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On (R, S)-Module Homomorphisms

D A Yuwaningsih[1], **I E Wijayanti**[2] **and P W Prasetyo**[1]

[1]Universitas Ahmad Dahlan, Jalan Ring Road Selatan, Yogyakarta, 55191 Indonesia
[2]Universitas Gadjah Mada, Sekip Utara, Yogyakarta, 55281 Indonesia

E-mail: `dian.ariesta@pmat.uad.ac.id`

Abstract. Let R and S be arbitrary rings. In the algebraic structure it is known that the R-module structure is a generalization of a vector space. As in the ring structure, in the R-module some previous researchers have defined R-module homomorphisms, the types of R-module homomorphisms, the properties of R-module homomorphisms, and the fundamental theorem of R-module isomorphisms. On the other hand, the R-module structure has been generalized to the (R, S)-module structure. However, research and discussion related to (R, S)-modules are still a bit worked out. Therefore, in this paper we present the definition of (R, S)-module homomorphisms, the types of (R, S)-module homomorphisms, the properties of (R, S)-module homomorphisms, and the fundamental theorem of (R, S)-module isomorphisms.

1. Introduction

All rings in this paper are arbitrary ring unless stated otherwise and M is an additive Abelian group. On ring theory, there exist a fuction that preserves the aditif and multiplication binary operation tha's called a ring homomorphism. Let $(T, +, \cdot)$ and $(T', +', \cdot')$ be rings with unity. Based on [1], a function $f : T \to T'$ is called a ring homomorphism if for each $a, b \in T$ satisfy $f(a + b) = f(a) +' f(b)$ and $f(a \cdot b) = f(a) \cdot' f(b)$. Moreover, [1] also discussed the types of ring homomorphism and its properties. The types of ring homomorphisms include ring monomorphisms, ring epimorphisms and ring isomorphisms. Furthermore, the discussions related to the ring homomorphism are always closed with the fundamental theorem of ring isomorphisms. Moreover, a deeper discussion related to ring homomorphisms have been discussed in [1–7].

In subsequent development, all theories related to ring theory were brought to module theory, including theories related to ring homomorphisms. Let T be a ring with unity. Given M and M' be left modules over T. Based on [2], a function $f : M \to M'$ is called a module homomorphism if for each $m, n \in M$ and $t \in T$ satisfy $f(m + n) = f(m) +' f(n)$ and $f(tm) = tf(m)$. As with the ring, the types of T-module homomorphisms include T-module monomorphisms, T-module epimorphisms and T-module isomorphisms. The properties of T-module homomorphism are the generalization of properties ring homomorphisms. Moreover, the study of T-module homomorphism is also closed with the fundamental theorem of T-module isomorphisms. Moreover, the notion of T-module homomorphism have been extensively studied by various authors such as [1–4, 6, 7].

Let R and S be arbitrary rings. Over time, the module has been generalized to (R, S)-module. An (R, S)-modules as a generalization of (R, S)-bimodules has been introduced in [8]. An (R, S)-module has an (R, S)-bimodule structure when both rings R and S have central idempotent elements. In [8], also defined (R, S)-submodules of M as an additive subgroup N of M such that $rns \in N$ for all $r \in R$, $n \in N$, and $s \in S$.

Furthermore, the research related to (R, S)-module has begun to be developed by other researchers. Among of them are research in [9–13]. However, there is no researcher has ever investigated about (R, S)-module homomorphism. Therefore in Section 2, we present the definition of an (R, S)-module homomorphism, some examples of (R, S)-module homomorphisms, and some properties of (R, S)-module homomorphisms.

In Section 3, we extend the types of T-module homomorphism and the fundamental theorem of T-module isomorphims to (R, S)-module. Moreover, we also present several example related to them and the natural (R, S)-module homomorphism.

2. Some Properties of (R, S)-Module Homomorphisms

Before we present the definiton of (R, S)-module homomorphisms, we present (R, S)-module structure and the (R, S)-submodule. This concept has been discussed in detail in [8].

Definition 2.1. *Let R and S be rings and M an abelian group under addition. Then M is an (R, S)-module if there is a function $\_ \cdot\_ \bullet\_ : R \times M \times S \longrightarrow M$ satisfying the following properties: for all $r, r' \in R$, $m, n \in M$, and $s, s' \in S$,*

(i) $r \cdot (m + n) \bullet s = r \cdot m \bullet s + r \cdot n \bullet s$

(ii) $(r + r') \cdot m \bullet s = r \cdot m \bullet s + r' \cdot m \bullet s$

(iii) $r \cdot m \bullet (s + s') = r \cdot m \bullet s + r \cdot m \bullet s'$

(iv) $r(r' \cdot m \bullet s)s' = (rr') \cdot m \bullet (ss')$

We usually abbreviate $r \cdot m \bullet s$ by rms. We may also say that M is an (R, S)-module under $+$ and $\_ \cdot\_ \bullet\_$.

Furthermore, given an (R, S)-module M and a nonempty set $N \subseteq M$. Then, N is called an (R, S)-submodule of M if N is an additive subgroup of M and also an (R, S)-module under the same operation that defined on M.

It is obvious that R is an (R, R)-module via usual multiplication the ring R. Moreover, any ideal of R are (R, R)-submodules of R. However, an (R, R)-submodule of the (R, R)-module R need not be an ideal of the ring R. An (R, R)-submodule of R is an ideal of R if R is a ring with unity.

Now, we define (R, S)-module homomorphisms as follow.

Definition 2.2. *Let M and M' be (R, S)-modules. A function $f : M \to M'$ is called an (R, S)-module homomorphism if for each $m_1, m_2 \in M$, $r \in R$, and $s \in S$ satisfy:*

(i) $f(m_1 + m_2) = f(m_1) + f(m_2)$

(ii) $f(r \cdot m_1 \cdot s) = r \cdot' f(m_1) \cdot' s$.

Before we consider more example of (R, S)-module homomorphisms, let us prove some basic properties of (R, S)-module homomorphisms.

Proposition 2.3. *Let $f : M \to M'$ be an (R, S)-module homomorphism. Then,*

(i) f preserves the neutral element, that is $f(0_M) = 0_{M'}$.

(ii) f preserves the inverse element of each element in M, that is $f(-m) = -f(m)$ for all $m \in M$.

(iii) If H is an (R, S)-submodule of M, then $f(H) = \{f(h) \mid h \in H\}$ is an (R, S)-submodule of M'.

(iv) If N be an (R, S)-submodule of M'. Then, $f^{-1}(N) = \{m \in M \mid f(m) \in N\}$ is an (R, S)-submodule of M.

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Proof. (i) Since f is an (R, S)-module homomorphism, $f(0_M) + f(0_M) = f(0_M + 0_M) = f(0_M) = f(0_M) + 0_{M'}$. This implies that $f(0_M) = 0_{M'}$ by the cancelation law.

(ii) Let $m \in M$. Then $f(m) + f(-m) = f(m + (-m)) = f(0_M) = 0_{M'}$. Similarly, $f(-m) + f(m) = f(-m + m) = f(0_M) = 0_{M'}$. Since $f(a)$ has a unique invers, $f(-m) = -f(m)$.

(iii) Let H be an (R, S)-submodule of M. Then, $0_M \in H$ and by (i), $f(0_M) = 0_{M'}$. Thus, $0_{M'} = f(0_M) \in f(H)$ and so $f(H) \neq \emptyset$. Let any $x, y \in f(H)$ where $x = f(h_1)$ and $y = f(h_2)$. Since f is an (R, S)-module homomorphism, then $x - y = f(h_1) - f(h_2) = f(h_1 - h_2) \in f(H)$. Furthermore, let any $r \in R$ and $s \in S$. Since f is an (R, S)-module homomorphism, then $rxs = rf(h_1)s = f(rh_1s) \in f(H)$. Hence,$f(H)$ is an (R, S)-submodule of M'.

(iv) Let N be an (R, S)-submodule of M'. It's clear that $f^{-1}(N) \neq \emptyset$ because $0_M \in f^{-1}(N)$. Let any $x, y \in f^{-1}(N)$. Then $f(x) \in N$ and $f(y) \in N$. Since f is an (R, S)-module homomorphism, then $f(x - y) = f(x) - f(y)$. Since $f(x) \in N$ and $f(y) \in N$, then $f(x) - f(y) \in N$, so $x - y \in f^{-1}(N)$. Furthermore, let any $r \in R$ dan $s \in S$. Since f is an (R, S)-module homomorphism then $f(rxs) = rf(x)s$. Since $f(x) \in N$ and N is an (R, S)-submodule then $rf(x)s \in N$, so $rxs \in f^{-1}(N)$. Hence, it's shown that $f^{-1}(N))$ is an (R, S)-submodule of M. □

Definition 2.4. *Let $f : M \to M'$ be an (R, S)-module homomorphism. The kernel of f, written $Ker(f)$, is defined to be the set*

$$Ker(f) = \{m \in M \mid f(m) = 0_{M'}\}.$$

Definition 2.5. *Let $f : M \to M'$ be an (R, S)-module homomorphism. The image of f, written $Im(f)$, is defined to be the set*

$$Im(f) = \{m' \in M' \mid (\exists m \in M)\ f(m) = m'\}.$$

Now, we give some example of (R, S)-module homomorphism.

Example 2.6. Let M and M' be (R, S)-modules and a zero function $\theta : M \to M'$ where $\theta(m) = 0_{M'}$ for each $m \in M$. Let any $m_1, m_2 \in M$, $r \in R$, and $s \in S$, then we obtain:

$$\theta(m_1 + m_2) = 0_{M'} = 0_{M'} + 0_{M'} = \theta(m_1) + \theta(m_2) \tag{1}$$

and

$$\theta(rm_1s) = 0_{M'} = r0_{M'}s = r\theta(m_1)s. \tag{2}$$

From (1) and (2), we shown that a zero function f is an (R, S)-module homomorphism. Now, it's clear that $Ker(\theta) = M$ and $Im(\theta) = \{0_{M'}\}$.

Example 2.7. Let $\mathbb{Z}$ and $\mathbb{Z}[X]$ be $(\mathbb{Z}, \mathbb{Z})$-modules. A function $\theta : \mathbb{Z} \to \mathbb{Z}[X]$ where $\theta(a) = aX^2$ for each $a \in \mathbb{Z}$ is an $(\mathbb{Z}, \mathbb{Z})$-module homomorphism since for all $r, s, a, b \in \mathbb{Z}$ satisfy:

(i) $\theta(a + b) = (a + b)X^2 = aX^2 + bX^2 = \theta(a) + \theta(b)$,

(ii) $\theta(ras) = (ras)X^2 = r(aX^2)s = r\theta(a)s$.

Now, we obtain $Ker(\theta) = \{0\}$ and $Im(\theta) = \{mX^2 \in \mathbb{Z}[X] \mid m \in \mathbb{Z}\}$.

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Example 2.8. Let $\mathbb{Z}$ be an $(2\mathbb{Z}, 2\mathbb{Z})$-module. A function $f : \mathbb{Z} \to \mathbb{Z}$ where $f(a) = 3a$ for each $a \in \mathbb{Z}$ is an $(2\mathbb{Z}, 2\mathbb{Z})$-module homomorphism. Let any $a, b \in \mathbb{Z}$ and $r, s \in 2\mathbb{Z}$, then we get:

$$f(a + b) = 3(a + b) = 3a + 3b = f(a) + f(b) \tag{3}$$

and

$$f(ras) = 3(ras) = r(3a)s = rf(a)s. \tag{4}$$

Therefore, from (3) and (4) we shown that f is an $(2\mathbb{Z}, 2\mathbb{Z})$-module homomorphism. Now, it's clear that $Ker(f) = \{0\}$ and $Im(f) = 3\mathbb{Z}$.

The following is given some properties showed that the kernel and the image of any (R, S)-module homomorphism is an (R, S)-submodule.

Proposition 2.9. *Let $f : M \to M'$ be an (R, S)-module homomorphism. Then, the kernel of f is an (R, S)-submodule of M.*

Proof. It's clear that $Ker(f) \subseteq M$. Since $f(0_M) = 0_{M'}$, then $0_M \in Ker(f)$ so we get $Ker(f) \neq \emptyset$. Let any $x, y \in Ker(f)$, then $f(x) = 0_{M'}$ and $f(y) = 0_{M'}$. Since f is an (R, S)-module homomorphism, then $f(x-y) = f(x)-f(y) = 0_{M'} - 0_{M'} = 0_{M'}$. Thus, $x-y \in Ker(f)$. Furthermore, let any $r \in R$ and $s \in S$. Since f is an (R, S)-module homomorphism, then $f(rxs) = rf(x)s = r0_{M'}s = 0_{M'}$. So, $rxs \in Ker(f)$. Thus, $Ker(f)$ is an (R, S)-submodule of M. □

Proposition 2.10. *Let $f : M \to M'$ be an (R, S)-module homomorphism. Then, the image of f is an (R, S)-submodule of M'.*

Proof. It's clear that $Im(f) \subseteq M'$. Since $f(0_M) = 0_{M'} \in Im(f)$, then $Im(f) \neq \emptyset$. Let any $x, y \in Im(f)$, then $x = f(a)$ and $y = f(b)$ where $a, b \in M$. Since f is an (R, S)-module homomorphism, then $x - y = f(a) - f(b) = f(a - b)$. Since $a, b \in M$, then $a - b \in M$ so we get $x - y \in Im(f)$. Furthermore, let any $r \in R$ and $s \in S$. Since f is an (R, S)-module homomorphism, then $rxs = rf(a)s = f(ras)$. Since $a \in M$, then $ras \in M$, so we obtain $rxs \in Im(f)$. Thus, $Im(f)$ is an (R, S)-submodule of M'. □

3. An (R, S)-Module Isomorphism and Fundamentals Theorems

In this section, we present the types of (R, S)-module homomorphisms, several examples related to (R, S)-module homomorphisms, the natural (R, S)-module homomorphisms, and the fundamental theorem of (R, S)-module isomorphisms.

On module theory, there are some types of R-module homomorphism such as R-module epimorphism, R-module monomorphism, and R-module isomorphism. These types of R-module homomorphism has been studied in [2]. Now, below we present the types of (R, S)-module homomorphism as follows.

Definition 3.1. *Let $f : M \to M'$ be an (R, S)-module homomorphism.*

(i) A function f is called an (R, S)-module monomorphism if f is an injective function.
(ii) A function f is called an (R, S)-module epimorphism if f is a surjective function.
(iii) A function f is called (R, S)-module isomorphism if f is a bijective function.

Moreover, the (R, S)-module M and (R, S)-module M' is called isomorphic if there exist an (R, S)-module isomorphism from M to M', denoted by $M \simeq M'$.

Here are given some examples of the types of (R, S)-module homomorphism.

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Example 3.2. Let $\mathbb{Z}$ be an $(2\mathbb{Z}, 2\mathbb{Z})$-module. A function $f : \mathbb{Z} \to \mathbb{Z}$ where $f(a) = -a$ for each $a \in \mathbb{Z}$ is an $(2\mathbb{Z}, 2\mathbb{Z})$-module isomorphism. Let any $a, b \in \mathbb{Z}$ and $r, s \in 2\mathbb{Z}$, then we obtain:

$$f(a + b) = -(a + b) = -a - b = -a + (-b) = f(a) + f(b) \tag{5}$$

and

$$f(ras) = -(ras) = r(-a)s = rf(a)s. \tag{6}$$

From (5) and (6) it's shown that f is an (R, S)-module homomorphism. Furthermore, if asume that $f(a) = f(b)$, then $-a = -b$, so $a = b$. Thus, f is an injective function. Next, for each $a \in \mathbb{Z}$ there exist $-a \in \mathbb{Z}$ such that $a = -(-a) = f(-a)$. Thus, f is a surjective function. Hence, f is an $(2\mathbb{Z}, 2\mathbb{Z})$-module isomorphism.

Example 3.3. Let $\mathbb{Z}$ be an $(2\mathbb{Z}, 2\mathbb{Z})$-module and an $(2\mathbb{Z}, 2\mathbb{Z})$-module homomorphism $f : \mathbb{Z} \to \mathbb{Z}$ where $f(a) = 3a$ for each $a \in \mathbb{Z}$. We can show that f is an $(2\mathbb{Z}, 2\mathbb{Z})$-module monomorphism. Let any $a, b \in \mathbb{Z}$ where $f(a) = f(b)$, then $3a = 3b$. Consequently, we obtain $a = b$, so f is an injective function.

Example 3.4. Let $\mathbb{Z}$ be an $(2\mathbb{Z}, 3\mathbb{Z})$-module and $2\mathbb{Z}$ be an $(2\mathbb{Z}, 3\mathbb{Z})$-submodule of $\mathbb{Z}$. The function $f : \mathbb{Z} \to \mathbb{Z}/2\mathbb{Z}$ where $f(a) = a + 2\mathbb{Z}$ for each $a \in \mathbb{Z}$ is an $(2\mathbb{Z}, 3\mathbb{Z})$-module epimorphism. Let any $a, b \in \mathbb{Z}$, $r \in 2\mathbb{Z}$, and $s \in 3\mathbb{Z}$. Then,

$$f(a + b) = (a + b) + 2\mathbb{Z} = (a + 2\mathbb{Z}) + (b + 2\mathbb{Z}) = f(a) + f(b) \tag{7}$$

and

$$f(ras) = (ras) + 2\mathbb{Z} = r(a + 2\mathbb{Z})s = rf(a)s. \tag{8}$$

From (7) and (8) it's proved that f is an $(2\mathbb{Z}, 3\mathbb{Z})$-module homomorphism. Next, let any $y + 2\mathbb{Z} \in \mathbb{Z}/2\mathbb{Z}$, then there exist $y \in \mathbb{Z}$ such that satisfy $fy) = y + 2\mathbb{Z}$. Hence, f is a surjective function. Hence, f is an $(2\mathbb{Z}, 3\mathbb{Z})$-module epimorphism.

The following proposition gives a necessary and sufficient condition for a monomorphism to be a injective mapping in terms of its kernel.

Proposition 3.5. *Let $f : M \to M'$ be an (R, S)-module homomorphism. Then f is injective if and only if $Ker(f) = \{0_M\}$.*

Proof. $(\Rightarrow)$. Let any $a \in Ker(f)$. Then, $f(a) = 0_{M'} = f(0_M)$. Since f is injective, we must have $a = 0_M$. Hence, $Ker(f) = \{0_M\}$.
$(\Leftarrow)$. Let any $a, b \in M$ and we may assume $f(a) = f(b)$. Then $f(a - b) = f(a) - f(b) = f(a) - f(a) = 0_{M'}$. Thus we get $a - b \in Ker(f)$. Since $Ker(f) = \{0_M\}$, then we get $a - b = 0_M$. i.e. $a = b$. Thus, it's proved that f is injective. □

In Proposition 2.9, we showed that if f is an (R, S)-module homomorphism of an (R, S)-module M in to an (R, S)-module M', then $Ker(f)$ is an (R, S)-submodule of M. In the following proposition, we show that every (R, S)-submodule N of M induces an (R, S)-module homomorphism g of M onto the quotient (R, S)-module M/N such that $Ker(g) = N$.

Proposition 3.6. *Let M be an (R, S)-module and N is an (R, S)-submodule of M. Define the function g from M onto the quotient (R, S)-module M/N by $g(a) = a + N$ for all $a \in G$. Then, g is an (R, S)-module homomorphism of M onto M/N and $Ker(g) = N$. Moreover, the (R, S)-module homomorphism g is called the* ***natural** (R, S)**-module homomorphism*** *of M onto M/H.*

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Proof. From the definition of g, it follows that g is a function from M onto M/N. To show g is homomorphism, let $a, b \in M$. Then $g(a+b) = (a+b)+N = (a+N)+(b+N) = g(a)+g(b)$. Hence, g is an (R, S)-module homomorphism of M onto M/N. Next, we will show that $Ker(g) = N$. Now $a \in Ker(g)$ if and only if $g(a) = 0_M + N$ if and only if $a + N = 0_M + N$ if and only if $a \in N$. Thus, $Ker(g) = N$. □

Now, we give a proposition that show us the relationship between (R, S)-module homomorphisms and quotient (R, S)-modules.

Proposition 3.7. *Let $f : M \to M'$ be an (R, S)-module homomorphism and N be an (R, S)-submodule of M contained in $Ker(f)$. Let g be the natural (R, S)-module homomorphism of M onto M/N. Then there exists a unique homomorphism h of M/N onto M' such that $f = h \circ g$. Furthermore, h is injective if and only if $N = Ker(f)$.*

Based on Proposition 3.7, it follows that if $N = Ker(f)$, then h is an (R, S)-module isomorphism and hence $M/Ker(f)$ is isomorphic to M'. So, we obtain that every homomorphism of a group M onto a grup M' induces an isomorphism of $M/Ker(f)$ onto M'. This result is known as **the fundamentals theorem of (R, S)-module homomorphism**. This result is also called the first (R, S)-module isomorphism theorem.

Theorem 3.8. *(**First** (R, S)**-Module Isomorphism Theorem**). Let M and M' be (R, S)-modules. Let $f : M \to M'$ be an (R, S)-module homomorphism, then $M/Ker(f) \simeq Im(f)$.*

Proof. Let $K = Ker(f)$ and define $\theta : M/K \to Im(f)$ by $\theta(m + K) = f(m)$ for all $m + K \in M/K$. Let any $a + K, b + K \in M/K$. Now $a + K = b + K$ if and only if $a - b \in K$ if and only if $f(a - b) = 0_{M'}$ if and only if $f(a) = f(b)$ if and only if $\theta(a + K) = \theta(b + K)$. Thus, θ is an injective function. Let any $y \in Im(f)$. Then, then $y = f(a)$ for some $a \in M$. Therefore, $\theta(a + K) = f(a) = y$. This show that θ is a surjective function. Finally, let any $a + K, b + K \in M/K$, $r \in R$, and $s \in S$. Then,

$$\theta((a + K) + (b + K)) = \theta((a + b) + K) = f(a + b) = f(a) + f(b) = \theta(a + K) + \theta(b + K)$$

and

$$\theta(r(a + K)s) = \theta((ras) + K) = f(ras) = rf(a)s = r\theta(a + K),$$

proving that θ is an (R, S)-module homomorphism. Consequently, $M/Ker(f) \simeq Im(f)$. □

If the function $f : M \to M'$ on First (R, S)-module Isomorphism Theorem is an (R, S)-module epimorphism, then it's clear that $M/Ker(f) \simeq M'$.

Theorem 3.9. *(**Second** (R, S)**-Module Isomorphism Theorem**). Let M be an (R, S)-module. If N and P be (R, S)-submodules of M, then $(N + P)/P \simeq N/(N \cap P)$.*

Proof. Let $\pi : M \to M/P$ be the natural (R, S)-module homomorphism and π_0 be the restriction of π to N. Then π_0 is an (R, S)-module homomorphism with $Ker(\pi_0) = N \cap P$ and $Im(\pi_0) = (N + P)/P$. The result then follows from the First (R, S)-Module Isomorphism Theorem. □

Theorem 3.10. *(**Third** (R, S)**-Module Isomorphism Theorem**). Let M be an (R, S)-module. If N and P be (R, S)-submodules of M with $P \subseteq N$, then $M/N \simeq (M/P)/(N/P)$.*

Proof. Define $f : M/P \to M/N$ by $f(m + P) = m + N$. This is well defined (R, S)-module homomorphism and $Ker(f) = \{m + P \mid m + N = N\} = \{m + P \mid m \in N\} = N/P$. The result then follows from the First (R, S)-Module Isomorphism Theorem. □

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4. Conclusion

In this paper we have discussed the (R, S)-module homomorphism, their properties, their types, and the fundamentals theorem of (R, S)-module isomorphisms. The result show that the definition and the properties of (R, S)-module homomorphisms can be generalized from the definition and the properties of module homomorphism. As well as module theory, the types of (R, S)-module homomorphism include (R, S)-module monomorphisms, (R, S)-module epimorphisms and (R, S)-module isomorphisms. Moreover, the investigation of this study is closed by the fundamentals theorem of (R, S)-module isomorphisms.

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