a reform mathematics curriculum. *Mathematics Educator*, 18(2), 11-22. Retrieved from <http://tme.journals.libs.uga.edu/index.php/tme/article/view/198>
- Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., ... Rowley, G. (2012). Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). In *International Association for the Evaluation of Educational Achievement*. Retrieved from <https://files.eric.ed.gov/fulltext/ED542380.pdf>
- Turnuklu, E., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, 1, 1-13. Retrieved from <http://www.k-12prep.math.ttu.edu/journal/1.contentknowledge/yesildere01/article.pdf>
- Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating preservice mathematics teachers' pedagogical content knowledge through microteaching. *REDIMAT-Journal of Research in Mathematics Education*, 9(1), 62-87. <https://doi.org/http://dx.doi.org/10.17583/redimat.2020.3353>
- Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.
- White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2013). Teachers learning from teachers. In M.A. Clements, A.J. Bishop, C. Keitel, J. Kilpatrick, & F.K.S. Leung (Eds.), *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13
- Widodo, & Tamimudin, H. M. (2014). Three training strategies for improving mathematics teacher competence in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Retrieved from <http://atcm.mathandtech.org/EP2014/index.html>
- Yeo, K. K. Y. (2008). Teaching area and perimeter : Mathematics-pedagogical-content knowledge-in-action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*. Retrieved from <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.471.9965&rep=rep1&type=pdf>
- Yin, R. K. (2014). *Case Study Research: Design and Methods* (5th ed.). Thousand Oaks: SAGE Publications.
- Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, 6(2), 41-62. <https://doi.org/10.26220/rev.1627>

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#13537 Review

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**“Indonesian Mathematics Teachers’ Knowledge of Content and Students of Area
and Perimeter of Rectangle”**

[Paper ID: 13537]



INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS OF AREA AND PERIMETER OF RECTANGLE

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Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. Research has shown teachers' Competencies and skills influence students' performances. Previous studies explored teachers' knowledge through testing. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items or tasks to assess mathematics teacher knowledge in the previous studies were dominated by subject matter knowledge problems. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' KCS through lesson plans developed by the teachers. To accommodate the gap in the previous studies, this study focuses on Knowledge of Content and Students (KCS) on the topic of area-perimeter through their designed lesson plans. Twenty-nine mathematics teachers attended a professional development activity and voluntarily participated in this study. Two teachers were selected to be the focus of this case study. Content analysis of the lesson plan and semi-structured interviews were conducted, and then data were analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible strategies and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Namun, dalam beberapa hal, penilaian guru belum sepenuhnya mencakup seluruh keterampilan dan kompetensi guru. Fokus penelitian ini adalah menyelidiki *Knowledge of Content and Students* (KCS) pada topik luas dan keliling melalui rancangan rencana pembelajaran mereka. Pengetahuan guru tentang topik luas dan keliling dan strategi pengajaran telah dinilai melalui pengujian. Secara umum, materi untuk menilai pengetahuan guru matematika didominasi oleh materi pengetahuan. Dengan demikian, penilaian tersebut tampaknya belum sepenuhnya mencakup seluruh pengetahuan dan kompetensi guru. Dalam penelitian ini, peneliti menginvestigasi KCS guru matematika melalui RPP yang dikembangkan oleh guru. Guru matematika mengikuti kegiatan pengembangan profesional dan secara sukarela berpartisipasi dalam penelitian ini. Analisis isi RPP dan wawancara semi-terstruktur dilakukan, dan data dianalisis. Hasil penelitian ini mengungkapkan bahwa guru yang berpartisipasi ditantang ketika membuat prediksi kemungkinan tanggapan siswa. Mereka tampaknya tidak menyadari strategi siswa biasa, yang digunakan untuk menyelesaikan memaksimalkan luas dari keliling tertentu. Dengan pengetahuan yang terbatas tentang kemungkinan metode dan kesalahan siswa, para guru ini kurang siap untuk mendukung pembelajaran siswa.

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

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Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research have shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2013, p.394).

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Loewenberg Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

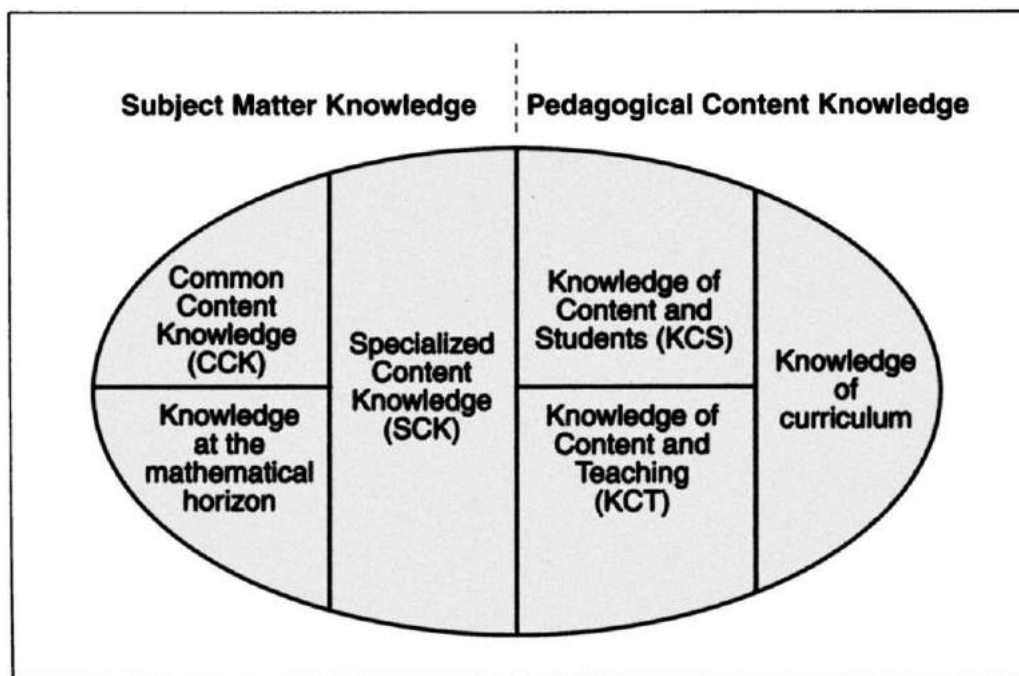


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). In the case of pre-service teachers, they have challenges with student's thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018;

Ünver et al., 2020). It makes sense as they have limited teaching experiences or even have not taught yet. For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COACTIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

As one of the ways, testing is used to assess teachers. The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin H, 2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Another study using testing, faced challenges in measuring teachers' knowledge (Fauskanger, 2015). An interesting finding of a study of pre-service teachers is that they possessed higher PCK scores than SMK from limited test items (Dwi Kristanto, Hariwangsa Panuluh, & Dian Atmajati, 2020). A case study in South Korea revealed that teachers with sufficient SMK of a certain competence/ topic faced challenges in incorporating KCS and KCT of that topic (Lee, Capraro, & Capraro, 2018). Therefore, testing to measure teachers' knowledge still face challenges.

Lesson plans are considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers can easily understand it when reading it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called 'Bansho', which anticipates and tries to elicit student mathematical thinking and student thinking schema for solving the given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed that the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). The students' engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; "Planning for Mathematics Instruction: A Model of Experienced Teachers' Planning Processes in the Context of a Reform Mathematics Curriculum," 2008). In other words, teacher classroom practices are influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions

Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

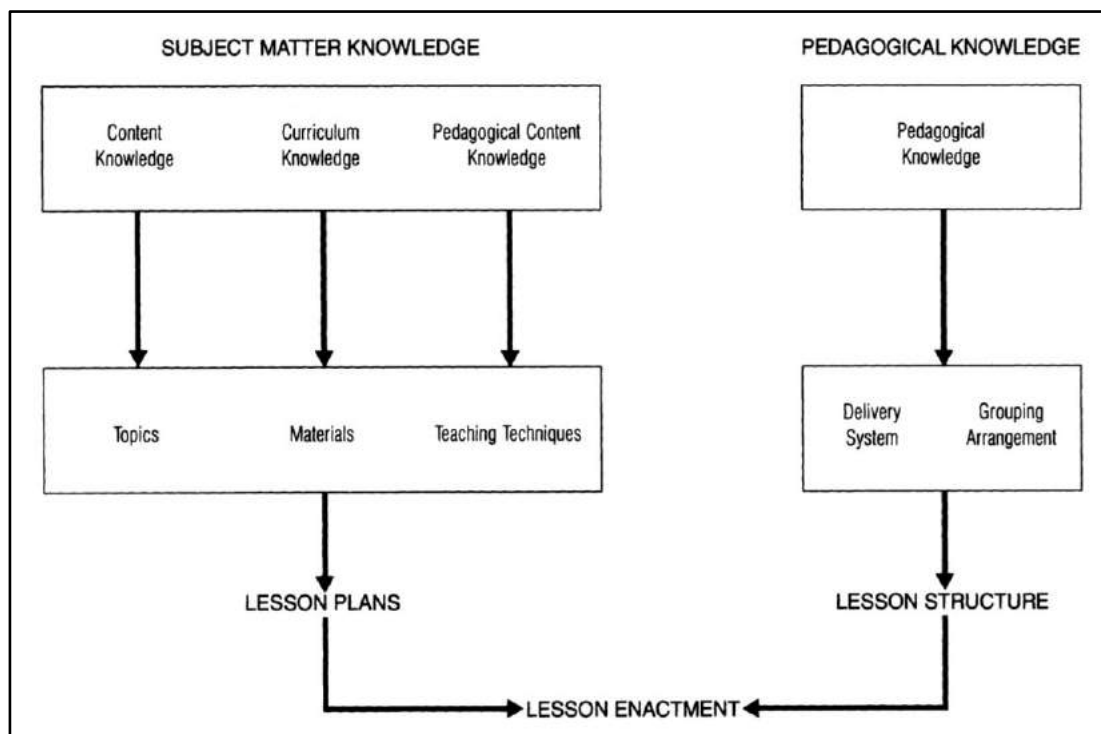


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zacahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013). It seems to make sense as they could manipulate and observe what changes happen by reshaping a figure. (Yunianto, 2015)

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Thus, we should not expect teachers to deliver mathematics well if they do not have mastered it and do not understand how to teach it. Yeo (2008) explored the importance of SMK

and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturu and Nason, (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Kow & Yeo, 2008).

This study intends to focus on a part of PCK, the KCS within lesson plans on the topic of area-perimeter. It is necessary to obtain a fuller insight into teacher knowledge as it influence students' performance. Beside testing, there might be alternative way such as lesson plans to investigate teachers' knowledge. How mathematics teachers prepare their lesson plans and how is PCK integrated in their lesson plans? How are the KCS integrated in the lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved humans and has been approved by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk) as this is a part of completion of the first author's dissertation. This study administrated a case study approach. This approach suits this study as it does not seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). The research subjects were the mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan as part of the whole PD. It was done somewhere in the middle of all complete sessions. As it is a case study, the researchers examined two selected lesson plans of two mathematics teachers. The remaining lesson plans have not been analyzed due to time limitation. The sample was chosen from twenty-nine teachers who attended a professional development (PD) session, and two teachers were selected for the lesson plan analysis and interview. Additionally, these teachers were selected based on their teaching experience; at least five years. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The topic that would be taught was area and perimeter for grade 7. The "Gold Rush/Mining" task was selected. This task was chosen because it is a problem-solving task and has several ways to be solved on area-perimeter of a rectangle (see

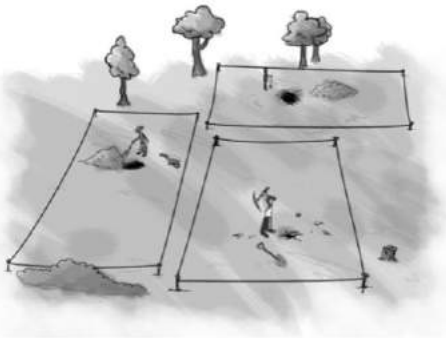
Figure 3). Additionally, the complete Gold Rush activity showed the mistakes that students might do. Thus, it is considered as a good activity to be explored to understand how teachers prepare this activity.


Gold Rush

In the 19th Century, many prospectors travelled to North America to search for gold.

A man named Dan Jackson owned some land where gold had been found.

Instead of digging for the gold himself, he rented plots of land to the prospectors.





Dan gave each prospector four wooden stakes and a rope measuring exactly 100 meters.

Each prospector had to use the stakes and the rope to mark off a rectangular plot of land.

1. Assuming each prospector would like to have the biggest plot, what should the dimensions of the plot be, once he places his stakes?
Explain your answer.

Figure 3. The Gold Rush problem (<https://www.map.mathshell.org/download.php?fileid=1637>)

To analyze the lesson plans, the researchers used content analysis. This method has the ‘potential to disclose many hidden aspects of what is being communicated through the written text’ (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers’ knowledge of students’ conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find themes by classifying instructions and KCS integrated in the lesson plans.

Table 1. Knowledge of Content and Student (KCS) (Loewenberg Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students’ emerging and incomplete thinking

By using Table 2, it is easy to differentiate instructions’ categories. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers

included information about what students would do to the task (KCS). The data were presented descriptively.

The two lesson plans were coded and analyzed. There were three types of instructions to refer to with the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. “what did you do and How did you do it? How are you convinced with your strategies?	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3). The percentage is from type of instruction per total instructions written on the lesson plans.

Indonesian teachers follow the prescribed template of a lesson plan by MoEC. The template consists of three main parts namely; introduction, main and closure. It also consists learning goals and how teachers and students would do in the classroom.

Table 3. Proportions of the instructions

Instruction	T1	T2
--------------------	-----------	-----------

GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	23 (100%)	19 (100%)

Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2’s SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	Introduction		Main		Closure	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	12	3	7	11	4	5

T1 put more details of what students would ask to her on her lesson plan. For instance: ‘Can I solve it freely?’ has been put on her lesson plan. This is a proof of PCK in the lesson plan, but not specific to KCS.

❖ **Main Activity 100 minutes**

PHASE: Organizing Students
 Students make up groups consisting of 4-5 students.

> **Observing**
 After receiving the worksheet (problem), students observe the problem within their groups.

> **Questioning**
 Students ask some questions related to the worksheet such as:
 ⚡ I still do not understand what the problem means.
 ⚡ Can I solve it freely?

PHASE: Guiding the individual and group investigation

> **Gathering Information/ Data/ Trying out**
 Students look for data and discuss the problem on the worksheet of Gold mining.

> **Reasoning/ Associating**
 Students conclude the result of their discussion.

PHASE: Developing and Presenting the result

> **Communicating**
 Students communicate their result in written or oral presentation. One of the members of the group presents the result and other groups respond to him.

❖ **Closure (10 minutes)**

PHASE: Analysing and Evaluating the process of problem solving

1. Teacher facilitates students to conclude what they have just learned)
2. Teacher facilitates students to identify the parts that they both understand and not
3. Teacher gives homework or assignment to students.
4. Teachers informs students that the next lesson would be about triangle.

Figure 4. Teacher 1 Lesson Plan

In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4). On the phase

of guiding the individual and group investigation, which should be rich of KCS. In this lesson plan, detail ways of students might solve it or make mistakes and how to facilitate it have not been depicted.

The T2 lesson plan of rectangle using Gold Rush task depicted detailed information about a possible student strategy. Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

Main Activity

- Teacher divide students into groups
- Teacher delivers the worksheet to be discussed
- Teacher facilitates the learning processes
- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
for instance, $p=10, l=... m$
 $p = 15 m, l = ... m$, then the area = $p=15 m, l= ... m$ etc
Students determine the largest area by themselves
- For the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan of Gold Rush

After finding the largest area of the rectangle, students had to find the largest area by joining two miners' ropes and how would they join it. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

- **Students evaluate and make generalisation into questioning.**
 - Teacher asks students to present in front of the class
 - Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Teachers T1 and T2 have more than five-years teaching experience each. Based on the questionnaire and interview, their schools are different in terms of location and students' background. These teachers themselves employed different abilities in solving the Gold Mining problem (

Figure 3). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : Are there other ways T1?

T1 : Yesterday, I just did that one.

T1 : ...just let students find the ways to solve it Then, I will let them know that there are some ways to solve it. I give that opportunity to students

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

*T2 : To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*

R : So, you (T2), induce them by using the table?

T2 : Yes, by the table.

R : What do you think, how many ways to solve it?

*T2 : To me, I did one way I know it directly it would be a square. **I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and..., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

*T2 : I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*

T2 : However, I have thought only one strategy, which is global to solve a task. ... I, I... know at least I understand my students' characteristic so that it will be difficult for my students. ... It is not possible to come up if I let them to do it freely. ... I am so careful to give it the various strategies because students would get confuse

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

T2 : By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.

- T2 : To me, I did one way I know it directly it would be a square. I knew it already
 T2 : ...instead of table, we can make the variable x , then I will be a quadratic function,
 R : Are there other ways to solve it?
 T2 : For the time being, not yet, making rectangles and to the square
 R : Do you think there are still other ways to solve that problem?
 T2 : I could use the graph ...

To some extent, from the lesson plan, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task (figure 5). However, by giving the students the strategy, he inadvertently is making the students dependent on him. Whereas, from the lesson plan, T1 is helping the students to make decisions themselves (figure 4). From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

- T1 : The task has missing information, it should be more, and some students would think that. So that they **have not thought** yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.
 T1 : 100. Maybe **they thought that** that's the only think they know.
 R : ... So, they would answer it 100, possibly
 T1 : Yeah, possibly
 T2 : ... for those who did not understand, **they would not know what 100 m rope** is to with the perimeter. So that the concept of perimeter, for those who understood, they already make it but later **they would not think** the rectangles can be varied.
 T2 : Students **would confuse** the meaning of maximum, which is the largest, **they have not thought about it**. So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7). The researcher proposed a possible mistake by a student of which the shape looks like a rectangle 25 x 26,5.

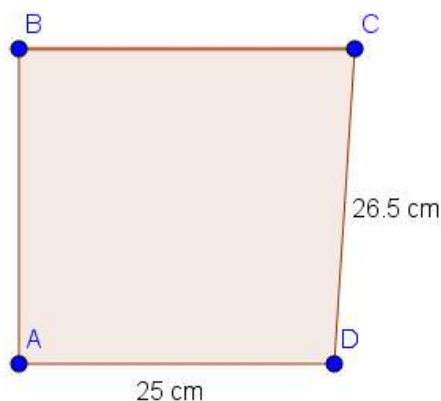


Figure 7. A student's possible mistake proposed by the researcher

If faced with a student mistake that they not have thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing

the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

- R : If it happens if you see this (showing)*
T1 : I would ask students back to try it then you calculate it as what being asked to you
R : They have not yet known the result!
T1 : Try, try it, by trialing they would know that it is different, this one is more, and that one is like that,
R : T2, what if your students did this? what would you do?
T2 : I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...

In this study, the lesson plans facilitated an insight into teachers' knowledge. In this case, it showed teacher's pedagogical knowledge as well as PCK. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid, and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Kow & Yeo, 2008). In Japanese lesson plans, they contain more detailed instructions (Nakahara & Koyama, 2000) which shows more information about teachers knowledge. In contrast, the two case of teachers in this study, have not yet shown detailed instructions but more in general instruction.

Teachers have different ways of supporting students to solve tasks (Yei, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task (possessing SMK and PCK). However, these results are not generalizable. The limited sample was not chosen and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in

Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS of a specific topic through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter of rectangle topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared in facing the situations during teaching. Developing problem solving skills and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment. Additionally, this study explored a rectangle topic, the result might vary in different topics. Therefore, further investigation on different topic could be conducted. This study is not generalizable as it used limited research subjects.

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REFERENCES

- An, S., Kulm, G., & Wu, Z. (2004). The Pedagogical Content Knowledge of Middle School, Mathematics Teachers in China and the U.S. *Journal of Mathematics Teacher Education*. <https://doi.org/10.1023/b:jnte.0000021943.35739.1c>
- Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*. <https://doi.org/10.1007/BF00376322>
- Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers: Results from the COACTIV Project*. https://doi.org/10.1007/978-1-4614-5149-5_2
- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., ... Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*. <https://doi.org/10.3102/0002831209345157>
- Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' Thinking About Instruction. *Remedial and Special Education*. <https://doi.org/10.1177/074193259001100609>

- Burns, R. B., & Lash, A. A. (1988). Nine Seventh-Grade Teachers' Knowledge and Planning of Problem-Solving Instruction. *The Elementary School Journal*. <https://doi.org/10.1086/461545>
- Butt, G. (2008). *Lesson Planning 3rd Edition*. Bloomsbury Publishing.
- Carle, S. M. (1993). *Student held misconceptions regarding area and perimeter of rectangles*.
- Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. In *Handbook of research on teaching*. <https://doi.org/10.1258/ijsa.2008.008228>
- Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from Lessons: Studying the Construction of Teacher Knowledge Catalysed by Purposefully-Designed Experimental Mathematics Lessons. *Mathematics Education Research Group of Australasia*.
- Denscombe, M. (2010). The Good Research Guide For Small Scale Research Projects. In *Open University Press*. <https://doi.org/10.1371/journal.pone.0017540>
- Dwi Kristanto, Y., Hariwangsa Panuluh, A., & Dian Atmajati, E. (2020). Development and validation of a test instrument to measure pre-service mathematics teachers' content knowledge and pedagogical content knowledge. *Journal of Physics: Conference Series*, 1470(1). <https://doi.org/10.1088/1742-6596/1470/1/012008>
- Fauskanger, J. (2015). Challenges in measuring teachers' knowledge. *Educational Studies in Mathematics*, 90(1). <https://doi.org/10.1007/s10649-015-9612-4>
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*. <https://doi.org/10.3102/00028312042002371>
- John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies*. <https://doi.org/10.1080/00220270500363620>
- Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience: 3rd Edition* (pp. 79–100). Routledge Taylor & Francis Group. <https://doi.org/10.4324/9780203844120>
- Korkmaz, H. I., & Şahin, Ö. (2019). Preservice Preschool Teachers' Pedagogical Content Knowledge on Geometric Shapes in Terms of Children's Mistakes. *Journal of Research in Childhood Education*, 1–21.
- Kow, K., & Yeo, J. (2008). Teaching Area and Perimeter : Mathematics-Pedagogical-Content Knowledge-in-Action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*.
- Lee, Y., Capraro, R. M., & Capraro, M. M. (2018). Mathematics Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge in Problem Posing. *International Electronic Journal of Mathematics Education*, 13(2). <https://doi.org/10.12973/iejme/2698>
- Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*. <https://doi.org/10.1177/0022487108324554>
- Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1*.

- Özerem, A. (2012). Misconceptions In Geometry And Suggested Solutions For Seventh Grade Students. *Procedia - Social and Behavioral Sciences*. <https://doi.org/10.1016/j.sbspro.2012.09.557>
- Planning for Mathematics Instruction: A Model of Experienced Teachers' Planning Processes in the Context of a Reform Mathematics Curriculum. (2008). *Mathematics Educator*.
- Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical Content Knowledge of Mathematics Pre-service Teachers: Do they know their students? *Journal of Physics: Conference Series*. <https://doi.org/10.1088/1742-6596/1097/1/012098>
- Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*. <https://doi.org/10.3102/0013189X015002004>
- Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education*. <https://doi.org/10.2307/749205>
- Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., ... Rowley, G. (2012). Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M). In *International Association for the Evaluation of Educational Achievement*.
- Turnuklu, E., & Yesildere, S. (2007). The Pedagogical Content Knowledge in Mathematics: Pre-Service Primary Mathematics Teachers' Perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*.
- Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating Preservice Mathematics Teachers' Pedagogical Content Knowledge through Microteaching. *REDIMAT-Journal of Research in Mathematics Education*, 9(1), 62–87.
- Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.
- Widodo, & Tamimudin H, M. (2014). Three Training Strategies for Improving Mathematics Teacher Competences in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Yogyakarta: Mathematics and Technology, LLC. Retrieved from <http://atcm.mathandtech.org/EP2014/index.html>
- Yin, R. K. (2014). Case study research: Design and methods (5th ed.). In *Thousand Oaks, CA: SAGE Publications*.
- Yunianto, W. (2015). Supporting Students' Understanding of Area Measurement Through Verknippen Applet. *Southeast Asian Mathematics Education Journal*, 5(1). <https://doi.org/10.46517/seamej.v5i1.34>
- Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, 6(2), 41–62.

Keputusan diterima pasca revisi pada tanggal 25 Maret 2021.

The screenshot shows a Gmail interface with a search bar containing 'jme@unsri.ac.id'. The left sidebar lists folders like 'Compose', 'Inbox' (406), 'Starred', 'Snoozed', 'Sent', 'Drafts', 'Academia dan Res...' (19), 'IConProCS' (204), 'Notes', 'Publons' (2), 'Rejected Email', 'Research Gate' (38), and 'Sejarah Matematik...' (24). The main content area displays an email from 'Jme Fkip Matematika' to 'Wahid, me, Cosette' dated 'Thu, Mar 25, 5:39 AM'. The subject is '[JME] Editor Decision'. The email body contains the following text: 'Dear Wahid Yudianto, Rully Charitas Indra Prahmana, and Cosette Crisan, We have decided on your submission entitled Indonesian Mathematics Teachers' Knowledge of Content and Students of Area and Perimeter of Rectangle. After reviewing your revised paper and all supporting documents, our editorial committee considers accepting your article. Herewith, we attach the invoice for your paper. Furthermore, we will proceed to the next step of the editing process after completing the publication fee's payment with the amount stated in the attached invoice. On behalf of the editorial board of the journal, we give a high appreciation for your contribution to our journal. We believe that this contribution will bring benefits to the worldwide mathematics education society. Should you have any questions, please do not hesitate to contact us. Thank you very much for your cooperation. We do really appreciate your work and look forward to welcoming further contributions. Kind regards, Editorial Team Journal on Mathematics Education'. An attachment titled 'Invoice - Publicatio...' is visible. At the bottom, there are three buttons: 'Well received with thanks.', 'Received, thank you.', and 'Noted with thanks.'

Konfirmasi hasil editing dan proofread pasca diterima pada tanggal 25 Maret 2021.

The screenshot shows a Gmail interface with a search bar containing 'jme@unsri.ac.id'. The left sidebar lists folders like 'Compose', 'Inbox' (406), 'Starred', 'Snoozed', 'Sent', 'Drafts', 'Academia dan Res...' (19), 'IConProCS' (204), 'Notes', 'Publons' (2), 'Rejected Email', 'Research Gate' (38), and 'Sejarah Matematik...' (24). The main content area displays an email from 'Jme Fkip Matematika' to 'Wahid, me, Cosette, zulkardi' dated 'Thu, Mar 25, 10:51 AM'. The subject is '[In Editing] Indonesian Mathematics Teachers' Knowledge of Content and Students of Area and Perimeter of Rectangle'. The email body contains the following text: 'Dear Wahid Yudianto, Rully Charitas Indra Prahmana, and Cosette Crisan, Your accepted manuscript has been edited, and our editorial team's layout needs to be proofread before being published. To ensure the timely publication of your document, we ask that you mark any typographical errors. It is important to us that your work reaches your intended readership as soon as possible. We ask that you respond to this email as an indication that you have received our request. If no response has been received within the next two working days, we will assume that you approve and accept the galley format and proceed to the next step of the publishing process. Please let the publisher know if an extension is required. Thank you very much for your cooperation and support. We do really appreciate it. Kind regards, Editorial Team Journal on Mathematics Education'. An attachment titled 'Layout - 02 - 1353...' is visible.

Paper in Editing Version

[Paper ID: 13537]



INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS OF AREA AND PERIMETER OF RECTANGLE

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Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. Research has shown teachers' competencies and skills influence students' performances. Previous studies explored teachers' knowledge through testing. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items or tasks to assess mathematics teacher knowledge in the previous studies were dominated by subject matter knowledge problems. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' Knowledge of Content and Students (KCS) through lesson plans developed by the teachers. To accommodate the gap in the previous studies, this study focuses on KCS on the topic of area-perimeter through their designed lesson plans. Twenty-nine mathematics teachers attended a professional development activity voluntarily participated in this study. Two teachers were selected to be the focus of this case study. Content analysis of the lesson plan and semi-structured interviews were conducted, and then data were analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible strategies and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Penelitian ini menunjukkan bahwa kompetensi dan keterampilan guru mempengaruhi performa siswa. Penelitian sebelumnya telah mengkaji pengetahuan guru melalui tes. Pengetahuan guru pada topik keliling-luas dan strategi pembelajaran juga telah dikaji melalui tes. Pada umumnya, banyaknya soal pada tes didominasi oleh soal-soal tentang pengetahuan subjek yang diajarkan. Oleh karena itu, asesmen seperti ini belum mencakup keseluruhan pengetahuan dan kompetensi guru. Pada studi ini, peneliti menginvestigasi pengetahuan guru matematika tentang KCS pada rencana pelaksanaan pembelajaran yang mereka kembangkan. Untuk mengakomodasi kesenjangan pada penelitian sebelumnya, penelitian kali ini berfokus pada pengetahuan tentang konten dan siswa (KCS) pada topik keliling-luas pada rencana pelaksanaan pembelajaran. Dua puluh Sembilan guru matematika yang sedang mengikuti pelatihan peningkatan kompetensi secara sukarela mengikuti penelitian ini. Dua guru matematika menjadi fokus penelitian studi kasus ini. Konten analisis dan interview semi terstruktur dilakukan dan datanya dianalisis. Terungkap bahwa peserta ini mengalami tantangan dalam memprediksi kemungkinan respon yang diberikan siswa. Mereka belum menyadari strategi siswa yang biasanya digunakan untuk menyelesaikan persoalan memaksimalkan luas dari keliling yang ditentukan. Dengan pengetahuan yang terbatas pada kemungkinan strategi siswa dan kesalahan siswa, guru ini kurang siap dalam mendukung siswanya

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

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Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research have shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2012, p.394).

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

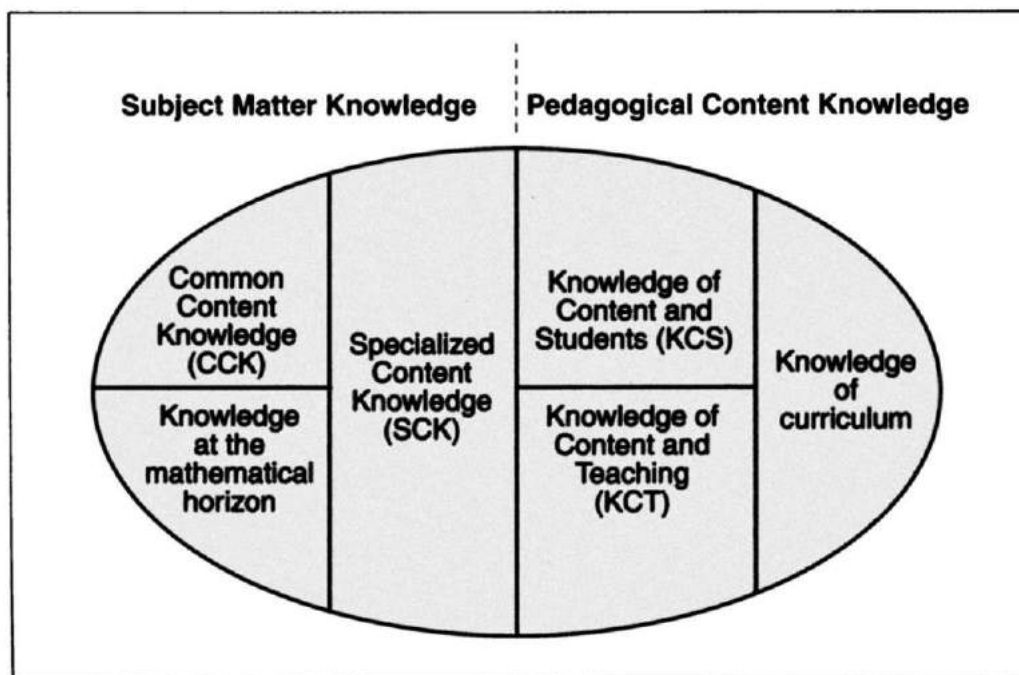


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). In the case of pre-service teachers, they have challenges with student's thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). It makes sense as they have limited teaching experiences or even have not taught

yet. For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COACTIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

As one of the ways, testing is used to assess teachers. The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin, 2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Another study using testing; faced challenges in measuring teachers' knowledge (Fauskanger, 2015). An interesting finding of a study of pre-service teachers is that they possessed higher PCK scores than SMK from limited test items (Kristanto, Panuluh, & Atmajati, 2020). A case study in South Korea revealed that teachers with sufficient SMK of a certain competence/ topic faced challenges in incorporating KCS and KCT of that topic (Lee, Capraro, & Capraro, 2018). Therefore, testing to measure teachers' knowledge still face challenges.

Lesson plans are considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers can easily understand it when reading it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called 'Bansho', which anticipates and tries to elicit student mathematical thinking and student thinking schema for solving the given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed that the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). The students' engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom practices are influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in [Figure 2](#).

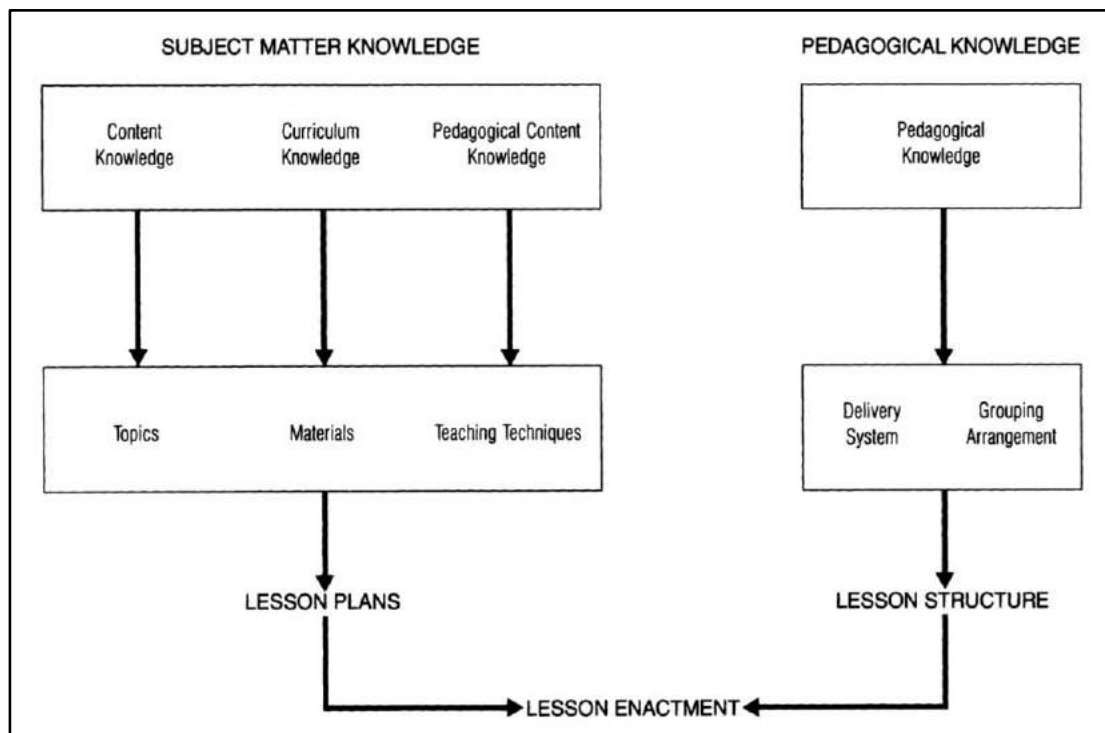


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zazahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013). It seems to make sense as they could manipulate and observe what changes happen by reshaping a figure (Yunianto, 2015).

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Thus, we should not expect teachers to deliver mathematics well if they do not have mastered it and do not understand how to teach it. Kow and Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason, (1996) evaluated first-year teacher education student understanding of subject matter

knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Kow & Yeo, 2008).

This study intends to focus on a part of PCK, the KCS within lesson plans on the topic of area-perimeter of a rectangle. It is necessary to obtain a fuller insight into teacher knowledge as it influence students' performance. Beside testing, there might be alternative way such as lesson plans to investigate teachers' knowledge. How mathematics teachers prepare their lesson plans and how is PCK integrated in their lesson plans? How are the KCS integrated in the lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

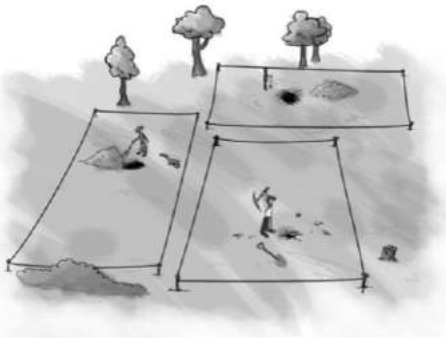
This research involved humans and has been approved by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk) as this is a part of completion of the first author's dissertation. This study administrated a case study approach. This approach suits this study as it does not seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). The research subjects were the mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan as part of the whole PD. It was done somewhere in the middle of all complete sessions. As it is a case study, the researchers examined two selected lesson plans of two mathematics teachers. The remaining lesson plans have not been analyzed due to time limitation. The sample was chosen from twenty-nine teachers who attended a professional development (PD) session, and two teachers were selected for the lesson plan analysis and interview. Additionally, these teachers were selected based on their teaching experience; at least five years. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The topic that would be taught was area and perimeter for grade 7. The "Gold Rush/Mining" task was selected. This task was chosen because it is a problem-solving task and has several ways to be solved on area-perimeter of a rectangle (see [Figure 3](#)). Additionally, the complete Gold Rush activity showed the mistakes that students might do. Thus, it is considered as a good activity to be explored to understand how teachers prepare this activity.


Gold Rush

In the 19th Century, many prospectors travelled to North America to search for gold.

A man named Dan Jackson owned some land where gold had been found.

Instead of digging for the gold himself, he rented plots of land to the prospectors.





Dan gave each prospector four wooden stakes and a rope measuring exactly 100 meters.

Each prospector had to use the stakes and the rope to mark off a rectangular plot of land.

1. Assuming each prospector would like to have the biggest plot, what should the dimensions of the plot be, once he places his stakes?
Explain your answer.

Figure 3. The Gold Rush problem (<https://www.map.mathshell.org/download.php?fileid=1637>)

To analyze the lesson plans, the researchers used content analysis. This method has the ‘potential to disclose many hidden aspects of what is being communicated through the written text’ (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers’ knowledge of students’ conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find themes by classifying instructions and KCS integrated in the lesson plans.

Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students’ emerging and incomplete thinking

By using Table 2, it is easy to differentiate instructions’ categories. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers

included information about what students would do to the task (KCS). The data were presented descriptively.

The two lesson plans were coded and analyzed. There were three types of instructions to refer to with the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?"	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3). The percentage is from type of instruction per total instructions written on the lesson plans.

Indonesian teachers follow the prescribed template of a lesson plan by MoEC. The template consists of three main parts namely; introduction, main and closure. It also consists learning goals and how teachers and students would do in the classroom.

Table 3. Proportions of the instructions

Instruction	T1	T2
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	23 (100%)	19 (100%)

Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2's SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	Introduction		Main		Closure	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	12	3	7	11	4	5

T1 put more details of what students would ask to her on her lesson plan. For instance: 'Can I solve it freely?' has been put on her lesson plan. This is a proof of PCK in the lesson plan, but not specific to KCS.

<p>❖ Main Activity 100 minutes</p> <p>PHASE: Organizing Students</p> <p>Students make up groups consisting of 4-5 students.</p> <p>➤ Observing</p> <p>After receiving the worksheet (problem), students observe the problem within their groups.</p> <p>➤ Questioning</p> <p>Students ask some questions related to the worksheet such as:</p> <ul style="list-style-type: none"> 👉 I still do not understand what the problem means. 👉 Can I solve it freely? <p>PHASE: Guiding the individual and group investigation</p> <p>➤ Gathering Information/ Data/ Trying out</p> <p>Students look for data and discuss the problem on the worksheet of Gold mining.</p> <p>➤ Reasoning/ Associating</p> <p>Students conclude the result of their discussion.</p> <p>PHASE: Developing and Presenting the result</p> <p>➤ Communicating</p> <p>Students communicate their result in written or oral presentation. One of the members of the group presents the result and other groups respond to him.</p> <p>❖ Closure (10 minutes)</p> <p>PHASE: Analysing and Evaluating the process of problem solving</p> <ol style="list-style-type: none"> 1. Teacher facilitates students to conclude what they have just learned) 2. Teacher facilitates students to identify the parts that they both understand and not 3. Teacher gives homework or assignment to students. 4. Teachers informs students that the next lesson would be about triangle.
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Figure 4. Teacher 1 Lesson Plan

In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4). On the phase of guiding the individual and group investigation, which should be rich of KCS. In this lesson plan, detail ways of students might solve it or make mistakes and how to facilitate it have not been depicted.

The T2 lesson plan of rectangle using Gold Rush task depicted detailed information about a possible student strategy (KCS). Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

Main Activity

- Teacher divide students into groups
- Teacher delivers the worksheet to be discussed
- Teacher facilitates the learning processes
- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
for instance, $p=10$, $l=... m$
 $p = 15 m$, $l = ...m$, then the area = $p=15 m$, $l= ... m$ etc
Students determine the largest area by themselves
- For the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan of Gold Rush

After finding the largest area of the rectangle, students had to find the largest area by joining two miners' ropes and how would they join it. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

- **Students evaluate and make generalisation into questioning.**
 - Teacher asks students to present in front of the class
 - Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or

which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Teachers T1 and T2 have more than five-years teaching experience each. Based on the questionnaire and interview, their schools are different in terms of location and students' background. These teachers themselves employed different abilities in solving the Gold Mining problem (Figure 3). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : Are there other ways T1?

T1 : Yesterday, I just did that one.

T1 : ...just let students find the ways to solve it Then, I will let them know that there are some ways to solve it. I give that opportunity to students

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

*T2 : To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*

R : So, you (T2), induce them by using the table?

T2 : Yes, by the table.

R : What do you think, how many ways to solve it?

*T2 : To me, I did one way I know it directly it would be a square. **I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and..., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

*T2 : I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*

T2 : However, I have thought only one strategy, which is global to solve a task. ... I, I... know at least I understand my students' characteristic so that it will be difficult for my students. ... It is not possible to come up if I let them to do it freely. ... I am so careful to give it the various strategies because students would get confuse

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

- T2 : *By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.*
- T2 : *To me, I did one way I know it directly it would be a square. I knew it already*
- T2 : *...instead of table, we can make the variable x , then I will be a quadratic function,*
- R : *Are there other ways to solve it?*
- T2 : *For the time being, not yet, making rectangles and to the square*
- R : *Do you think there are still other ways to solve that problem?*
- T2 : *I could use the graph ...*

To some extent, from the lesson plan, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task (Figure 5). However, by giving the students the strategy, he inadvertently is making the students dependent on him. Whereas, from the lesson plan, T1 is helping the students to make decisions themselves (Figure 4). From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

- T1 : *The task has missing information, it should be more, and some students would think that. So that they **have not thought** yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
- T1 : *100. Maybe **they thought that** that's the only think they know.*
- R : *... So, they would answer it 100, possibly*
- T1 : *Yeah, possibly*
- T2 : *... for those who did not understand, **they would not know what 100 m rope is to with the perimeter.** So that the concept of perimeter, for those who understood, they already make it but later **they would not think** the rectangles can be varied.*
- T2 : *Students **would confuse** the meaning of maximum, which is the largest, **they have not thought about it.** So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7). The researcher proposed a possible mistake by a student of which the shape looks like a rectangle 25 x 26,5.

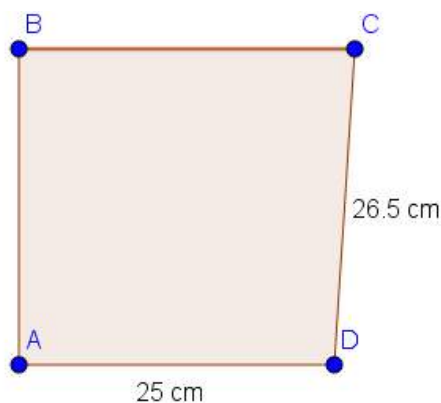


Figure 7. A student's possible mistake proposed by the researcher

If faced with a student mistake that they have not thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R : If it happens if you see this (showing)

T1 : I would ask students back to try it then you calculate it as what being asked to you

R : They have not yet known the result!

T1 : Try, try it, by trialing they would know that it is different, this one is more, and that one is like that,

R : T2, what if your students did this? what would you do?

T2 : I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...

In this study, the lesson plans facilitated an insight into teachers' knowledge. In this case, it showed teacher's pedagogical knowledge as well as PCK. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid, and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Kow & Yeo, 2008). In Japanese lesson plans, they contain more detailed instructions (Nakahara & Koyama, 2000) which shows more information about teachers knowledge. In contrast, the two case of teachers in this study, have not yet shown detailed instructions but more in general instruction.

Teachers have different ways of supporting students to solve tasks (Yeo, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task (possessing SMK and PCK). However, these results are not generalizable. The limited sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this

study might not show detail information on their lesson plans and have not fully been aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS of a specific topic through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter of rectangle topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared in facing the situations during teaching. Developing problem solving skills and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment. Additionally, this study explored a rectangle topic, the result might vary in different topics. Therefore, further investigation on different topic could be conducted. This study is not generalizable as it used limited research subjects.

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REFERENCES

- An, S., Kulm, G., & Wu, Z. (2004). The Pedagogical Content Knowledge of Middle School Mathematics Teachers in China and the U.S. *Journal of Mathematics Teacher Education*, 7, 145-172. <https://doi.org/10.1023/b:jmte.0000021943.35739.1c>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>
- Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, 31, 235-268. <https://doi.org/10.1007/BF00376322>
- Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers*:

Results from the COACTIV Project (pp. 25-48). Boston: Springer. https://doi.org/10.1007/978-1-4614-5149-5_2

- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., ... Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 133-180. <https://doi.org/10.3102/0002831209345157>
- Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' Thinking About Instruction. *Remedial and Special Education*, 11(6), 40-49. <https://doi.org/10.1177/074193259001100609>
- Burns, R. B., & Lash, A. A. (1988). Nine Seventh-Grade Teachers' Knowledge and Planning of Problem-Solving Instruction. *The Elementary School Journal*, 88(4), 369-386. <https://doi.org/10.1086/461545>
- Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.
- Carle, S. M. (1993). Student held misconceptions regarding area and perimeter of rectangles. *Critical and Creative Thinking Capstones Collection*, 46. http://scholarworks.umb.edu/cct_capstone/46
- Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136-143). Adelaide: Australian Association of Mathematics Teachers.
- Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. *Handbook of research on teaching*. New York: MacMillan
- Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from Lessons: Studying the Construction of Teacher Knowledge Catalysed by Purposefully-Designed Experimental Mathematics Lessons. *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165-172). Sunshine Coast: MERGA
- Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.
- Fauskanger, J. (2015). Challenges in measuring teachers' knowledge. *Educational Studies in Mathematics*, 90, 57-73. <https://doi.org/10.1007/s10649-015-9612-4>
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372-400. <https://www.jstor.org/stable/pdf/40539304>
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406. <https://doi.org/10.3102/00028312042002371>
- John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies*, 38(4), 483-498. <https://doi.org/10.1080/00220270500363620>
- Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience: 3rd Edition* (pp. 79-100). London: Routledge Taylor & Francis Group. <https://doi.org/10.4324/9780203844120>
- Korkmaz, H. I., & Şahin, Ö. (2019). Preservice Preschool Teachers' Pedagogical Content Knowledge on Geometric Shapes in Terms of Children's Mistakes. *Journal of Research in Childhood Education*, 34(3), 385-405. <https://doi.org/10.1080/02568543.2019.1701150>
- Kow, K., & Yeo, J. (2008). Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action. *Proceedings of the 31st Annual Conference of the Mathematics Education*

Research Group of Australasia.

- Kristanto, Y. D., Panuluh, A. H., & Atmajati, E. D. (2020). Development and validation of a test instrument to measure pre-service mathematics teachers' content knowledge and pedagogical content knowledge. *Journal of Physics: Conference Series*, 1470(1), 012008. <https://doi.org/10.1088/1742-6596/1470/1/012008>
- Lee, Y., Capraro, R. M., & Capraro, M. M. (2018). Mathematics Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge in Problem Posing. *International Electronic Journal of Mathematics Education*, 13(2), 75-90. <https://doi.org/10.12973/iejme/2698>
- Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1.*
- Özerem, A. (2012). Misconceptions In Geometry And Suggested Solutions For Seventh Grade Students. *Procedia - Social and Behavioral Sciences*, 55, 720-729. <https://doi.org/10.1016/j.sbspro.2012.09.557>
- Superfine, A. M. C. (2008). Planning for Mathematics Instruction: A Model of Experienced Teachers' Planning Processes in the Context of a Reform Mathematics Curriculum. *The Mathematics Educator*, 18(2), 11-22. <https://ojs01.galib.uga.edu/tme/article/view/1925/1830>
- Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical Content Knowledge of Mathematics Pre-service Teachers: Do they know their students? *Journal of Physics: Conference Series*, 1097(1), 012098. <https://doi.org/10.1088/1742-6596/1097/1/012098>
- Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*, 15(2), 4-14. <https://doi.org/10.3102/0013189X015002004>
- Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education*, 26(2), 114-145. <https://doi.org/10.2307/749205>
- Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., ... Rowley, G. (2012). *Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M)*. Amsterdam: International Association for the Evaluation of Educational Achievement.
- Turnuklu, E., & Yesildere, S. (2007). The Pedagogical Content Knowledge in Mathematics: Pre-Service Primary Mathematics Teachers' Perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, 1, 1-13.
- Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating Preservice Mathematics Teachers' Pedagogical Content Knowledge through Microteaching. *REDIMAT-Journal of Research in Mathematics Education*, 9(1), 62-87. <http://dx.doi.org/10.17583/redimat.2020.3353>
- Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.
- White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2012). Teachers learning from teachers. In *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13
- Widodo, & Tamimudin, M. (2014). Three Training Strategies for Improving Mathematics Teacher Competences in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Yogyakarta: Mathematics and Technology, LLC. Retrieved from <http://atcm.mathandtech.org/EP2014/index.html>

- Yeo, K. K. Y. (2008). Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 621-627). Brisbane: The University of Queensland. Retrieved from https://repository.nie.edu.sg/bitstream/10497/14397/1/MERGA-2008-621-YeoKK_a.pdf
- Yin, R. K. (2014). *Case study research: Design and methods*. Thousand Oaks: SAGE Publications.
- Yunianto, W. (2015). Supporting Students' Understanding of Area Measurement Through Verknippen Applet. *Southeast Asian Mathematics Education Journal*, 5(1), 73-82. <https://doi.org/10.46517/seamej.v5i1.34>
- Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, 6(2), 41–62. <https://doi.org/10.26220/rev.1627>

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[JME] Your article has been published Inbox x

Jme Fkip Matematika
to Wahid, me, Cosette, zulkardi

Fri, Mar 26, 2:03 PM

Dear Wahid Yudianto, Rully Charitas Indra Prahmana, and Cosette Crisan,

Congratulations!

After our editorial staff revised your paper based on your remark and finalization of your payment process, your article entitled **Indonesian Mathematics Teachers' Knowledge of Content and Students of Area and Perimeter of Rectangle** has just been published and is available at the following link:

<https://ejournal.unsri.ac.id/index.php/jme/article/view/13537>

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Thank you again for publishing with the Journal on Mathematics Education. We look forward to your future contributions.

Kind regards,
Editorial Office
Journal on Mathematics Education

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Crisan, Cosette <c.crisan@ucl.ac.uk>
to Wahid, me

Fri, Mar 26, 4:48 PM

Dear Wahid and Indra,

This is such good news!
Many thanks for persevering with the publication of this paper. I have added it to my research profile - very proud!

It would be great if we had a Zoom chat at some point? We are interested in encouraging MA students to publish their dissertation work - Wahid, if you have 40min or so, you could come and talk to our current cohort.

with best wishes, Cosette

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INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS OF AREA AND PERIMETER OF RECTANGLE

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Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. Research has shown teachers' competencies and skills influence students' performances. Previous studies explored teachers' knowledge through testing. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items or tasks to assess mathematics teacher knowledge in the previous studies were dominated by subject matter knowledge problems. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' Knowledge of Content and Students (KCS) through lesson plans developed by the teachers. To accommodate the gap in the previous studies, this study focuses on KCS on the topic of area-perimeter through their designed lesson plans. Twenty-nine mathematics teachers attended a professional development activity voluntarily participated in this study. Two teachers were selected to be the focus of this case study. Content analysis of the lesson plan and semi-structured interviews were conducted, and then data were analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible strategies and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Penelitian ini menunjukkan bahwa kompetensi dan keterampilan guru mempengaruhi performa siswa. Penelitian sebelumnya telah mengkaji pengetahuan guru melalui tes. Pengetahuan guru pada topik keliling-luas dan strategi pembelajaran juga telah dikaji melalui tes. Pada umumnya, banyaknya soal pada tes didominasi oleh soal-soal tentang pengetahuan subjek yang diajarkan. Oleh karena itu, asesmen seperti ini belum mencakup keseluruhan pengetahuan dan kompetensi guru. Pada studi ini, peneliti menginvestigasi pengetahuan guru matematika tentang KCS pada rencana pelaksanaan pembelajaran yang mereka kembangkan. Untuk mengakomodasi kesenjangan pada penelitian sebelumnya, penelitian kali ini berfokus pada pengetahuan tentang konten dan siswa (KCS) pada topik keliling-luas pada rencana pelaksanaan pembelajaran. Dua puluh Sembilan guru matematika yang sedang mengikuti pelatihan peningkatan kompetensi secara sukarela mengikuti penelitian ini. Dua guru matematika menjadi fokus penelitian studi kasus ini. Konten analisis dan interview semi terstruktur dilakukan dan datanya dianalisis. Terungkap bahwa peserta ini mengalami tantangan dalam memprediksi kemungkinan respon yang diberikan siswa. Mereka belum menyadari strategi siswa yang biasanya digunakan untuk menyelesaikan persoalan memaksimalkan luas dari keliling yang ditentukan. Dengan pengetahuan yang terbatas pada kemungkinan strategi siswa dan kesalahan siswa, guru ini kurang siap dalam mendukung siswanya

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

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Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research have shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2012, p.394).

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

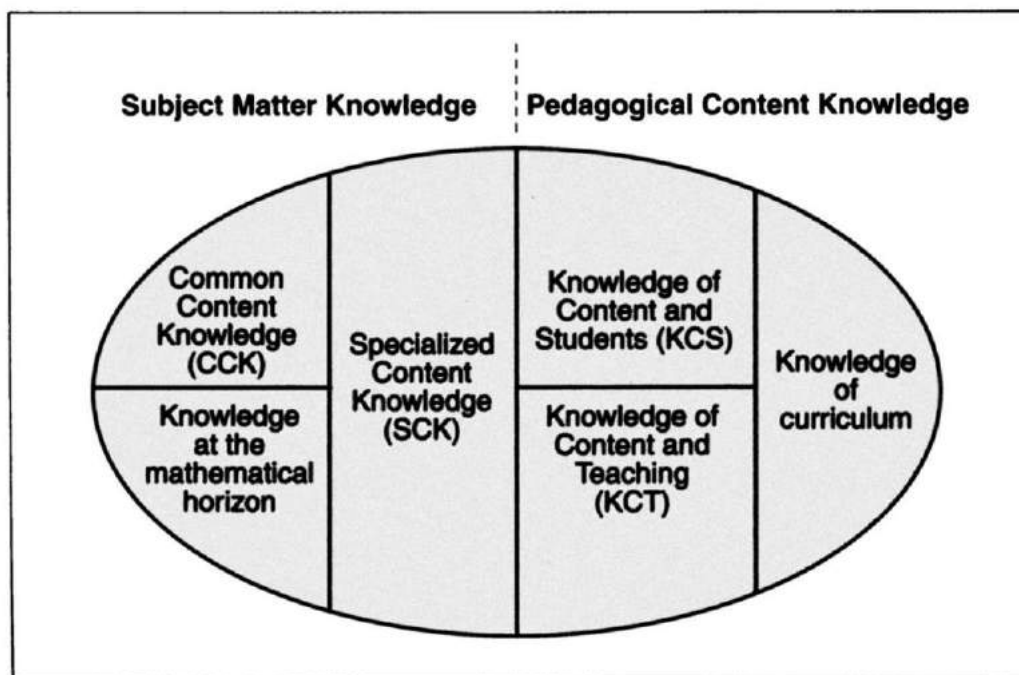


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). In the case of pre-service teachers, they have challenges with student's thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). It makes sense as they have limited teaching experiences or even have not taught

yet. For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COACTIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

As one of the ways, testing is used to assess teachers. The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin, 2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Another study using testing faced challenges in measuring teachers' knowledge (Fauskanger, 2015). An interesting finding of a study of pre-service teachers is that they possessed higher PCK scores than SMK from limited test items (Kristanto, Panuluh, & Atmajati, 2020). A case study in South Korea revealed that teachers with sufficient SMK of a certain competence/ topic faced challenges in incorporating KCS and KCT of that topic (Lee, Capraro, & Capraro, 2018). Therefore, testing to measure teachers' knowledge still face challenges.

Lesson plans are considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers can easily understand it when reading it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called, 'Bansho', which anticipates and tries to elicit student mathematical thinking and student thinking schema for solving the given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed that the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). The students' engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom practices are influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in [Figure 2](#).

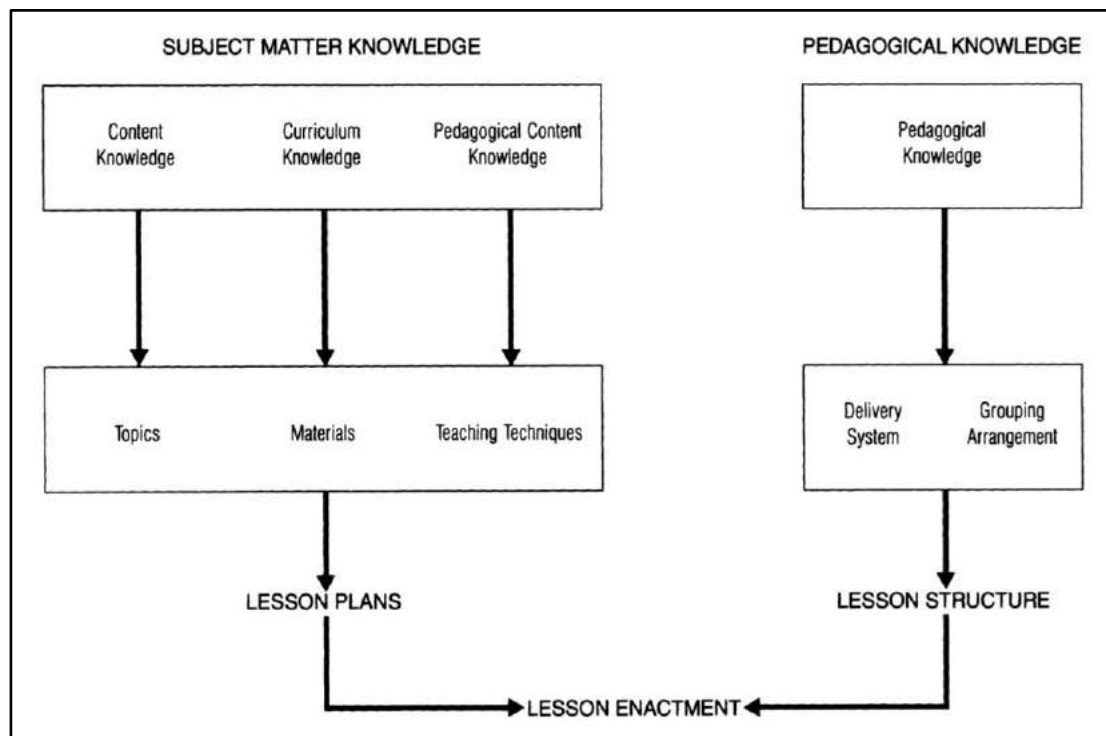


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zazahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013). It seems to make sense as they could manipulate and observe what changes happen by reshaping a figure (Yunianto, 2015).

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Thus, we should not expect teachers to deliver mathematics well if they do not have mastered it and do not understand how to teach it. Kow and Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason (1996) evaluated first-year teacher education student understanding of subject matter

knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Kow & Yeo, 2008).

This study intends to focus on a part of PCK, the KCS within lesson plans on the topic of area-perimeter of a rectangle. It is necessary to obtain a fuller insight into teacher knowledge as it influence students' performance. Beside testing, there might be alternative way such as lesson plans to investigate teachers' knowledge. How are mathematics teachers prepare their lesson plans and how is PCK integrated in their lesson plans? How are the KCS integrated in the lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

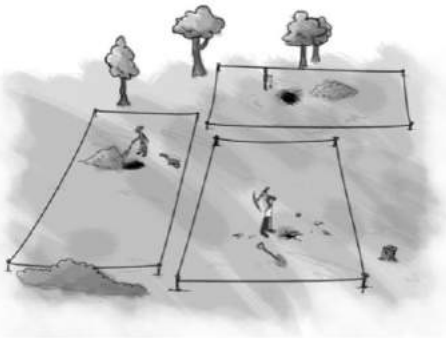
This research involved humans and has been approved by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk) as this is a part of completion of the first author's dissertation. This study administrated a case study approach. This approach suits this study as it does not seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). The research subjects were the mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan as part of the whole PD. It was done somewhere in the middle of all complete sessions. As it is a case study, the researchers examined two selected lesson plans of two mathematics teachers. The remaining lesson plans have not been analyzed due to time limitation. The sample was chosen from twenty-nine teachers who attended a professional development (PD) session, and two teachers were selected for the lesson plan analysis and interview. Additionally, these teachers were selected based on their teaching experience; at least five years. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The topic that would be taught was area and perimeter for grade 7. The "Gold Rush/Mining" task was selected. This task was chosen because it is a problem-solving task and has several ways to be solved on area-perimeter of a rectangle (see [Figure 3](#)). Additionally, the complete Gold Rush activity showed the mistakes that students might do. Thus, it is considered as a good activity to be explored to understand how teachers prepare this activity.


Gold Rush

In the 19th Century, many prospectors travelled to North America to search for gold.

A man named Dan Jackson owned some land where gold had been found.

Instead of digging for the gold himself, he rented plots of land to the prospectors.





Dan gave each prospector four wooden stakes and a rope measuring exactly 100 meters.

Each prospector had to use the stakes and the rope to mark off a rectangular plot of land.

1. Assuming each prospector would like to have the biggest plot, what should the dimensions of the plot be, once he places his stakes?
Explain your answer.

Figure 3. The Gold Rush problem (<https://www.map.mathshell.org/download.php?fileid=1637>)

To analyze the lesson plans, the researchers used content analysis. This method has the ‘potential to disclose many hidden aspects of what is being communicated through the written text’ (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers’ knowledge of students’ conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find themes by classifying instructions and KCS integrated in the lesson plans.

Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students’ emerging and incomplete thinking

By using Table 2, it is easy to differentiate instructions’ categories. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers

included information about what students would do to the task (KCS). The data were presented descriptively.

The two lesson plans were coded and analyzed. There were three types of instructions to refer to with the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?"	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3). The percentage is from type of instruction per total instructions written on the lesson plans.

Indonesian teachers follow the prescribed template of a lesson plan by MoEC. The template consists of three main parts namely; introduction, main and closure. It also consists learning goals and how teachers and students would do in the classroom.

Table 3. Proportions of the instructions

Instruction	T1	T2
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	23 (100%)	19 (100%)

Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2's SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	Introduction		Main		Closure	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	12	3	7	11	4	5

T1 put more details of what students would ask to her on her lesson plan. For instance: 'Can I solve it freely?' has been put on her lesson plan. This is a proof of PCK in the lesson plan, but not specific to KCS.

<p>❖ Main Activity 100 minutes</p> <p>PHASE: Organizing Students</p> <p>Students make up groups consisting of 4-5 students.</p> <p>➤ Observing</p> <p>After receiving the worksheet (problem), students observe the problem within their groups.</p> <p>➤ Questioning</p> <p>Students ask some questions related to the worksheet such as:</p> <ul style="list-style-type: none"> 👉 I still do not understand what the problem means. 👉 Can I solve it freely? <p>PHASE: Guiding the individual and group investigation</p> <p>➤ Gathering Information/ Data/ Trying out</p> <p>Students look for data and discuss the problem on the worksheet of Gold mining.</p> <p>➤ Reasoning/ Associating</p> <p>Students conclude the result of their discussion.</p> <p>PHASE: Developing and Presenting the result</p> <p>➤ Communicating</p> <p>Students communicate their result in written or oral presentation. One of the members of the group presents the result and other groups respond to him.</p> <p>❖ Closure (10 minutes)</p> <p>PHASE: Analysing and Evaluating the process of problem solving</p> <ol style="list-style-type: none"> 1. Teacher facilitates students to conclude what they have just learned) 2. Teacher facilitates students to identify the parts that they both understand and not 3. Teacher gives homework or assignment to students. 4. Teachers informs students that the next lesson would be about triangle.
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Figure 4. Teacher 1 Lesson Plan

In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4). On the phase of guiding the individual and group investigation which be rich of KCS. In this lesson plan, detail ways of students might solve it or make mistakes and how to facilitate it have not been depicted.

The T2 lesson plan of rectangle using Gold Rush task depicted detailed information about a possible student strategy (KCS). Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

<p>Main Activity</p> <ul style="list-style-type: none"> <input checked="" type="checkbox"/> Teacher divide students into groups <input checked="" type="checkbox"/> Teacher delivers the worksheet to be discussed <input checked="" type="checkbox"/> Teacher facilitates the learning processes ○ For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m. for instance, $p=10, l=... m$ $p = 15 m, l = ...m$, then the area = $p=15 m, l= ... m$ etc Students determine the largest area by themselves ○ For the second question, after students have solved the largest area for one miner, then how if it is for 2 miners? Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area? How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan of Gold Rush

After finding the largest area of the rectangle, students had to find the largest area by joining two miners' ropes and how would they join it. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

<ul style="list-style-type: none"> ○ Students evaluate and make generalisation into questioning. <ul style="list-style-type: none"> <input checked="" type="checkbox"/> Teacher asks students to present in front of the class <input checked="" type="checkbox"/> Other students respond the presenter
--

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or

which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Teachers T1 and T2 have more than five-years teaching experience each. Based on the questionnaire and interview, their schools are different in terms of location and students' background. These teachers themselves employed different abilities in solving the Gold Mining problem (Figure 3). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : Are there other ways T1?

T1 : Yesterday, I just did that one.

T1 : ...just let students find the ways to solve it Then, I will let them know that there are some ways to solve it. I give that opportunity to students

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

*T2 : To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*

R : So, you (T2), induce them by using the table?

T2 : Yes, by the table.

R : What do you think, how many ways to solve it?

*T2 : To me, I did one way I know it directly it would be a square. **I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and..., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

*T2 : I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*

T2 : However, I have thought only one strategy, which is global to solve a task. ... I, I... know at least I understand my students' characteristic so that it will be difficult for my students. ... It is not possible to come up if I let them to do it freely. ... I am so careful to give it the various strategies because students would get confuse

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

- T2 : By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.
- T2 : To me, I did one way I know it directly it would be a square. I knew it already
- T2 : ...instead of table, we can make the variable x , then I will be a quadratic function,
- R : Are there other ways to solve it?
- T2 : For the time being, not yet, making rectangles and to the square
- R : Do you think there are still other ways to solve that problem?
- T2 : I could use the graph ...

To some extent, from the lesson plan, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task (Figure 5). However, by giving the students the strategy, he inadvertently is making the students dependent on him. Whereas, from the lesson plan, T1 is helping the students to make decisions themselves (Figure 4). From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

- T1 : The task has missing information, it should be more, and some students would think that. So that they **have not thought** yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.
- T1 : 100. Maybe **they thought that** that's the only think they know.
- R : ... So, they would answer it 100, possibly
- T1 : Yeah, possibly
- T2 : ... for those who did not understand, **they would not know what 100 m rope is to with the perimeter**. So that the concept of perimeter, for those who understood, they already make it but later **they would not think** the rectangles can be varied.
- T2 : Students **would confuse** the meaning of maximum, which is the largest, **they have not thought about it**. So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7). The researcher proposed a possible mistake by a student of which the shape looks like a rectangle 25 x 26,5.

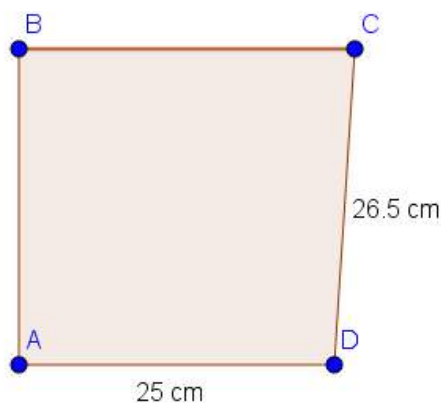


Figure 7. A student's possible mistake proposed by the researcher

If faced with a student mistake that they have not thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R : If it happens if you see this (showing)

T1 : I would ask students back to try it then you calculate it as what being asked to you

R : They have not yet known the result!

T1 : Try, try it, by trialing they would know that it is different, this one is more, and that one is like that,

R : T2, what if your students did this? what would you do?

T2 : I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...

In this study, the lesson plans facilitated an insight into teachers' knowledge. In this case, it showed teacher's pedagogical knowledge as well as PCK. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid, and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Kow & Yeo, 2008). In Japanese lesson plans, they contain more detailed instructions (Nakahara & Koyama, 2000) which shows more information about teachers knowledge. In contrast, the two case of teachers in this study, have not yet shown detailed instructions but more in general instruction.

Teachers have different ways of supporting students to solve tasks (Yeo, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task (possessing SMK and PCK). However, these results are not generalizable. The limited sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this

study might not show detail information on their lesson plans and have not fully been aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS of a specific topic through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter of rectangle topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared in facing the situations during teaching. Developing problem solving skills and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment. Additionally, this study explored a rectangle topic, the result might vary in different topics. Therefore, further investigation on different topic could be conducted. This study is not generalizable as it used limited research subjects.

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REFERENCES

- An, S., Kulm, G., & Wu, Z. (2004). The Pedagogical Content Knowledge of Middle School Mathematics Teachers in China and the U.S. *Journal of Mathematics Teacher Education*, 7, 145-172. <https://doi.org/10.1023/b:jmte.0000021943.35739.1c>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>
- Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, 31, 235-268. <https://doi.org/10.1007/BF00376322>
- Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In *Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers*:

Results from the COACTIV Project (pp. 25-48). Boston: Springer. https://doi.org/10.1007/978-1-4614-5149-5_2

- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., ... Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 133-180. <https://doi.org/10.3102/0002831209345157>
- Borko, H., Livingston, C., & Shavelson, R. J. (1990). Teachers' Thinking About Instruction. *Remedial and Special Education*, 11(6), 40-49. <https://doi.org/10.1177/074193259001100609>
- Burns, R. B., & Lash, A. A. (1988). Nine Seventh-Grade Teachers' Knowledge and Planning of Problem-Solving Instruction. *The Elementary School Journal*, 88(4), 369-386. <https://doi.org/10.1086/461545>
- Butt, G. (2008). *Lesson Planning 3rd Edition*. London: Bloomsbury Publishing.
- Carle, S. M. (1993). Student held misconceptions regarding area and perimeter of rectangles. *Critical and Creative Thinking Capstones Collection*, 46. http://scholarworks.umb.edu/cct_capstone/46
- Cavanagh, M. (2007). Year 7 students' understanding of area measurement. In K. Milton, H. Reeves, & T. Spencer (Eds.), *Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 136-143). Adelaide: Australian Association of Mathematics Teachers.
- Clark, C. M., & Peterson, P. L. (1986). Teachers' thought process. *Handbook of research on teaching*. New York: MacMillan
- Clarke, D., Clarke, D., Roche, A., & Chan, M. C. E. (2015). Learning from Lessons: Studying the Construction of Teacher Knowledge Catalysed by Purposefully-Designed Experimental Mathematics Lessons. *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165-172). Sunshine Coast: MERGA
- Denscombe, M. (2010). *The Good Research Guide For Small Scale Research Projects*. Berkshire: Open University Press.
- Fauskanger, J. (2015). Challenges in measuring teachers' knowledge. *Educational Studies in Mathematics*, 90, 57-73. <https://doi.org/10.1007/s10649-015-9612-4>
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372-400. <https://www.jstor.org/stable/pdf/40539304>
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406. <https://doi.org/10.3102/00028312042002371>
- John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies*, 38(4), 483-498. <https://doi.org/10.1080/00220270500363620>
- Jones, K., & Edwards, R. (2010). Planning for mathematics learning. In *Learning to Teach Mathematics in the Secondary School: A Companion to School Experience: 3rd Edition* (pp. 79-100). London: Routledge Taylor & Francis Group. <https://doi.org/10.4324/9780203844120>
- Korkmaz, H. I., & Şahin, Ö. (2019). Preservice Preschool Teachers' Pedagogical Content Knowledge on Geometric Shapes in Terms of Children's Mistakes. *Journal of Research in Childhood Education*, 34(3), 385-405. <https://doi.org/10.1080/02568543.2019.1701150>
- Kow, K., & Yeo, J. (2008). Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action. *Proceedings of the 31st Annual Conference of the Mathematics Education*

Research Group of Australasia.

- Kristanto, Y. D., Panuluh, A. H., & Atmajati, E. D. (2020). Development and validation of a test instrument to measure pre-service mathematics teachers' content knowledge and pedagogical content knowledge. *Journal of Physics: Conference Series*, 1470(1), 012008. <https://doi.org/10.1088/1742-6596/1470/1/012008>
- Lee, Y., Capraro, R. M., & Capraro, M. M. (2018). Mathematics Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge in Problem Posing. *International Electronic Journal of Mathematics Education*, 13(2), 75-90. <https://doi.org/10.12973/iejme/2698>
- Nakahara, T., & Koyama, M. (2000). *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (PME)(24th, Hiroshima, Japan, July 23-27, 2000), Volume 1.*
- Özerem, A. (2012). Misconceptions In Geometry And Suggested Solutions For Seventh Grade Students. *Procedia - Social and Behavioral Sciences*, 55, 720-729. <https://doi.org/10.1016/j.sbspro.2012.09.557>
- Superfine, A. M. C. (2008). Planning for Mathematics Instruction: A Model of Experienced Teachers' Planning Processes in the Context of a Reform Mathematics Curriculum. *The Mathematics Educator*, 18(2), 11-22. <https://ojs01.galib.uga.edu/tme/article/view/1925/1830>
- Setyaningrum, W., Mahmudi, A., & Murdanu. (2018). Pedagogical Content Knowledge of Mathematics Pre-service Teachers: Do they know their students? *Journal of Physics: Conference Series*, 1097(1), 012098. <https://doi.org/10.1088/1742-6596/1097/1/012098>
- Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*, 15(2), 4-14. <https://doi.org/10.3102/0013189X015002004>
- Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education*, 26(2), 114-145. <https://doi.org/10.2307/749205>
- Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., ... Rowley, G. (2012). *Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M-M)*. Amsterdam: International Association for the Evaluation of Educational Achievement.
- Turnuklu, E., & Yesildere, S. (2007). The Pedagogical Content Knowledge in Mathematics: Pre-Service Primary Mathematics Teachers' Perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, 1, 1-13.
- Ünver, S. K., Özgür, Z., & Güzel, E. B. (2020). Investigating Preservice Mathematics Teachers' Pedagogical Content Knowledge through Microteaching. *REDIMAT-Journal of Research in Mathematics Education*, 9(1), 62-87. <http://dx.doi.org/10.17583/redimat.2020.3353>
- Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based Guidance for Ages 9-19* (1st ed.). Oxford: Oxford University Press.
- White, A. L., Jaworski, B., Agudelo-Valderrama, C., & Gooya, Z. (2012). Teachers learning from teachers. In *Third International Handbook of Mathematics Education* (pp. 393-430). New York: Springer. https://doi.org/10.1007/978-1-4614-4684-2_13
- Widodo, & Tamimudin, M. (2014). Three Training Strategies for Improving Mathematics Teacher Competences in Indonesia. *Electronic Proceedings of the 19th Asian Technology Conference in Mathematics*. Yogyakarta: Mathematics and Technology, LLC. Retrieved from <http://atcm.mathandtech.org/EP2014/index.html>

- Yeo, K. K. Y. (2008). Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 621-627). Brisbane: The University of Queensland. Retrieved from https://repository.nie.edu.sg/bitstream/10497/14397/1/MERGA-2008-621-YeoKK_a.pdf
- Yin, R. K. (2014). *Case study research: Design and methods*. Thousand Oaks: SAGE Publications.
- Yunianto, W. (2015). Supporting Students' Understanding of Area Measurement Through Verknippen Applet. *Southeast Asian Mathematics Education Journal*, 5(1), 73-82. <https://doi.org/10.46517/seamej.v5i1.34>
- Zacahros, K., & Chassapis, D. (2012). Teaching suggestions for the measurement of area in Elementary School. Measurement tools and measurement strategies. *Review of Science, Mathematics and ICT Education*, 6(2), 41–62. <https://doi.org/10.26220/rev.1627>

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Indonesian mathematics teachers' knowledge of content and students of area and perimeter of rectangle (Article) [\(Open Access\)](#)

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Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. Research has shown teachers' competencies and skills influence students' performances. Previous studies explored teachers' knowledge through testing. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items or tasks to assess mathematics teacher knowledge in the previous studies were dominated by subject matter knowledge problems. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' Knowledge of Content and Students (KCS) through lesson plans developed by the teachers. To accommodate the gap in the previous studies, this study focuses on KCS on the topic of area-perimeter through their designed lesson plans. Twenty-nine mathematics teachers attended a professional development activity voluntarily participated in this study. Two teachers were selected to be the focus of this case study. Content analysis of the lesson plan and semi-structured interviews were conducted, and then data were analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible strategies and mistakes, these teachers were poorly prepared to support student learning. © 2021 Sriwijaya University. All rights reserved.

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- 1 An, S., Kulm, G., Wu, Z.
The Pedagogical Content Knowledge of Middle School, Mathematics Teachers in China and the U.S (2004) *Journal of Mathematics Teacher Education*, 7, pp. 145-172. Cited 155 times.
<https://doi.org/10.1023/b:jmte.0000021943.35739.1c>
- 2 Loewenberg Ball, D., Thames, M.H., Phelps, G.
[Content knowledge for teaching: What makes it special?](#) [\(Open Access\)](#)
(2008) *Journal of Teacher Education*, 59 (5), pp. 389-407. Cited 1747 times.
doi: 10.1177/0022487108324554
[View at Publisher](#)
- 3 Baturu, A., Nason, R.
[Student teachers' subject matter knowledge within the domain of area measurement](#)
(1996) *Educational Studies in Mathematics*, 31 (3), pp. 235-268. Cited 65 times.
www.kluweronline.com/issn/0013-1954/current
doi: 10.1007/BF00376322
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