# STRONGLY SUMMABLE VECTOR-VALUED SEQUENCE SPACES DEFINED BY 2-MODULAR

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# Strongly Summable Vector Valued Sequence Spaces Defined by 2 Modular

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**ABSTRACT**

Summability is an important concept in sequence spaces. One summability concept is strongly Cesaro summable. In this paper, we study a subset of the set of all vector-valued sequence in 2-modular space. Some facts that we investigated in this paper include linearity, the existence of modular and completeness with respect to these modular.

**Keywords**: Strongly; Summable; Sequence Spaces; 2-modular

## INTRODUCTION

Summability is an important concept in sequence spaces. The familiar example of sequence spaces that using the summability concept is $\ell^p$ spaces. In [1], it is explained that Kutner discusses spaces of strongly Cesaro summable sequences, and furthermore, Maddox generalizes this concept. If $\omega$ denote the set of all infinite sequence of real/complex numbers, then the set

$$w = \left\{(x_k) \in \omega : \exists L, \ni \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} |x_k - L| = 0\right\},$$

denote the space of strongly Cesaro summable sequence [2] [3].

Let $X$ be a real linear space of dimension $d \geq 2$. A 2-norm on $X$ is a function $\|.,.\|: X \times X \to \mathbb{R}$, where for all $x, y, z \in X$, satisfy

(i) $\|x, y\| = 0$ if and only if $x$ and $y$ are linearly dependent
(ii) $\|x, y\| = \|y, x\|$
(iii) $\|\alpha x, y\| = |\alpha|\|x, y\|$, $\alpha \in \mathbb{R}$
(iv) $\|x + y, z\| \leq \|x, z\| + \|y, z\|$.

The pair $(X, \|.,.\|)$ is then called a 2-normed space [4]. The concept is initially introduced by Gahler [5] in the middle of 1963. Furthermore, in 1989, Misiak generalized the 2-normed concept to be $n$-normed [6]. Since then, many kinds research on 2-normed ($n$-normed) spaces, include research on strongly Cesaro summable vector-valued sequences or the generalize in 2-normed ($n$-normed) spaces [7] [8] [9] [10] [11].

In 1950, Nakano developed modular function and it was generalized by Musielak and Orlicz [12] [13]. Modular is the generalization of the norm. Let $Y$ be a real linear space, a functional $g: Y \to \mathbb{R}^*$ is said tobe modular if it satisfies the following conditions:

(i) $g(x) = 0$ if and if $x = 0$
(ii) $g(-x) = g(x)$
(iii) $g(\alpha x + \beta y) \leq g(x) + g(y)$, every $x, y \in Y$, $\alpha, \beta \geq 0$, $\alpha + \beta = 1$.

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The pair $(Y, g)$ is then called a modular space. Following the 2-norm ($n$-norm) concept, K. Nourouzi and S. Shabanian in 2009 initially introduced the $n$-modular concept [14] [15]. Let $X$ be a real linear space of dimension $d \geq 2$. A 2-modular on $X$ is a function $\rho(.,.): X \times X \to \mathbb{R}^*$ where for all $x, y, z \in X$, satisfy

(i) $\rho(x, y) = 0$ if and only if $x$ and $y$ are linearly dependent
(ii) $\rho(x, y) = \rho(y, x)$
(iii) $\rho(-x, y) = \rho(x, y)$,
(iv) $\rho(\alpha x + \beta y, z) \leq \rho(x, z) + \rho(y, z)$, every $\alpha, \beta \geq 0, \alpha + \beta = 1$.

The pair $(X, \|.,.\|)$ is then called a 2-modular space. The 2-modular space, with $\rho$ satisfies $\Delta_2$-condition, if there exist $L > 0$, such that

$$\rho(2x, y) \leq L\rho(x, y),$$

for all $x, y \in X$. A sequence $(x_k)$ in $X$ is said to be 2-modular convergent to $x_0 \in X$ if

$$\lim_{k\to\infty} \rho(x_k - x_0, y) = 0, \forall y \in X.$$

It means that for every $\epsilon > 0$, there exists an $k_0 \in \mathbb{N}$, such that for any $k \in \mathbb{N}$, $k \geq k_0$, we have

$$\rho(x_k - x_0, y) < \epsilon, \forall y \in X.$$

Furthermore, a sequence $(x_k)$ in $X$ is called 2-modular Cauchy sequence if, for all $y \in X$, we have

$$\lim_{k,l\to\infty} \rho(x_k - x_l, y) = 0.$$

The standard example of a 2-modular space is $X = \mathbb{R}^2$, with 2-modular on $\mathbb{R}^2$ define by

$$\rho(\bar{x}, \bar{y}) = \sqrt{\left|\det\begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix}\right|},$$

Where $\bar{x} = (x_1, x_2), \bar{y} = (y_1, y_2) \in \mathbb{R}^2$. Clearly that $\rho$ satisfies $\Delta_2$-condition and the sequence $\left(\left(\frac{1}{n}, 0\right)\right)$ in $\mathbb{R}^2$ is 2-modular convergent to $(0,0) \in \mathbb{R}^2$.

This paper will be constructed t spaces of strongly Cesaro summable vector-valued sequences in 2-modular spaces based on the facts presented above.

## METHODS

Let $(X, \rho)$ be a 2-modular space, with $\rho$ satisfies $\Delta_2$-condition and the dimension of $X$ greater than one. We define

$$X_\rho = \{x \in X: \rho(x, y) < \infty, \forall y \in X\}.$$

Because $\rho$ satisfies $\Delta_2$-condition, then there exists $K > 0$, such that for all $x, y \in X_\rho, z \in X$ and $\alpha \in \mathbb{R}$, we have

$$\begin{aligned} \rho(x + y, z) &= \rho\left(\frac{2x + 2y}{2}, z\right) \\ &\leq \rho(2x, z) + \rho(2y, z) \\ &\leq K\rho(x, z) + K\rho(y, z) \\ &< \infty \end{aligned}$$

Based on Archimedean property, there exists $n_0 \in \mathbb{N}$, such that $\alpha \leq 2^{n_0}$

$$\begin{aligned} \rho(\alpha x, z) &\leq \rho(2^{n_0} x, z) \\ &\leq K^{n_0} \rho(x, z) \\ &< \infty. \end{aligned}$$

Hence, we have that $X_\rho$ is a subspace linear of $X$. Furthermore $(X_\rho, \rho)$ is a 2-modular space too.

The notation $\omega(X_\rho)$ will donate as the set of all sequences in $X_\rho$

$$\omega(X_\rho) = \{(x_k): x_k \in X, k \in \mathbb{N}\} \quad (1)$$

where linear space operations are defined coordinatewise,

$$(x_k) + (y_k) = (x_k + y_k), \qquad \alpha(x_k) = (\alpha x_k)$$

for all $(x_k), (y_k) \in \omega(X_\rho)$ and $\alpha \in \mathbb{R}$.

The goal of this paper is that we want to extend the concept of strongly Cesaro summable to 2-modular spaces valued sequences, defined as

$$w_0^\rho(X_\rho) = \left\{(x_k) \in \omega(X_\rho): \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \rho(x_k, y) = 0, \forall y \in X_\rho\right\} \quad (2)$$

$$w^\rho(X_\rho) = \left\{(x_k) \in \omega(X_\rho): \exists x_0 \in X_\rho, \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \rho(x_k - x_0, y) = 0, \forall y \in X_\rho\right\} \quad (3)$$

Furthermore, we also studied the properties of $w_0^\rho(X_\rho)$ and $w^\rho(X_\rho)$.

## RESULTS AND DISCUSSION

Henceforth, if not specified then $X$ is a 2-modular space with 2-modular $\rho$, that satisfies the $\Delta_2$-conditions.

First, we will prove that the mean Cesaro theorem applies to 2-modular space.

**Theorem 1.** Let sequence $(x_k)$ in $X_\rho$ 2-modular convergent to $x_0 \in X_\rho$, then

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \rho(x_k - x_0, y) = 0, \forall y \in X_\rho$$

**Proof.** Since the sequence $(x_k)$ in $X_\rho$ 2-modular convergent to $x_0 \in X_\rho$, then for all $\epsilon > 0$, there exists $n_\epsilon \in \mathbb{N}$, such that for all $k \geq n_\epsilon$, we have

$$\rho(x_k - x_0, y) < \frac{\epsilon}{2},$$

for all $y \in X$. Note that, for all $n \geq n_\epsilon$, we have

$$\begin{aligned}
\frac{1}{n}\sum_{k=1}^{n} \rho(x_k - x_0, y) &= \frac{1}{n}\sum_{k=1}^{n_\epsilon} \rho(x_k - x_0, y) + \frac{1}{n}\sum_{k=n_\epsilon+1}^{n} \rho(x_k - x_0, y) \\
&\leq \frac{1}{n}\sum_{k=1}^{n_\epsilon} \max_{1\leq k\leq n_\epsilon} \rho(x_k - x_0, y) + \frac{1}{n}\sum_{k=n_\epsilon+1}^{n} \max_{n_\epsilon+1\leq k\leq n} \rho(x_k - x_0, y) \\
&= \frac{\max_{1\leq k\leq n_\epsilon} \rho(x_k - x_0, y)}{n}\sum_{k=1}^{n_\epsilon} 1 + \frac{\max_{n_\epsilon+1\leq k\leq n} \rho(x_k - x_0, y)}{n}\sum_{k=n_\epsilon+1}^{n} 1 \\
&= \max_{1\leq k\leq n_\epsilon} \rho(x_k - x_0, y)\frac{n_\epsilon}{n} + \max_{n_\epsilon+1\leq k\leq n} \rho(x_k - x_0, y)\frac{n - n_\epsilon}{n} \\
&= \max_{1\leq k\leq n_\epsilon} \rho(x_k - x_0, y)\frac{n_\epsilon}{n} + \max_{n_\epsilon+1\leq k\leq n} \rho(x_k - x_0, y) \\
&= \max_{1\leq k\leq n_\epsilon} \rho(x_k - x_0, y)\frac{n_\epsilon}{n} + \frac{\epsilon}{2}.
\end{aligned}$$

By Archimedean property, there exists $n' \geq n_\epsilon$ , such that for all $n \geq n'$, we have

$$\max_{1\leq k\leq n_\epsilon} \rho(x_k - x_0, y)\frac{n_\epsilon}{n} < \frac{\epsilon}{2}.$$

Hence, for all $n \geq n'$, we have

$$\frac{1}{n}\sum_{k=1}^{n} \rho(x_k - x_0, y) < \epsilon.$$

In other words, the proof is complete. ■

Based on Theorem 1, we can say that for all 2-modular convergent sequence $(x_k)$ in $X_\rho$ is an element of $w^\rho(X_\rho)$.

**Theorem 2.** The set $w^\rho(X_\rho)$ is a linear subspace of $\omega(X_\rho)$.

**Proof.** Note that for all $(x_k),(y_k) \in w^\rho(X_\rho)$ and $\alpha \in \mathbb{R}$, there exsist $x_0, y_0 \in X_\rho$ so that for all $y \in X_\rho$, we have

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \rho(x_k - x_0, y) = 0, \text{and} \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \rho(x_k - y_0, y) = 0.$$

Therefore, $\rho$ satisfy $\Delta_2$-condition, then there exists $L > 0$ and $n_0 \in \mathbb{N}$ so that

$$\begin{aligned} 0 \leq \rho\big((x_k + y_k) - (x_0 + y_0), y\big) &= \rho\big((x_k - x_0) + (y_k - y_0), y\big) \\ &\leq \rho(2(x_k - x_0), y) + \rho(2(y_k - y_0), y) \\ &\leq L\rho\big((x_k - x_0), y\big) + L\rho\big((y_k - y_0), y\big) \end{aligned}$$

and

$$\begin{aligned} 0 \leq \rho(\alpha x_k - \alpha l, y) &= \rho(\alpha(x_k - l), y) \\ &\leq \rho(2^{n_0}(x_k - x_0), y) \\ &\leq L^{n_0}\rho(x_k - x_0, y). \end{aligned}$$

Hence, we have

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \rho((x_k + y_k) - (x_0 + y_0), y) = 0$$

and

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \rho(\alpha x_k - \alpha x_0, y) = 0.$$

In other words $(x_k) + (y_k)$, $\alpha(x_k) \in w^\rho(X_\rho)$, and we proof that $w^\rho(X_\rho)$ is a subspace linear of $\omega(X_\rho)$.■

**Theorem 3.** If $(x_k) \in w^\rho(X_\rho)$, then for all $y \in X_\rho$, $\left(\frac{1}{n}\sum_{k=1}^{n} \rho(x_k, y)\right)$ is a bounded sequence of real numbers.

**Proof.** If $(x_k) \in w^\rho(X_\rho)$, then there exist $x_0 \in X_\rho$, such that for all $y \in X_\rho$, we have

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \rho(x_k - x_0, y) = 0.$$

Hence, there exist $n_0 \in \mathbb{N}$, such that for all $n \in \mathbb{N}$, with $n \geq n_0$ we have

$$\frac{1}{n}\sum_{k=1}^{n}\rho(x_k - x_0, y) \leq 1.$$

Since $\rho$ satisfies the $\Delta_2$-conditions, there exist $L > 0$, for all $y \in X_\rho$, we have

$$\rho(x_k, y) = \rho\left(\frac{2(x_k - x_0)}{2} + \frac{2x_0}{2}, y\right) \leq L\rho(x_k - x_0, y) + L\rho(x_0, y).$$

It implies,

$$\frac{1}{n}\sum_{k=1}^{n}\rho(x_k, y) \leq \frac{L}{n}\sum_{k=1}^{n}\rho(x_k - x_0, y) + L\rho(x_0, y).$$

If we set

$$M = \sup\left\{\rho(x_1 - x_0, y), \frac{1}{2}\sum_{k=1}^{2}\rho(x_k - x_0, y), \cdots, \frac{1}{n_0 - 1}\sum_{k=1}^{n_0-1}\rho(x_1 - x_0, y), 1\right\}$$

then it follows that we have $K = L(M + \rho(x_0, y))$, such that

$$\frac{1}{n}\sum_{k=1}^{n}\rho(x_k, y) \leq K,$$

for all $n \in \mathbb{N}$. This implies that for all $y \in X_\rho$, $\left(\frac{1}{n}\sum_{k=1}^{n}\rho(x_k, y)\right)$ is a bounded sequence. ∎

**Theorem 4.** Function

$$g((x_k)) = \sup\left\{\frac{1}{n}\sum_{k=1}^{n}\rho(x_k, z), \forall z \in X_\rho\right\} \quad (5)$$

is a modular on $w^\rho(X_\rho)$.

**Proof.** If $(x_k) = \mathbf{0}$ is the zero sequence. Then it is clear that $g((x_k)) = 0$. Conversely, if $((x_k)) = 0$, then we have

$$\sup\left\{\frac{1}{n}\sum_{k=1}^{n}\rho(x_k, z), \forall z \in X_\rho\right\} = 0.$$

Hence, it implies for all $n \in \mathbb{N}$ and $z \in X_\rho$, we have

$$\frac{1}{n}\sum_{k=1}^{n}\rho(x_k, y_k) = 0 \Leftrightarrow \rho(x_k, z) = 0 \Leftrightarrow x_k = 0, \forall k \in \mathbb{N}.$$

Thus, it is evident that $(x_k) = \mathbf{0}$.

Since $\rho(-x, y) = \rho(x, y)$ applies, for all $x, y \in X_\rho$, consequently, it is clear that $g(-(x_k)) = g((x_k))$. Finally, for all $\alpha, \beta \geq 0$ with $\alpha + \beta = 1$, the for all $(x_k), (y_k) \in w^\rho(X_\rho)$ we have,

$$g(\alpha(x_k) + \beta(y_k)) = \sup\left\{\frac{1}{n}\sum_{k=1}^{n}\rho(\alpha x_k + \beta\, y_k, z), \forall z \in X_\rho\right\}$$

$$= \sup\left\{\frac{1}{n}\sum_{k=1}^{n}\left(\rho(x_k, z) + \rho(y_k, z)\right), \forall z \in X_\rho\right\}$$

$$\leq \sup\left\{\frac{1}{n}\sum_{k=1}^{n}\rho(x_k, z), \forall z \in X_\rho\right\} + \sup\left\{\frac{1}{n}\sum_{k=1}^{n}\rho(y_k, z), \forall z \in X_\rho\right\}$$
$$= g((x_k)) + g((y_k)).$$

This completes the proof. ■

**Theorem 5.** If $X_\rho$ 2-modular complete, then $\left(w^\rho(X_\rho), g\right)$ is a modular complete.

**Proof.** Let $n \in \mathbb{N}$ and $\left(x^i\right)$ be a 2-modular Cauchy sequence in $w^\rho(X_\rho)$, where $x^i = (x_k^i)$, for all $i \in \mathbb{N}$. Hence, for all $\epsilon > 0$, there exists $n_0 \in \mathbb{N}$, such that for all $i, j \in \mathbb{N}$, with $i, j \geq n_0$, we have

$$g\left(x^i - x^j\right) = \sup\left\{\frac{1}{n}\sum_{k=1}^{n}\rho\left(x_k^i - x_k^j, z\right), \forall z \in X_\rho\right\} < \epsilon.$$

It implies that, for all $i, j \geq n_0$, we have

$$\frac{1}{n}\sum_{k=1}^{n}\rho\left(x_k^i - x_k^j, z\right) < \epsilon, \forall z \in X_\rho,$$

or

$$\sum_{k=1}^{n}\rho\left(x_k^i - x_k^j, z\right) < n\epsilon, \forall z \in X_\rho,$$

such that,

$$\rho\left(x_k^i - x_k^j, z\right) < n\epsilon, \forall z \in X_\rho.$$

Hence, for all $k \in \mathbb{N}$, $(x_k^i)$ is a $\rho$-Cauchy sequence in $X_\rho$. Since $X_\rho$ complete 2-modular, then $(x_k^i)$ is 2-modular convergent in $X_\rho$, for all $k \in \mathbb{N}$. Therefore, for $k \in \mathbb{N}$, there exist $x_k \in X_\rho$, such that for all $z \in X_\rho$, we have

$$\lim_{i\to\infty}\rho\left(x_k^i - x_k, z\right) = 0.$$

Since, for all $i, j \geq n_0$, we have

$$\frac{1}{n}\sum_{k=1}^{n}\rho\left(x_k^i - x_k, z\right) = \lim_{j\to\infty}\frac{1}{n}\sum_{k=1}^{n}\rho\left(x_k^i - x_k^j, z\right) < \epsilon, \forall z \in X_\rho,$$

then

$$g\left(\left(x_k^i\right) - (x_k)\right) = \sup\left(\frac{1}{n}\sum_{k=1}^{n}\rho\left(x_k^i - x_k, z\right)\right) < \epsilon, \text{for all } i \geq n_0,$$

such that

$$\rho\left(x_k^i - x_k, z\right) < n\epsilon, \text{for all } i \geq n_0$$

Therefore $(x^i)$ modular convergent to $(x_k)$, and $\left(x_k^i - x_k\right) \in w\left(X_\rho\right)$. Since $\left(x_k^i\right) \in w\left(X_\rho\right)$ and $w\left(X_\rho\right)$ is a linear spaces, so we have

$$(x_k) = \left(x_k^i\right) - \left(x_k^i - x_k\right) \in w\left(X_\rho\right).$$

This complete the proof that $\left(w^\rho(X_\rho), g\right)$ is a complete modular ($\rho$-complete). ■

## CONCLUSIONS

If $(X, \rho)$ is a 2-modular space, with $\rho$ satisfies $\Delta_2$-condition, then we can construct $w^\rho\left(X_\rho\right) \subset w\left(X_\rho\right)$ is the space of strongly Cesaro summable vector-valued sequences in 2-modular $\left(X_\rho, \rho\right)$. It certainly can be shown that $w^\rho\left(X_\rho\right)$ is a linear space. Furthermore, if $(x_k) \in w^\rho\left(X_\rho\right)$, then we can prove that for all $y \in X_\rho$, $\left(\frac{1}{n}\sum_{k=1}^{n}\rho(x_k, y)\right)$ is a bounded sequence of real numbers. This fact provides a guarantee for us to be able to build a modular $g$ on $w^{\rho}(X_{\rho})$. Finally, we proved that $(w^{\rho}(X_{\rho}), g)$ is modular complete, if $(X_{\rho}, \rho)$ is a 2-modular complete.

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