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On the existing of fully invariant submodule

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On the existing of fully invariant submodule

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Abstract. Let M be a nonzero $R-$module, where R is a ring. A submodule U of M is called a fully invariant submodule if $f(U) \subseteq U$ for every $f \in S$, where $S = End_R(M)$. Moreover, M is called an $\oplus-$supplemented module if every submodule N of M there exists a submodule K of M such that K is a direct summand of M, $M = N + K$, and $N \cap K$ is small in M. Furthermore, M is called a $cms-$module if for every cofinite submodule K of M, there exist submodules P and Q of M such that P is a supplement of K, $P + Q = M$, and $P \cap Q$ is a small submodule in Q. In fact, factor module of a $\oplus-$supplemented module (respectively, $cms-$module) is not $\oplus-$supplemented (respectively, is not cms) in general. In this paper, we show that factor module of $\oplus-$supplemented module (respectively, $cms-$module) determined by fully invariant submodule is also $\oplus-$supplemented (respectively, cms). Moreover, we generate a fully invariant submodule by using radical of a module.

1. Introduction

Throughout this paper, all rings are considered to be associative with identity and all modules are unitary left module. Let A be any ring and let M be any $A-$module. A submodule N of M is called a small submodule if $N + L \subset M$ for any proper submodule L of M. Some notions on small submodules were described in [1]. A submodule N is called a prime submodule if N is a proper submodule of M and for every $a \in A, m \in M$, if $am \in N$ then $m \in N$ or $a \in (N : M)_A$ where $(N : M)_A = \{a \in A | aM \subseteq N\}$. If $\{0_M\}$ is an $A-$submodule of M, then M is called a prime $A-$module. A prime submodule N of M is called $s-$prime if $R/(N : M)$ contains no nonzero nil ideals. Furthermore, a proper submodule N of M is called an $l-$prime submodule of M if for any I ideal of A, L submodule of M and for every finite subset I_n of I there exists a natural number m such that for any product $a_{1i}a_{2i}...a_{mi}$ of elements from I_n, $(a_{1i}a_{2i}...a_{mi})L \subseteq N$ gives $L \subseteq N$ or $I \subseteq (N : M)$. The definition of $l-$prime submodule is derived from the definition of $l-$prime ideal. An ideal I of a ring R is $l-$prime if given $p, q \in I$, there exists elements $p_1, p_2, ..., p_n \in \langle p \rangle$ and $q_1, q_2, ..., q_n \in \langle q \rangle$ such that for every $p > 1$ there exists a product of $N \geq p$ factors which consist of $p_i's$ and $b_j's$ such that $p_i's$ and $b_j's$ are not in I. The relationship between prime, $s-$prime, and $l-$prime submodules are presented in [2].

The Jacobson (respectively, the prime, the Levitzki, the upper nil) radical of M is denoted by $\mathcal{J}(M)$ (respectively, $\mathcal{P}(M), \mathcal{L}(M), \mathcal{N}(M)$) and it is defined as the the sum of all small submodules of M(respectively, intersection of all prime submodules, intersection of all $l-$prime submodules, intersection of all $s-$prime submodules). In fact, in commutative case, let M be any $A-$module. It has been already known in [2] that $\mathcal{P}(M) = \mathcal{L}(M) = \mathcal{N}(M)$ when A is a commutative ring.

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On the other hand, a submodule N of an $A-$module M is called a supplement for a submodule L of M if N is minimal with respect to the property $N + L = M$. If every submodule of M has a supplement, then the module M is called a complemented module or a supplemented module. Moreover, if every submodule of M has a supplement which is a direct summand of M, then M is called an $\oplus-$supplemented module.

Let D be an integral domain and let F be the field fractions of D. The integral domain D is called a valuation ring if for every $x \in F$ implies $x \in D$ or $x^{-1} \in D$. A ring A is said to be local if A has a uniqe maximal ideal. It follows from the Example 2.2 in [1] that there exists a finitely presented indecomposable module $M = A^n/K$ which is not an $\oplus-$supplemented module for some natural numbers $n \geq 2$, where A is a commutative local ring which is not a valuation ring. In fact, the ring A^n is an $\oplus-$supplemented module. Hence, every module factor of any $\oplus-$supplemented module is not an $\oplus-$supplemented module in general. Advanced properties of $\oplus-$supplemented modules are given in [3]. On the other hand, more about the basic properties of rings and modules can be found in [4–13]. In this paper, we give a direct consequence of the existing of a fully invariant submodule.

2. Results and Discussion

The following proposition explains the property of a submodule X of an $R-$module M such that $f(X) \subseteq X$ for every endomorphism f of M.

Proposition 1. *[1] Let M be a nonzero $R-$module and let X be a submodule of M such that $f(X) \subseteq X$ for every $f \in S$, where $S = End_R(M)$ is the set of all endomorphism of M. If $M = M_1 \oplus M_2$, then $X = (X \cap M_1) \oplus (X \cap M_2)$.*

Proof. See Lemma 2.4 in [1] □

Proposition 2. *[1] Let M be a nonzero module and let X be a submodule of M such that $f(X) \subseteq U$ for every $f \in S$, where $S = End_R(M)$ is the set of all endomorphism of M. If M is $\oplus-$supplemented, then so is M/X. Furthermore, if X is a direct summand of M, then X is also $\oplus-$supplemented.*

Proof. See Proposition 2.5 in [1] □

Recall the definition of a fully submodule. A submodule N of an $R-$module M is called fully invariant submodule if $f(N) \subseteq N$ for every endomorphism f of M. Hence, we therefore have the following theorem as a direct consequence of the existing of a fully invariant submodule.

Theorem 1. *Let M be a nonzero $R-$module and let U be a fully invariant submodule of M and $M = M_1 \oplus M_2$, then $U = (U \cap M_1) \oplus (U \cap M_2)$.*

Proof. Let M be any nonzero $R-$module and assume that U is a fully invariant submodule of M. Suppose $M = M_1 \oplus M_2$. Then there exist canonical projections, $\alpha_1 : M \to M_1$ and $\alpha_2 : M \to M_2$. Since $M = M_1 \oplus M_2, x = \alpha_1(x) + \alpha_2(x)$ for every $x \in M$. It is clear that $\alpha_1, \alpha_2 \in End(M)$. Furthermore, since U is a fully invariant submodule of M, $\alpha_1(U) \subseteq U$ and $\alpha_2(U) \subseteq U$. Therefore, $\alpha_1(x) \in M_1 \cap U$ and $\alpha_2(x) \in M_2 \cap U$. Hence U is a submodule of $M_1 \cap U \oplus M_2 \cap U$. Conversely, the opposite, it is obvious. So, $U = M_1 \cap U \oplus M_2 \cap U$ □

As a direct consequence of Proposition 2, we have the following theorem.

Theorem 2. *Let M be a nonzero $R-$module and let U be a fully invariant submodule of M. If M is $\oplus-$supplemented, then so is M/U. Moreover, if U is a direct summand of M, then U is also $\oplus-$supplemented.*

Proof. Suppose M is a $\oplus-$supplemented modul, let U be a fully invariant of M, and let K be a submodule of M containing U. Since M is a $\oplus-$supplemented module, there exist P, Q submodules of M such that $M = P \oplus Q, M = K + P$ and $K \cap P$ is a small submodule in M. This gives $P + U/U$ is a supplement of K/U in M/U. Now, we have to show that $P + U/U$ is a direct summand of M/U. It follows from 1 that $U = U \cap P \oplus U \cap Q$. This gives $(P+U) \cap (Q+U)$ is a submodule of $(P + U + Q) \cap U + (P + U + U) \cap Q$. Therefore, $(P + U) \cap (Q + U)$ is a submodule of $U + (P + U \cap P + U \cap Q) \cap Q$. Furthermore, $(P + U) \cap (Q + U)$ is a submodule of U. Since $((P + U)/U) \oplus (Q + U/U)) = M/U$, $(P + U)$ is a direct summand of M/U. Therefore, M/U is $\oplus-$supplemented.

Moreover, suppose that U is a direct summand of M and let N be a submodule of U, we have to show that N has a supplement which is a direct summand of U. Since M is an $\oplus-$supplemented module, there exist submodules K and L of M such that $M = K \oplus L, M = K + L$ and $N \cap K$ is small in K. This gives $U = V + U \cap K$. However, $U = U \cap K \oplus U \cap L$. Therefore, $U \cap K$ is a direct summand of U. Furthermore, $N \cap (U \cap K)$ is a small submodule in K. Hence, $V \cap (U \cap K)$ is small in $U \cap K$. This implies $U \cap K$ is a supplement of N in U. So, we can infer that U is also an $\oplus-$supplemented module. □

Let M be an $R-$module and let K and L be any submodules of M such that $M = K+L$. The submodule L of M is called a generalized supplement of K if $(K \cap L \subseteq \mathcal{J}(L))$. An $R-$module M is called generalized $\oplus-$supplemented module if every supmodule of M has a generalized supplement which is a direct summand of M. Some properties of generalized $\oplus-$supplemented modules are given in [14].

Theorem 3. *Let M be a nonzero $R-$module and let U be a fully invariant submodule of M. If M is generalized $\oplus-$supplemented, then so is M/U. Furthermore, if U is a direct summand of M, then U is also generalized $\oplus-$supplemented.*

Proof. Suppose M is an $R-$module which is a generalized $\oplus-$supplemented module and let U is a fully invariant submodule of M. It follows from Proposition 3.5 (1) described in [14] that U satisfies the assumption. Hence M/U is a generalized $\oplus-$supplemented module. Moreover, if U is a direct summand of M. By using Proposition 3.5 (2) in [14], we may infer that U is also a generalized $\oplus-$supplemented module. □

The most important things in abstract algebra is a concrete example which satisfies the assumption. When the example does not exist, the assumption is only imagination. Now, we give some examples of fully invariant submodules. Some submodules are naturally fully invariants.

Theorem 4. *Let M be a nonzero $R-$module. Then $\mathcal{J}(M)$, $\mathcal{N}(M)$, $\mathcal{L}(M)$ and $\mathcal{P}(M)$ are fully invariant submodule of M, where $\mathcal{J}(M)$ is the Jacobson radical of M, $\mathcal{N}(M)$ is the Nilradical of M, $\mathcal{L}(M)$ is the Levitzki radical of M, and $\mathcal{P}(M)$ is the prime radical of M.*

Proof. Let M be a nonzero $R-$module and let f be any members of the set $End(M)$ of all endomorphism of M. Then $f(\mathcal{J}(M)) = \mathcal{J}(f(M)) \subseteq \mathcal{J}(M)$. Analogously for the Niradical $\mathcal{N}(M)$ of M, the prime radical $\mathcal{P}(M)$ of M, and the Levitzki radical $\mathcal{L}(M)$ of M. □

As a direct consequence of Theorem 4, we have the following theorem.

Theorem 5. *Let M be a nonzero $R-$module. If M is $\oplus-$supplemented module, then $M/\mathcal{J}(M), M/\mathcal{P}(M), M/\mathcal{N}(M), M/\mathcal{L}(M)$ are $\oplus-$supplemented module.*

Proof. Since $\mathcal{J}(M)$ and $\mathcal{P}(M)$ are fully invariant submodules of M, it follows from Theorem 4, $M/\mathcal{J}(M), M/\mathcal{P}(M), M/\mathcal{N}(M), M/\mathcal{L}(M)$ are $\oplus-$supplemented module. □

On the other hand, every ring can be viewed as a module over itself. In the following proposition, a necessary and sufficient condition for a submodule of an $A-$module M to be $l-$prime submodule and a necessary and sufficient condition for a ring with identity R to be $l-$prime ring are given.

Proposition 3. *[2] Let M be any $A-$module. A submodule N of M is an $l-$prime submodule of M if and only if P is a prime submodule of M and $(P : M)$ is an $l-$prime ideal of R.*

Proof. See Proposition 2.2 in [2]. □

Proposition 4. *[2] If R is a ring with identity, then R is $l-$prime ring if and only if R is an $l-$prime left $R-$module.*

Proof. See Proposition 2.4 in [2]. □

It follows from Proposition 2.7 in [2] that every $s-$prime submodule is $l-$prime and every $l-$prime submodule is prime. This condition implies every $s-$prime module is $l-$prime and every $l-$prime module is prime. However, the converse is not generally true. An $A-$module is called an $l-$prime module if $\{0\}$ is an $l-$prime submodule of M. In general, if P is an $l-$prime submodule of M, then M/P is an $l-$prime module. The Example 2.10 in [2] showed that a prime module which is not $l-$prime exists. Furthermore, the Example 2.11 in [2] showed that an $l-$prime module which is not $s-$prime exists. Moreover, in the case of commutative ring, we have the following consequence.

Corollary 1. *Let M be a nonzero $R-$module. If M is $\oplus-$supplemented module and R is a commutative ring, then $M/\mathcal{P}(M), M/\mathcal{N}(M), M/\mathcal{L}(M)$ are $\oplus-$supplemented modules such that $M/\mathcal{P}(M) = M/\mathcal{N}(M) = M/\mathcal{L}(M)$.*

Proof. It follows from Theorem 2.12 in [2] that if M is a module over commutative ring, then $s-$prime $\Leftrightarrow$ $l-$prime $\Leftrightarrow$ prime. Therefore, $\mathcal{P}(M) = \mathcal{N}(M) = \mathcal{L}(M)$. This gives $M/\mathcal{P}(M) = M/\mathcal{N}(M) = M/\mathcal{L}(M)$. □

A submodule of an $R-$module M is called cofinite in M if the factor module M/N is finitely generated. Then, M is called an $\oplus-$cofinitely supplemented if every cofinite submodule of M has a supplement which is a direct summand of M. Furthermore, M is called a $cms-$module if for every cofinite submodule K of M, there exist submodules P and Q of M such that P is a supplement of K, $P + Q = M$, and $P \cap Q$ is a small submodule in Q. A necessary and sufficient condition for a module to be $\oplus-$cofinitely supplemented is given in [15]. In fact, every $\oplus-$supplemented module is a $cms-$module. However, it follows from the Example 2.2 in [15] that the converse is not generally true. We therefore have the following corollary.

Corollary 2. *Let M be a nonzero $R-$module. If M is $cms-$module, then $M/\mathcal{J}(M), M/\mathcal{P}(M), M/\mathcal{N}(M), M/\mathcal{L}(M)$ are $cms-$modules.*

Proof. It is follows from 4 that $\mathcal{J}(M), \mathcal{P}(M), \mathcal{N}(M), \mathcal{L}(M)$ are fully invariant submodules of M. Then, it follows from Theorem 2.6 in [15] that $M/\mathcal{J}(M), M/\mathcal{P}(M), M/\mathcal{N}(M), M/\mathcal{L}(M)$ are $cms-$modules. □

3. Conclusion

Given an $\oplus-$supplemented (respectively, $cms-$module) $A-$module. In fact, for every nonzero submodule N of M, the factor module M/N is not generally an $\oplus-$supplemented (respectively, $cms-$module) $A-$module. On the other hand, we may classify some submodules of M generating the factor modules which are $\oplus-$supplemented (respectively, $cms-$module). The fully invariant submodules meet the requirement to be submodules generating factor modules which are $\oplus-$supplemented (respectively, $cms-$modules). Some submodules are naturally fully invariant submodules, for example, $\mathcal{J}(M), \mathcal{P}(M), \mathcal{N}(M), \mathcal{L}(M)$ of any $A-$module M are fully invariant submodules.

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References

[1] Tribak R and Idelhaj A 2003 On Some Properties of $\oplus$-Supplemented Modules *International Journal of Mathematics and Mathematical Sciences* **69** 4373-87
[2] Groenewald N J and Ssevviiri D 2014 On The Levitzki Radical of Modules *International Electronic Journal of Algebra* **15** 77-89
[3] Ungor B, Halicioglu S and Harmanci A 2014 On A Class of $\oplus$-Supplemented Modules *Contemporary Mathematica* **609** 123-36
[4] Adkins W A and Wintraub S 1992 *Algebra, An Approach Via Module Theory* (New York: Springer-Verlag)
[5] Anderson F W and Fuller K R 2006 *Rings and Categories of Modules, 2nd Edition* (New York: Springer-Verlag)
[6] Dauns J 2004 *Modules and Rings* (Cambridge: Cambridge University Press)
[7] Dummit D and Foote R 2004 *Abstract Algebra, Third Edition* (USA: John Willey and Sons)
[8] Kaplansky I 1974 *Commutative Rings* (London: University of Chicago Press)
[9] Lomp C 2000 *On Dual Goldie Dimension* (Dusseldorf: Heinrich-Heine Universitat Dusseldorf)
[10] Lomp C, Clark J O, Vanaja N and Wisbauer R 2006 *Lifting Modules, Supplements and Projevtivity in Module Theory* (Basel: Birkhauser-Verlag)
[11] Wisbauer R 1991 *Foundations of Module and Ring Theory* (Reading: Gordon and Breach Science Publishers)
[12] Fleury P 1974 Hollow Modules and Local Endomorphism *Pasific Journal of Mathematics* **53** 379-85
[13] Zoschinger H 1974 Komplememtiere Moduln uber Dedekindringen *J. Algebra* **29** 43-70
[14] Calisici H and Turkmen E 2010 Generalized $\oplus-$supplemented Modules *Algebra and Discrete Mathematics* **10** 10-18
[15] Kosar B and Turkmen B 2016 A Generalization of $\oplus$-Cofinitely Supplemented Modules *Bull. Iranian Math. Soc* 91-99