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Alternative Solution Optimization Quadratic Form with Fuzzy Number Parameters

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ABSTRACT

The fuzzy number is one of the alternatives to maximize solving a quadratic model programming problem. Based on that statement, this paper provides one of the methods to solve the quadratic model programming problem. It started with a general discussion on quadratic model programming and continued by transposing a basic form into a fuzzy quadratic programming equation and giving a reference to solve that problem. Finally, a few examples are provided to analyze how accurate this method works.

Keywords: optimization; quadratic; fuzzy program; fuzzy triangular numbers; Karush-Kuhn-tucker conditions

A. INTRODUCTION

Optimization is a branch of mathematics learning which has an essential function in mathematical modeling. In general, optimization is classified into two aspects, which are constrained and unconstrained optimization. Based on that classification, the main component in optimization are objective function and constrained function. Meanwhile, if we look into the result which wanted to find, optimization also could be divided into two, whether looking for maximum value or minimum value.

On many aspects, optimization involves non-linear function equations, whether in its objective function or constrained function. Therefore, the optimization problem needs to be solved by using transformations in quadratic form.

Quadratic programming has an essential role in non-linear programming. The non-linear problems become easier to solve if they can be brought into quadratic programming. The non-linear optimization problem is maximizing or minimizing the objective function in the form of a quadratic function. The objective function depends on the linear constraint function and non-negative variables constraints (Peressini et al., 1988). Problems and tests for quadratic programming are found in (Floudas et al., 1999; Gould and Toint, 2000; Hock and Schittkowski, 1981; Maros and Meszaros, 1997; Schittkowski, 1987). Examples of this problem can be found in game theory, economic problems, location problems and facility allocation, engineering modeling, design, and control. Portfolios, logistics, and others (Silvia et al., 2009).

Quadratic programming problem is mostly used in a real problem. Besides, ambiguity and uncertainty are mostly presented in such an optimization problem. Therefore, the applicable fuzzy set theory is used to model. Fuzzy quadratic programming became interested in researchers as Abbasi Molai in 2012 (Mahdavi-Amiri et al., 2009), and then In 2017, Mirmohseni (Mirmohseni and Nasseri, 2017) carried out a program-related development. This research focuses on quadratic programs with fuzzy triangular numbers.

This paper is composed of 5 parts. In the next section, it contains basic definitions regarding quadratic programming and arithmetic fuzzy number problems. Section 3 contains the optimization of fuzzy quadratic programming. Section 4 contains numerical examples to illustrate how to apply and solve problems. The last section, namely section 5, contains the conclusions of this paper.

B. THE BASIC DEFINITION

1. Quadratic Programming

The optimization problem can be written as follows:

$$\begin{aligned} &Min\ f(x) \\ &s.t\ g_i(x) \leq 0 \\ &\boldsymbol{x} \in \boldsymbol{C} \subset \mathbb{R}^{\boldsymbol{n}} \end{aligned} \qquad (1.1)$$

where the function $f: \mathbb{R}^n \to \mathbb{R}$ function is a real value function, which is the minimum value. Eq. 1.1 is called the cost function. The vector $\mathbf{x}$ is an independent variable with n- vector. The variables $x_1, x_2, \dots, x_n$ it is decision variables.

The above problem is a decision problem to find the "best" $\mathbf{x}$ value from all possible factors' decision variables. The minimization vector of the function f is the best vector. The point $\boldsymbol{x} \in \boldsymbol{C}$, which satisfies all the constraints of equation (1.1), is called the feasible point, and the set F of all feasible points for equation (1.1) is the feasible region.

As previously explained, the objective function of a non-linear function can be transformed into a quadratic form, which is contained in the function x_j^2 and $x_i x_j (i \neq j)$ and is defined as follows:

Definition 2.1 (Chong and Zak, 2001) *A quadratic form* $f: \mathbb{R}^n \to \mathbb{R}$ *is a function*

$$f(\mathbf{x}) = \mathbf{x}^T Q \mathbf{x} \qquad (1.2)$$

where Q is a real matrix $n \times n$.

This does not reduce generality if it is assumed that Q is a symmetrical matrix, namely $Q = Q^T$. If Q is not symmetrical, it will be converted to a symmetrical matrix using the following equation,

$$Q = \frac{1}{2}(Q_0 + Q_0^T). \qquad (1.3)$$

So for an asymmetrical

$$\mathbf{x}^T Q \mathbf{x} = \mathbf{x}^T \left(\frac{1}{2} Q_0 + \frac{1}{2} Q_0^T\right) \mathbf{x}. \qquad (1.4)$$

The most typical forms of quadratic program problems are as follows:

$$Min\ Z = \sum_{j=1}^{n} c_j x_j + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ij} x_i x_j$$

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_{i,} \, i = 1,2, \dots, m \tag{1.5}$$

$$s.t. \qquad x_j \geq 0, \;\; j = 1,2,3, \dots n$$

Using vector and matrix notation, equation (1.5) can be modeled as follows:

$$\text{Min } Z = \mathbf{cx} + \mathbf{x}^T\mathbf{Qx} \tag{1.6}$$
$$\text{s.t } \mathbf{Ax} \leq \mathbf{b}$$
$$\mathbf{x} \geq 0$$

where $\mathbf{x} = (x_j; j = 1, \dots, n)$ is the vector variable and $\mathbf{c} = (c_j\,;\, j = 1, \dots, n)$, $\mathbf{b} = \mathbf{(}b_i\,;\, i = 1, \dots, m\mathbf{)}$ is a cost coefficients row vector right- hand side vector, Q is asymmetric and positive semi-definite, and A is the constraint coefficient matrix.

2. Arithmetic Fuzzy and Numbers

This section shares some of the required definitions of fuzzy set theory (Mirmohseni and Nasseri, 2017; Schittkowski, 1987; Mahdavi-Amiri et al., 2009; Nasseri and Ebrahimnejad, 2010; Nasseri and Mahdavi-Amiri, 2009).

Definition 2.2.1 (Mirmohseni and Nasseri, 2017) (Mirmohseni and Nasseri, 2017) If R is a real line, then the fuzzy set A in R is defined as the set of ordered pairs $A = \{(x,\ \mu_A(x)) \mid x \in R\}$, where $\mu_A(x)$ it is a fuzzy set membership function that maps each R element to a membership value between 0 and 1. Mathematically it can be written as follows:

$$\mu_A(x) : X \to [0,1] \tag{2.1}$$

Definition 2.2.2 (Wang, 1997) The concept of support, height, $\alpha - cut$, convex, triangular fuzzy number, and Arithmetic in Triangle Fuzzy Numbers are defined as below.

a. Support for the fuzzy set A in the universal set U is a crisp set consisting of all elements of U that have non-zero membership value in A, namely

$$supp(A) = \{x \in U | \mu_A(x) > 0\} \tag{2.2}$$

b. The largest membership value achieved at any point is called the height of the fuzzy set. If hgt $\mu_A = 1$, then A is called normal; otherwise, eating A is called subnormal.

c. A crisp set A_α consisting of all U elements with a membership value in A greater than or equal to α is called a slice $\alpha - cut$ of the fuzzy set A. In mathematics, it can be written as follows:

$$A_\alpha = \{x \in U | \mu_A(x) \geq \alpha\} \tag{2.3}$$

d. Fuzzy set A in R is convex if for any x, $x, y \in R$ and $\lambda \in [0,1]$ meet

$$\mu_A(\lambda x + (1-\lambda)y) \geq \min\{\mu_A(x), \mu_A(y)\}. \tag{2.4}$$

e. A fuzzy number $\tilde{a}$ on R is called a form of the triangular fuzzy number if there exist real numbers $l, r \geq 0$ such that

$$\tilde{a}(x) = \begin{cases} \dfrac{x}{l} + \dfrac{l-s}{l}, & x \in [s-l, s] \\ \dfrac{-x}{r} + \dfrac{s+r}{r}, & x \in [s, s+r] \\ 0, & o.w \end{cases} \tag{2.5}$$

Where $\bar{x}$ is the capital value of fuzzy numbers and $l < x < r$., $\tilde{a} =< s, l, r >$.

f. Arithmetic in Triangle Fuzzy Numbers

Let $\tilde{a} = \langle \bar{x}_a, l_a, r_a \rangle$ and $\tilde{b} = \langle \bar{x}_b, l_b, r_b \rangle$ Are fuzzy triangular numbers and $x \in \mathbb{R}$. Addition, subtraction, and multiplication can be defined as follows:

$$\tilde{a} + \tilde{b} = \langle \bar{x}_a + \bar{x}_b, l_a + l_b, r_a + r_b \rangle \tag{2.6}$$

$$\tilde{a} - \tilde{b} = \langle \bar{x}_a - \bar{x}_b, l_a - l_b, r_a - r_b \rangle \tag{2.7}$$

$$x\tilde{a} = \begin{cases} \langle x\bar{x}_a, xl_a, xr_a \rangle, & x \geq 0 \\ \langle x\bar{x}_a, -xr_a, -xl_a \rangle, & x < 0 \end{cases} \tag{2.8}$$

3. Optimization Fuzzy Numbers Quadratic Program

The quadratic programming problem (1.5), if some or all the parameters were fuzzy numbers, the problem can be fuzzy quadratic programming, so the general form of fuzzy quadratic programming as follows:

$$\begin{aligned} &\text{Min } Z = \sum_{j=1}^{n} \tilde{c}_j x_j + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{q}_{ij} x_i x_j \\ &\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i, \quad i = 1, \dots, m \\ \text{s.t} \quad & x_j \geq 0, \quad j = 1, \dots, n \end{aligned} \tag{3.1}$$

where $\tilde{c}_j, \tilde{q}_{ij}, \tilde{a}_{ij}$, and $\tilde{b}_i$ they are fuzzy numbers.

In this study, the fuzzy numbers used are fuzzy triangular numbers. Thus, in fuzzy quadratic programming (3.1), it can be written as follows:

$$\text{Min} \quad Z = \sum_{j=1}^{n} \langle c_j, p_j, t_j \rangle x_j + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \langle q_{ij}, s_{ij}, w_{ij} \rangle x_i x_j$$

$$\text{s.t} \quad \sum_{j=1}^{n} \langle a_{ij}, l_{ij}, r_{ij} \rangle x_j \leq \langle b_i, u_i, v_i \rangle, \quad i = 1, \dots, m \tag{3.2}$$

$$x_j \geq 0, \quad j = 1, \dots, n$$

The optimization programming above consists of an objective function and a constraint function. In quadratic programming (3.1), both functions contain fuzzy triangular numbers. So by using addition and multiplication in fuzzy triangular numbers, the objective function in the fuzzy quadratic program (3.2) can be written as follows:

$$\text{Min} \begin{cases} z_m = \sum_{j=1}^{n} c_j x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} x_i x_j \\ z_m - z_l = \sum_{j=1}^{n} (c_j - p_j) x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (q_{ij} - s_{ij}) x_i x_j \\ z_m + z_r = \sum_{j=1}^{n} (c_j + t_j) x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (q_{ij} + w_{ij}) x_i x_j \end{cases} \tag{3.3}$$

Meanwhile, the constraint function can be written as

$$\text{s.t.} \begin{cases} \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i \in N_m \\ \sum_{j=1}^{n} (a_{ij} - l_{ij}) x_j \leq (b_i - u_i), \quad i \in N_m \\ \sum_{j=1}^{n} (a_{ij} + r_{ij}) x_j \leq (b_i + v_i), \quad i \in N_m \end{cases} \tag{3.4}$$

The solution Eq (3.4) changes in the objective function with fuzzy parameters in the quadratic program (3.1) into three objective functions with firm number parameters in equation (3.2). Each of these functions is then optimized with some of the same function constraints, namely equation (3.4). Thus, the fuzzy quadratic program in equation (3.1) can be converted into three simple quadratic programs as follows,

a. Quadratic Program I

$$\text{Min } z_m = \sum_{j=1}^{n} c_j x_j + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} q_{ij} x_i x_j$$

$$\text{s.t.} \begin{cases} \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i \in N_m \\ \sum_{j=1}^{n} (a_{ij} - l_{ij}) x_j \leq (b_i - u_i), \quad i \in N_m \\ \dots \end{cases}$$

Displayed equation (3.5)

b. Quadratic Program II

$$\text{Min } z_m - z_l = \sum_{j=1}^{n}(c_j - p_j)x_j + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(q_{ij} - s_{ij})x_i x_j$$

$$\text{s.t.} \begin{cases} \sum_{j=1}^{n} a_{ij}x_j \le b_i, \quad i \in N_m \\ \sum_{j=1}^{n}(a_{ij} - l_{ij})x_j \le (b_i - u_i), \quad i \in N_m \\ \sum_{j=1}^{n}(a_{ij} + r_{ij})x_j \le (b_i + v_i), \quad i \in N_m \end{cases} \tag{3.6}$$

c. Quadratic Program III

$$\text{Min } z_m + z_r = \sum_{j=1}^{n}(c_j + t_j)x_j + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(q_{ij} + w_{ij})x_i x_j$$

$$\text{s.t.} \begin{cases} \sum_{j=1}^{n} a_{ij}x_j \le b_i, \quad i \in N_m \\ \sum_{j=1}^{n}(a_{ij} - l_{ij})x_j \le (b_i - u_i), \quad i \in N_m \\ \sum_{j=1}^{n}(a_{ij} + r_{ij})x_j \le (b_i + v_i), \quad i \in N_m \end{cases} \tag{3.7}$$

The quadratic program in equations (3.5), (3.6), and (3.7) is a simple quadratic program. Therefore, to solve three quadratic programs above , we can use Karush-Kuhn-Tucker method.

The results obtained from this method are in the form of three optical values, namely z_m, $(z_m - z_l)$, and $(z_m + z_r)$. The three values are components of the triangular fuzzy number from $\tilde{z} = \langle z_m, z_l, z_r \rangle$.

C. RESULT AND DISCUSSION

In this section, the applicability of our proposed method for demonstrated by solving numerical examples.

$$\text{Min } Z = \langle -5,1,1\rangle x_1 + \langle 1.5,0.5,0.5\rangle x_2 + \frac{1}{2}(\langle 6,2,2\rangle x_1^2 + \langle -4,2,2\rangle x_1 x_2 + \langle 4,2,2\rangle x_2^2)$$

$$s.t \begin{cases} x_1 + \langle 1,0.5,0.5\rangle x_2 \le \langle 2,1,1\rangle \\ \langle 2,1,1\rangle x_1 + \langle -1,1,0.5\rangle x_2 \le \langle 4,1,1\rangle \\ x_1, x_2 \ge 0 \end{cases}$$

Displayed equation (4.1)

Then the quadratic fuzzy (4.1) can be converted into three simple programs as follows:

a. Quadratic Program I

$$\text{Min } z_m = -5x_1 + 1.5x_2 + 3x_1^2 - 2x_1x_2 + 2x_2^2$$

$$\text{s.t.} \begin{cases} x_1 + x_2 \leq 2 \\ x_1 + 0.5x_2 \leq 1 \\ x_1 + 1.5x_2 \leq 3 \\ 2x_1 - x_2 \leq 4 \\ x_1 - 2x_2 \leq 3 \\ 3x_1 - 0.5x_2 \leq 5 \\ x_1, x_2 \geq 0 \end{cases} \tag{4.2}$$

b. Quadratic Program II

$$\text{Min } z_m - z_l = -6x_1 + x_2 + 2x_1^2 - 3x_1x_2 + x_2^2$$

$$\text{s.t.} \begin{cases} x_1 + x_2 \leq 2 \\ x_1 + 0.5x_2 \leq 1 \\ x_1 + 1.5x_2 \leq 3 \\ 2x_1 - x_2 \leq 4 \\ x_1 - 2x_2 \leq 3 \\ 3x_1 - 0.5x_2 \leq 5 \\ x_1, x_2 \geq 0 \end{cases} \tag{4.3}$$

c. Quadratic Program III

$$\text{Min } z_m + z_r = -4x_1 + 2x_2 + 4x_1^2 - x_1x_2 + 3x_2^2$$

$$\text{s.t.} \begin{cases} x_1 + x_2 \leq 2 \\ x_1 + 0.5x_2 \leq 1 \\ x_1 + 1.5x_2 \leq 3 \\ 2x_1 - x_2 \leq 4 \\ x_1 - 2x_2 \leq 3 \\ 3x_1 - 0.5x_2 \leq 5 \\ x_1, x_2 \geq 0 \end{cases} \tag{4.4}$$

The three quadratic programs above generate results $z_m = -2.0875$, $z_m - z_l = -4.0833$, and $z_m + z_r = -1$. So that the optimal objective function value obtained is $\tilde{z} = \langle z_m, z_l, z_r \rangle = \langle -2.0875, 1.9958, 1.0875 \rangle$.

D. CONCLUSION

Quadratic programming problem is a very important field in operation research and is mostly used for the optimization problem.

This study proposes a new method to provide a solution to fuzzy quadratic programming using the triangular fuzzy number approach for the objective and cost functions. The Karush-Kuhn-Tucker method is used to solve any simple quadratic program. The results obtained from three simple quadratic programs are three optimal values in fuzzy triangular numbers.

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