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A proton-neutron QRPA Calculation of the β^- Decay of Nd^{152} using the Fayans Energy Density Functional

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Abstract. This paper presents a calculation of the β^- decay of the neutron-rich nucleus Nd^{152} using the proton-neutron Quasi-particle Random Phase Approximation (pnQRPA) model. The single-particle ground states are calculated self-consistently using the Fayans energy density functional in a Hartree-Fock-Bogoliubov (HFB) scheme for deformed nuclei. The quasi-particle states which are needed for the pnQRPA calculation are calculated from the HFB single-particle ground states using the Bardeen-Cooper-Schriefer (BCS) approximation. Only Gamow-Teller β^- excitations are considered, and two kinds of excitation force are used to excite the daughter nucleus to produce the β^- decay: (a) a pure Migdal force, and (b) a Migdal force plus a correction term obtained self-consistently from the BCS quasi-particle basis. The calculations with the pure Migdal force and the corrected Migdal force give half-lives of 5.92 min and 6.91 min, respectively, which are about one half of the experimental value of 11.6(7) min, and decay energies of 1.38 MeV and 1.33 MeV, respectively, both close to the experimental decay energy of 1.12 MeV. The correction to the Migdal excitation force turns out to improve the accuracy of the calculations.

1. Introduction

The theoretical study of nuclear beta decay has various applications in a number of branches of physics, including nuclear physics, astrophysics and reactor physics. To give an example in astrophysics, the *r*-process, though to be responsible for the origin of about half of the elements heavier than iron in the universe, is a series of neutron captures competing with β decays taking place in extreme neutron-rich stellar environments [1]. As an example in reactor physics, the so-called delayed neutrons play an important role in the safe control of nuclear reactors, due to the delay which takes place between a neutron-induced fission event and their eventual emission [2,3,4].

The theory of beta decay was first formulated by Enrico Fermi in 1933, which explains beta decay of a neutron by direct coupling of a neutron with an electron, a neutrino (later determined to be an antineutrino) and a proton [5]. In a nuclear beta decay calculation one needs to use a complicated many-body approach, and all existing approaches for the calculation of nuclear beta decay can be considered to fall into three basic methods. The first method is the gross theory which was first developed Takahashi and Yamada in 1969 [5,6], and it is basically a statistical theory (not a microscopic theory) and not a consistent theory since the Q value is obtained from an empirical mass formula. The second method is the shell model which was first developed Brown and Wildenthal in

1985 [7], and quite contrary to the gross model, the shell model is a fully microscopic theory which can explain the detailed states of the decay strength. The main limitation of the shell model is that its application is limited to light nuclei, whereas many nuclei of interest, e. g. in nuclear astrophysics, have large masses. An interesting compromise between the two methods is the so-called proton-neutron Quasi-particle Random Phase Approximation (pnQRPA) which was developed in 1967 by Halbleib and Sorensen [8], since it is a microscopic theory, i. e. based on the basic interaction between nucleons, so reliable predictions may be expected in yet unobserved regions of the nuclear chart, but it is also more practical than the shell model because of the smaller configuration space involved. However, the pnQRPA method lacks accuracy. To give a specific example, Staudt et al. [9] using the so-called second-generation microscopic pnQRPA was successful in reproducing half-lives for almost all known neutron-rich nuclei with half-lives shorter than 60 s within a factor of 10, but it used a simple separable Gamow-Teller force, which is not satisfactory from the theoretical point of view.

To get reliable predictions in "exotic" nuclear regions of current interest, e.g. in superheavy and extremely superheavy nuclei [10], it is desirable to use nuclear self-consistent single-particle energy states, since no outer mass parameters are needed. For this purpose, an energy density functional is required. The Fayans energy density functional for spherical nuclei [11,12] has been extended to the self-consistent calculation of ground state properties in deformed nuclei using cylindrical coordinates [13], treating the pairing in the general Hartree-Fock-Bogoliubov (HFB) approach [14]. The Fayans energy density functional has recently gained attention for the global description of nuclear charge radii [15] and to calculate alpha decay energies of superheavy nuclei [16].

Besides a single-particle basis, a residual interaction is also needed in a beta decay calculation to obtain the matrix elements of the charge-changing transition operator in the QRPA basis. As a consequence, the accuracy of a beta decay calculation in a nucleus is obviously governed by the accuracy of the parent nucleus (ground state) as well as the daughter nucleus (excited state). The self-consistent deformed nuclear ground states obtained by the HFB method using the Fayans energy density functional [13] have been applied by Borzov et al. [17] to the calculation of beta decay half-lives in spherical nuclei using the pnQRPA method, where the pairing in the ground state calculation has been treated by taking only diagonal matrix elements, an approach called the Bardeen-Cooper-Schriefer (BCS) model. Subsequently in an attempt similar to the work of Borzov et al. [17], the author [18] calculated the β^- decay of the neutron-rich nucleus Dy^{168} using the pnQRPA model, where the single-particle ground states were also calculated self-consistently using the Fayans energy density functional in a HFB scheme for deformed nuclei [13], and the pnQRPA quasi-particle states were built on the HFB single-particle ground states using the BCS approximation, however the subnucleonic interaction was neglected and a different excitation force, the Migdal force [19] was used, and only Gamow-Teller β^- excitations were considered.

To investigate further the use of a Gamow-Teller force for the microscopic description of beta decay in deformed nuclei, the present paper reports a β^- decay calculation using the same approach as Ref. [18], but now two kinds of excitation force are used to excite the daughter nucleus: (a) a pure Migdal force [19], and (b) a Migdal force plus a correction term obtained self-consistently from the BCS quasi-particle basis. As in Ref. [18], the subnucleonic excitation is neglected since we deal only with low energy excitations in the order of 1 MeV. The β^- decay from Nd^{152} to Pm^{152} is attractive since it contains, among others, Gamow-Teller transitions, with an overall decay energy of 1.12 MeV and a half-life of 11.6(7) min, as can be seen from Figure 1 [20,21]. It shows at least two Gamow-Teller $0^+ \rightarrow 1^+$ transitions: one transition to the daughter ground state and another transition to an excited state at 0.2946 MeV above the daughter ground state.

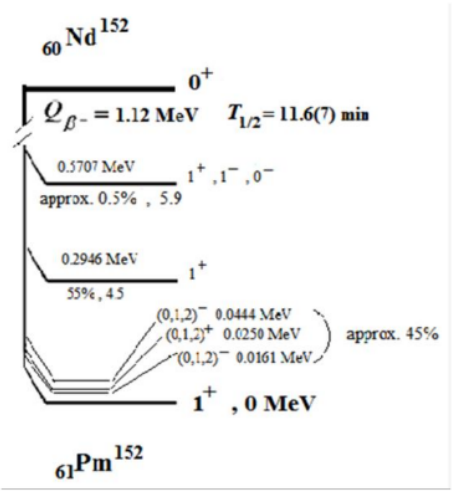

Figure 1. Experimental β^- decay of Nd^{152} to Pm^{152}. The percentages show intensities of different transitions and each number next to intensity shows the corresponding $\log ft$ value.

2. Theory

2.1. The Fayans energy density functional

The Fayans energy density functional is a functional of the particle normal density ρ and the anomalous density (ν,ν^+), such that the nuclear total energy can be expressed as

$$E\left[\rho,\nu,\nu^+\right]=\int d^3\boldsymbol{r}\{\varepsilon_{\text{kin}}+\varepsilon_{\text{vol}}+\varepsilon_{\text{sur}}+\varepsilon_{\text{coul}}+\varepsilon_{\text{so}}+\varepsilon_{\text{pair}}\}. \tag{1}$$

The first term in the integrand is kinetic energy density, while the second term, ε_{vol} and the third, ε_{sur} are the volume and surface energy densities, respectively. The first three terms are known already in the liquid drop model [22]. The last three terms are relatively small corrections due to the Coulomb, spinorbit and pairing interactions, respectively. The Fayans energy density functional contains Fermi nuclear matter parameters, the sum and the difference between proton and neutron relative densities

$$x_{\pm}=\frac{\rho^{\text{n}}\pm\rho^{\text{p}}}{2\rho_0}, \tag{2}$$

It also contains functions and differentials of the sum and the difference between proton and neutron relative densities in equation (2)

$$f_{\pm}=\frac{1-h_{1\pm}x_{\pm}}{1+h_{2\pm}x_{\pm}}, \qquad f'=\frac{d}{dx_{+}}f(x_{+}), \tag{3}$$

where $h_{1\pm}$ and $h_{2\pm}$ are parameters.

The single-particle potential and the strength of the two-nucleon particle-hole (p-h) interaction may be calculated as the differential of the interaction energy density (the sum of all energy density terms except kinetic energy density) with respect to the particle density ρ, while the particle-particle (p-p) interaction may be calculated as the differential of the pairing energy with respect to pairing

(anomalous) densities (ν,ν^{+}). The (two-nucleon) charge exchange interaction between a proton and a neutron can be calculated as the differential of the sum of surface energy and volume energy with respect to the difference $\rho_{-}=\rho^{\mathrm{n}}-\rho^{\mathrm{p}}$ [11].

2.2. Self-consistent ground state calculation

2.2.1. The pairing model

Before the introduction of pairing models such as the BCS model, the nuclear Hartree-Fock (HF) model has been used to calculate nuclear single-particle energies without taking into account pairing between nucleons. The nuclear BCS model is the first theory which introduces the idea of pairing to explain some empirical findings in nuclei which can not be described by the HF model, e.g. the large energy gaps in even-even nuclei [23].

In the BCS model, we can consider a constant pairing force G working on N_2-N_1+1 single-particle levels (each with double spin degeneracy) occupied by N_{int} nucleons. The level indices k fulfill $k=N_1,N_1+1,\cdots,N_2$.The pairing gap parameters Δ_k and the Fermi level ε_F are determined from two simultaneous nonlinear equations

$$N_{tot}=2\sum_{k=N_1}^{N_2}{v_k}^2+2(N_1-1), \tag{4}$$

$$\frac{2}{G}=\sum_{k=N_1}^{N_2}\frac{1}{\sqrt{(\varepsilon_k-\varepsilon_F)^2+{\Delta_k}^2}}, \tag{5}$$

where ε_k are the single-particle energies. The numbers u_k,v_k fulfill the normalization condition ${u_k}^2+{v_k}^2=1$, and ${v_k}^2$ is the occupation probability of level k. The quasi-partikel energies E_k are

$$E_k=\sqrt{(\varepsilon_k-\varepsilon_F)^2+{\Delta_k}^2},\quad k=N_1,N_1+1,\cdots,N_2\ . \tag{6}$$

Note that there is a different pairing gap Δ_k for each single-particle level k, so a constant pairing gap Δ can be considered as an average of the Δ_k values over all levels.

The BCS model has been frequently used in many nuclear structure calculations; however, it has a disadvantage that for large single-particle energy intervals near the Fermi level there is no nontrivial solution. By taking into account effects associated with particle number fluctuations, the so-called Lipkin-Nogami model can improve the BCS model and avoid such a “collapse” [24]. The Lipkin-Nogami model introduces a new parameter called the number fluctuation constant, λ_2, and it is the quantity

$$\Delta_{\mathrm{LN}}(k)=\Delta_k+\lambda_2, \tag{7}$$

rather than the BCS pairing gap parameter Δ_k in equation (4), which is associated with the so-called odd-even mass difference effect. The quantity $\Delta_{\mathrm{LN}}(k)$ in equation (7) is called Lipkin-Nogami pairing gap parameter.

2.2.2. The HFB model

The HFB model generalizes and unites the HF and the BCS models [14] such that the Hamiltonian reduces to two average potentials. The first average potential is the self-consistent field

Γ, which is known already in the HF theory, and contains all long-range p-h correlations which eventually lead to a deformed ground state. The second potential is a pairing field, Δ, which is known in the BCS model. Although in HFB theory the self-consistent potential Γ (and also $h = t + \Gamma$) is not diagonal, it is useful to define single-particle energies by

$$\varepsilon_k = h_{k\bar{k}}. \tag{8}$$

Similarly one can define single-particle energy gap parameters by

$$\Delta_k = \Delta_{k\bar{k}}, \tag{9}$$

such that we can define an average pairing gap in the neighborhood of the Fermi level by taking the mean of Δ_k values directly above the Fermi level and below the Fermi level,

$$\Delta_F = \frac{\Delta_{F+1} + \Delta_{F-1}}{2}. \tag{10}$$

This average gap parameter is used in the present study to be compared with the average Lipkin-Nogami, Δ_{LN} in equation (7) as reference gap parameter.

The HFB ground state eigenfunctions are obtained by solving the HFB equations

$$\begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}, \tag{11}$$

where $h = \varepsilon + \Gamma - \lambda$, and E_k are the HFB quasi-particle energies, and the (sub)matrices **U** and **V** now define uniquely the HFB quasi-particle operator.

The simple constant zero-range force was used in the form

$$F_{\text{p-p}}^{\text{p/n}} = -C_0 f_{\text{p/n}} \delta(\vec{r}_1 - \vec{r}_2), \tag{12}$$

where C_0, f_p and f_n are a force constant, proton-proton p-p pairing force strength parameter and neutron-neutron p-p pairing force strength parameter, respectively (all taken to be positive).

2.3. Beta decay calculation

The daughter nuclear states can be obtained from the parent nuclear states using an excitation by a charge exchange force. The following describes the pnQRPA method for an even-even parent nucleus, assuming only Gamow-Teller β^- transitions. The selection rules for a Gamow-Teller transition are that the magnitude of the change in spin and isospin must equal 0 or 1 (not $0 \rightarrow 0$), and that the nuclear parity must be conserved. The matrix elements of the nuclear Gamow-Teller transitions for a β^- decay from an initial nuclear state $|i\rangle$ to the final state $|f\rangle$ may be expressed as

$$M_{fi}^{-} = \langle f | \sum_{j=1}^{A} \vec{\sigma}(j) \vec{\tau}^{-}(j) | i \rangle. \tag{13}$$

In spherical coordinates, the Gamow-Teller matrix elements may be resolved into a geometrical factor and a reduced matrix element

$$M_{fi}^{-} = -c_A(J_i M_{i1}\mu \mid J_f M_f)\frac{\langle f \mid \sum_{j=1}^{A}\vec{\sigma}(j)\vec{\tau}^{-}(j) \mid i\rangle}{\sqrt{2J_i+1}}, \quad (14)$$

where the axial vector renormalization constant is taken to be $c_A = 1.26$ [25]. The parent nuclear spin is $J_i = 0$ in this study. The reduced transition probability B_{GT} is defined as

$$B_{GT} = \frac{|M_{fi}^{-}|^2}{2J_i+1}. \quad (15)$$

It is to calculate the reduced transition probability in equation (15) that the pnQRPA method is needed. The β^- decay total half-life $T_{1/2}$ may be obtained by summing over all n energetically allowed transitions to final nuclear states $|n\rangle$,

$$\frac{1}{T_{1/2}} = \sum_n \frac{1}{t_n} = \frac{c_A^2}{D}\sum_n f_n B_{GT}(n), \quad (16)$$

where $D = 6146.6 \pm 7$ s [26]. In equation (16), f_n is the corrected Fermi integral for the n-th Gamow-Teller transition and can be obtained from a $\log f$ table for axial transitions [27] and a vector transition correction factor [28]

$$\frac{f_V}{f_A} = 1 - \frac{2}{15}W_0 R\alpha Z - \frac{4}{105}(W_0 R_\alpha)^2, \quad (17)$$

where W_0 and R are the maximum energy release and the nuclear radius (in natural units), respectively, and $\alpha = 1/137$. The sum of all reduced transition probabilities is given by the Ikeda sum rule [29]

$$S_\beta^- - S_\beta^+ \equiv \sum_f B_{GT}^- - \sum_{f'} B_{GT}^+ = 3(N_i - Z_i), \quad (18)$$

which for very neutron-rich nuclei ($S_\beta^+ \approx 0$) can be approximated by

$$S_\beta^- - S_\beta^+ \approx S_\beta^- = 3(N_i - Z_i), \quad (19)$$

where N_i and Z_i are the neutron number and atomic number of the parent nucleus, respectively. For the nucleus Nd^{152} with $Z = 60$ and $N = 92$, we have the full Ikeda sum rule equal to $S_\beta^- - S_\beta^+ \approx 96$. In the pnQRPA model, quasi-particle operators α, α^+ are used and the excitation from the parent nucleus (ground state) to the daughter nucleus (excited state) is mediated through the creation of pnQRPA phonons [17]

$$A^+{}_\omega = \sum_{pn}\left(X^{pn}{}_{\omega\mu}\alpha^+{}_p\alpha^+{}_{\bar{n}} - Y^{pn}{}_{\omega\mu}\alpha_p\alpha_{\bar{n}}\right). \quad (20)$$

where X and Y are called forward (p-h) and backward (h-p) amplitudes, respectively, ω are the phonon energies and μ the phonon multipolarity (equal to 1^+ since we deal with Gamow-Teller transitions).

The present work uses a BCS+pnQRPA model, where the pnQRPA matrix equation has the following form

$$\begin{pmatrix} A & B \\ B* & A* \end{pmatrix}\begin{pmatrix} X_\nu \\ Y_\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} X_\nu \\ -Y_\nu \end{pmatrix}, \tag{21}$$

where ω_ν is the ν-th eigenvalue of the excitation energy, and the matrix elements of the matrices **A** and **B** *(k<k', l<l')* are calculated on the BCS basis. The p-h force is taken to be a Migdal force

$$F_{\text{p-h}} = C_0\delta(\vec{r}_1 - \vec{r}_2)\{f + g'\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\tau}_1\cdot\vec{\tau}_2\}, \tag{22}$$

where g' is the Gamow-Teller parameter, and C_0 a force constant. The parameter f depends on density, while g' is a density-independent parameter. Another force, a "corrected" Migdal force is used by combining the Migdal force in equation (22) with a p-h term taken self-consistently from the Fayans energy density functional,

$$F_{\text{p-h}} = F_{sc} + C_0 g'\delta(\vec{r}_1 - \vec{r}_2)\ \vec{\sigma}_1\cdot\vec{\sigma}_2\ \vec{\tau}_1\cdot\vec{\tau}_2\,. \tag{23}$$

The idea to add the self-consistent force in equation (23) comes from the fact that the Fayans energy density functional actually lacks terms involving spin-exchange, which is just the unique property of Gamow-Teller transitions, since its expectation value vanishes for an even-even (zero-spin) nucleus [12]. The first term in Equation (23), F_{sc} is the self-consistent p-h force which can be obtained by differentiating the interaction energy in the Fayans energy density functional (2) with respect to the normal density, ρ, whereas the second term, which is not self-consistent, is required because its expectation value at ground state is equal to zero. In the p-p channel, the constant proton-neutron pairing of zero range is used

$$F_{\text{p-p}} = -C_0 f_{\text{p-p}}\delta(\vec{r}_1 - \vec{r}_2)\,, \tag{24}$$

where $f_{\text{p-p}}$ is the proton-neutron p-p force strength parameter, but the choice of this parameter does not affect significantly the result of decay calculation. The value of $f_{\text{p-p}}$ is taken as the average of the proton-proton p-p force strength parameter, f_p and the neutron-neutron p-p force strength parameter, f_n. The QRPA matrix equation (21) is solved to obtain the excitation energy eigenvalues ω(i) and Gamow-Teller transition amplitudes. A partial decay energy for the i-th transition, $Q_\beta(i)$ is calculated from

$$Q_\beta(i) = \lambda_{\text{n}} - \lambda_{\text{p}} + \Delta m_{\text{nH}} - \omega(i)\,, \tag{25}$$

where $\Delta m_{\text{nH}} = 0.782$ MeV, and $\lambda_{\text{n}} - \lambda_{\text{p}}$ is the difference of chemical potentials of neutrons and protons. Only transitions with positive energies are taken into account.

3. Some Numerical Details

In the present work, the calculation of HFB quasi-particle basis is performed iteratively together with the calculation of single-particle basis until a self-consistency or convergence is achieved. The self-consistent calculation of the ground state is based on the MIX code [13] using the following Fayans energy-density functional parameter set

$$
\begin{aligned}
&a_{+}^{v} = -6{,}42, \quad h_{1+}^{v} = 0{,}163, \quad h_{2+}^{v} = 0{,}724,\\
&a_{-}^{v} = 5{,}42\ , \quad h_{1-}^{v} = 0{,}0, \quad h_{2-}^{v} = 3{,}0\ ,\\
&a_{+}^{s} = -11.1\ , \quad h_{+}^{s} = 0.0\ ,\\
&a_{-}^{v} = -4.0\ , \quad h_{-}^{s} = 0.0\ ,\\
&r_0 = 1.147\ \text{fm}, R = 0.35\ \text{fm},\\
&\kappa_{pp} = 0.285, \quad \kappa_{\text{pn}} = 0.135.
\end{aligned}
\tag{26}
$$

The proton-proton (f_p) and the neutron-neutron (f_n) p-p pairing force parameters for the ground state calculation, which also leads to the proton-neutron QRPA force strength parameter $f_{\text{p-p}}$ in equation (24), are so adjusted to produce a pairing gap parameter around the Fermi level (equation 10) which is as close as possible to the average Lipkin-Nogami gap parameter, Δ_{LN} as tabulated in Ref. [30].

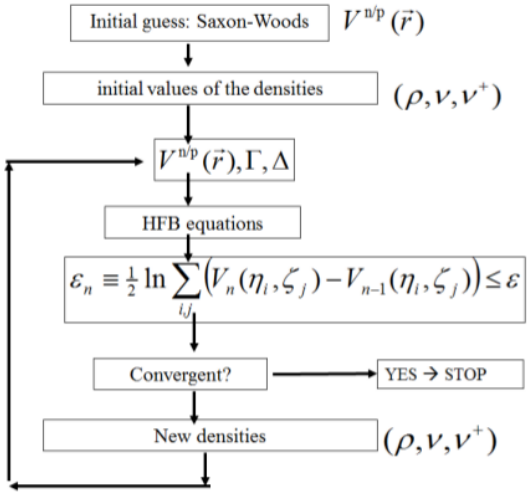

Figure 2. Iteration loop in the self-consistent ground state calculation.

As shown in Figure 2, the iteration to obtain a self-consistent solution of the HFB equations (11) starts by taking the Saxon-Woods single-particle potential as an initial guess for $V^{\text{n/p}}(\vec{r})$, to give the initial values of the densities (ρ,ν,ν^{+}), which in turn is used for the calculation of the next approximation to the single-particle potential $V^{\text{n/p}}(\vec{r})$, pairing force in the p-h and p-p channels and the HFB matrix. The diagonalisation of the HFB matrix gives HFB quasi-particle energy eigenvectors and eigenvalues. Next, the single-particle Hamiltonian using the single-particle potential $V^{\text{n/p}}(\boldsymbol{r})$ is diagonalized to obtain new densities (ρ,ν,ν^{+}), and so on. The iteration is stopped when the sum of the single-particle potential differences at all quadrature points between two successive iterations, e_n is equal or smaller than a certain small value ε,

$$
e_n \equiv \tfrac{1}{2}\ln\sum_{ij}\left(V_n(\eta_i,\zeta_j)-V_{n-1}(\eta_i,\zeta_j)\right)\le e,
\tag{27}
$$

where $V_n(\eta_i,\zeta_j)$ is the single-particle potential in cylindrical coordinates in the n-th iteration calculated at the (i,j) quadrature point, and e is small number. The maximum number of iteration steps allowed is 150, and e in equation (27) is taken equal to 0.1.

In the pnQRPA calculation, large matrices are involved, therefore two simplifications are performed. First, only the diagonal pairing matrix elements are taken in the ground state calculation, and a calculation of quasi-particle levels are performed in the BCS approximation by solving equations (4) and (5). Second, a selection of single-particle matrix elements in the ground state calculation is performed, and yet the theoretical Ikeda Gamow-Teller sum rule in equation (19) is fulfilled as close as possible. Using $\hbar\omega_0 = 41A^{-1/3}$, the matrix elements are selected in two steps: all matrix elements whose pair components have single-particle energies within $\varepsilon_F \pm \hbar\omega_0$ are included, and only the largest matrix elements are included so that the total number of selected matrix elements does not exceed 900.

4. Results and Discussion

The self-consistent HFB ground state calculation is convergent after 57 iterations, and the calculated gap parameters and nuclear binding energy are shown in Table 1. The reference Lipkin-Nogami gap parameters in Nd^{152} for proton and neutron are 1.12 and 0.92 MeV, respectively, and Table 1 shows that the gap parameters are reproduced correctly corresponding to $f_p = -0.737$, $f_n = -0.674$. The calculated binding energy is 1243 MeV, while the most recent experimental data is 8224.01 ± 0.16 keV per nucleon, corresponding to total binding energy of 1250.050 ± 0.024 MeV [31], so the calculated binding energy agrees well within experimental error with the experimental value at a deviation of about 0.6 per cent.

Table 1. The calculated gap parameters and nuclear binding energy.

Gap parameters (MeV)		Binding energy, E_b (MeV)	
Δ_p	Δ_n	Calc.	Exp.
1.120	0.923	1243	1250.050 ± 0.024[33]

The β^- decay calculations using the Migdal and corrected Migdal forces succeeds in producing the correct order of magnitude of the important transition probabilities, but the position of the energy levels in the daughter nucleus undergoing these transitions are shifted in some cases relative to the observed levels. As shown in Table 2, the calculations with the pure Migdal force and the corrected Migdal force give half-lives of 5.92 min and 6.91 min, respectively, which are about one half of the experimental value of 11.6(7) min, while the calculated decay energies are 1.38 MeV and 1.33 MeV, respectively. Ref. [20] gives an experimental decay energy of 1.12 MeV, while the most recent value is 1.105 ± 0.019 MeV [31], so the calculated decay energies using both forces are higher than the experimental energies with a deviation of about 23-24 %. It can be concluded that the correction to the Migdal excitation force improves the accuracy of the calculations.

Table 2. The calculated half-lives and decay energies using the Migdal and corrected Migdal excitation forces as compared to experimental data.

Half-life (min)			Decay energy (MeV)		
Migdal	Corrected Migdal	Exp.	Migdal	Corrected Migdal	Exp.
5.92	6.91	11.6(7)	1.38	1.33	1.12

The reduced transition probability B_{GT} for each transision n may also be obtained from experimental log ft data using $B_{GT}(n) = D/(c_A^2 ft_n)$ [27], and the calculated values corresponding to the Gamow-Teller transitions in Figure 1 are shown in Table 3. The first column is the index for the QRPA excitation energy, and the second column gives the QRPA excitation energy relative to the lowest excitation energy.

Table 3. Experimental β^- Gamow-Teller transition probabilities.

i	E_x (MeV)	Q_β (MeV)	Intensity (%)	Log ft	B_{GT}
1	0	1.12 [a]	≤ 45	-	-
2	0.0250	1.095 [b]	-	-	-
3	0.2946	0.8254	55	4.5	0.122
4	0.5707	0.5493	≈ 0.5	5.9	0.005

[a] according to Ref. [20], or 1.105 ± 0.019 MeV according to Ref. [31]

[b] uncertain spin-parity (I^π) designation

The large discrepancy between the calculated half-lives and the experimental half-life may be explained by considering the calculated (lower pane) and experimental (upper pane) reduced Gamow-Teller transition intensities, B_{GT} in Figure 3. The BCS+pnQRPA model using the Migdal and corrected Migdal excitation force succeed in producing the correct order of magnitude of the important transition probabilities, i.e. the lowest three levels, but the position of the second transition is shifted upwards by about 0.2 MeV relative to the observed levels. Both Migdal and corrected Migdal calculations underestimate the transition probability by a factor of 6. Both calculations also produce additional transitions with minor probabilities at higher excitation energies (starting from i = 4) which are not observed in the experimental data.

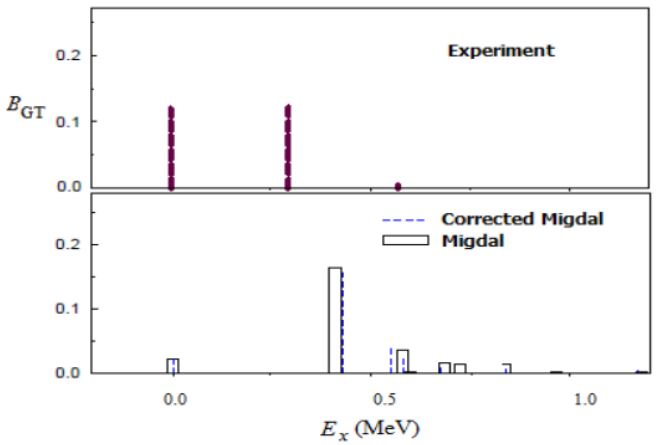

Figure 3. Calculated (lower pane) and experimental (upper pane) reduced Gamow-Teller transition intensities.

It is interesting to note that the experimental total decay energy is produced accurately by the present calculations but not the total decay half-life. This can be explained by noting that the total decay energy is only determined by the final and initial nuclear states as reflected by equation (25), while it is the reciprocal of the total decay half-life and not the total decay half-life itself which adds

algebraically, i.e. the additional transitions produced by the present calculations tend to make the total decay half-life significantly smaller than the experimental value.

To check whether the Ikeda sum rule equation (19) is fulfilled, the Gamow-Teller strength function, i.e. the running sum of B_{GT} values over excitation energies E_x has also been calculated, and to produce a continuous function, the probabilities are enveloped by a Gaussian distribution having full half-width of 1.5 MeV and summed up over energies in 1 MeV intervals. These can be compared, in principle at least, with the experimental Gamow-Teller strength function, which is unfortunately not available. In this work, all BCS+pnQRPA calculations exhaust practically the full Ikeda values (actually around 101 %, but the 1 % shoot-up is assumed to be due to purely numerical error).

Conclusions

The calculations with the pure Migdal force and the corrected Migdal force give half-lives which are about one half of the experimental value, and decay energies which are close to the experimental decay energy. The correction to the Migdal excitation force proves to improve the accuracy of the calculations.

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