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OPTIMIZATION OF PORTFOLIO USING FUZZY SELECTION

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Abstract. *The problem of portfolio optimization concerns the allocation of the investor's wealth between several security alternatives so that the maximum profit can be obtained. One of the methods used is Fuzzy Portfolio Selection to understand it better. This method separates the objective function of return and the objective function of risk to determine the limit of the membership function that will be used. The goal of this study is to understand the application of the Fuzzy Portfolio Selection method over shares that have been chosen on a portfolio optimization problem, understand return and risk, and understand the budget proportion of each claim. The subject of this study is the shares of 20 companies included in Bursa Efek Indonesia from 1 January 2021 until 1 January 2022. The result of this study shows that from 20 shares, there are 10 shares that is suitable in the forming of optimal portfolio, those are ADRO (0%), ANTM (43.3%), ASII (0%), BBCA (0%), BBRI (0%), BBTN (0%), BRPT (0%), BSDE (0%), ERAA (16%), and INCO (40.7%). The expected return from the portfolio is 0.0878895207 or 8.8% for the return and 0.0226022117 or 2.3% for the risk.*

Keywords: *fuzzy portofolio selectio, stocks, portofolio optimization, Indonesia Stock Exchange.*

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1. INTRODUCTION

Multi-Criteria Decision Making (MCDM) is a topic focused on determining the best alternative decision from other alternatives based on specific criteria as consideration. Based on its purpose, MCDM is divided into *Multi-Attribute Decision Making* (MADM) and *Multi-Objective Decision Making* (MODM), where MADM is used to solve problems in discreet areas. MADM often is used to do an evaluation or selection of several alternatives in the limited amount [1], % while MODM is the problem of decision making in which the main characteristic of MODM problem is the said decision needs to achieve many purposes, but there are contradicting purposes inside it. *Multi-Objective Linear Programming* (MOLP) is one of the critical forms for explaining MODM problems, where the purpose of the linear function that will be optimized, whether maximized or minimized, is depended on the set of linear problem[2]. For several last years, there has been a lot of research about MODM, including [3], [4], [5], and [6]. Based on the results from said research, the MODM method is integrated with fuzzy to optimize the wanted goal.

Multi-Objective Decision Making (MODM) is the problem of decision making in which the main characteristic of the MODM problem is the existence of contradicting purposes. MODM model considers the variable vector of decision, objective function, and constraint. The decision-making will be done by either maximizing or minimalizing the objective function. Since this problem is rarely seen and has a unique solution, the decision-making is hoped to be able to choose the solution over a group of efficient solutions (as an alternative).

Fuzzy Multi Objective Decision Making (FMODM) can be applied in various fields, one of which is a stock investment. The stock has become an investment instrument that is growing in popularity and is preferred by the public. Stock can be formed into an optimal portfolio. [7] has defined a portfolio as a group of assets in the shape of investment owned by an investor or company. Those investments can be found in the form of deposits, gold, stock, property, obligation, etc. There are two functions in a stock portfolio that are maximizing return and minimalizing risk. [8] Has shown that the investment of portfolio risk is dependent on covariant between assets. The average *return* will determine the portfolio *return*, while the risk is dependent on covariant between assets in the portfolio.

Numerous research has been done to solve and improve the Markowitz portfolio model. It's done to adapt the existing model to the finance market condition and demand from the capital market player. One of the focuses of the research in portfolio selection is the amount of return, risk, and budget proportion that is allocated in choosing the optimal portfolio [9],[10], and [11]. This can be understood because the more significant the involved security value in portfolio selection, the bigger chance of an optimal portfolio being formed. Many complicated securities in portfolio selection can be solved by classifying the data based on decided criteria. A security that is unable to fulfill the fixed criteria will not be used in the formation of an optimal portfolio [12],[13], dan [14].

In the several last years, lots of research about MODM on portfolio are being done, including [15], [16] and [17]. Based on the results of said research, MODM that is modeled using mean-variant of Markowitz in portfolio selection with new modification will result in various function variants of risk and *return,* so maximizing *return* a minimalizing portfolio valued as fuzzy can be done as other alternatives.

In the new contribution to past studies, this paper will use FMODM that is integrated with *the fuzzy* approach in which the proposed model is *fuzzy bi-objective* portfolio selection model that maximizes portfolio return and minimizes portfolio risk [18],[19],[20], and [18]. The writer uses *a fuzzy* interactive approach to solve the model, so the level of the desired goal in decision making is in relation to the purpose of *return,* and risk is achieved as close as possible.

The problem of the *bi-objective* portfolio optimization model is a squared programming problem. Considering the fact that in the application of portfolio selection in real life in which the decisions are often arranged around the unclear aspiration of investor in the desired portfolio concerning *return* and *risk*, the framework of *fuzzy* is used to predict the need for linguistic type information in portfolio selection problem. It is assumed that the investor shows aspiration based on experience and prior knowledge, and the linear membership function is used to define the level of unclear aspiration of the investor.

2. RESEARCH METHODS

The research method used is as follows.

2.1 Data Source

The data that is used in this study is the data of monthly stock close price that is registered in Bursa Efek Indonesia (BEI) in the span of 1 January 2020 until 1 January 2021, where the companies are included in the LQ45 market from the website https://finance.yahoo.com.

2.2 Multi-Objective Decision Making (MODM)

Multi-Objective Decision Making (MODM) considers the variable vector of decision, objective function, and constraint. Generally, the MODM problem can be written in the following formula:

$$\max f_k(\boldsymbol{x}) \quad (1)$$
$$s.t\ \boldsymbol{x} \in \boldsymbol{X} = \{\boldsymbol{x} \in R^n \mid g(\boldsymbol{x}) \le b, \boldsymbol{x} \ge 0\}$$

where $f_k(\boldsymbol{x})$ symbolizes k-objective function that is contradictory to each other, $g(\boldsymbol{x}) \le b$ symbolizes m-constraint and x is a n-vector which is a return from the decision variable with $\boldsymbol{x} \in R^n$[2].

Stock is security ownership of the company's assets that publish shares. By owning the share of a certain company, the investor has the right over the company's income and wealth after being subtracted by payment of all company's duties. It is a securities type that is quite popular in the capital market. The reason is that compared to other types of investments, the stock can give a bigger *return* or profit in a relatively short period [14].

The return can be defined as the result obtained after an investment. To calculate *return* can be used by formula written below [21]:

$$R_{it} = \frac{p_t - p_{(t-1)}}{p_{(t-1)}} \quad (3)$$

where

R_{it} : *Return* of share i in the time of t

p_t : *Close price* of share in the time of t

$p_{(t-1)}$: *Close price* of share in the time of t-1

And the *expected return* of share has a formula of[22]:

$$r_i = E[R_i] = \frac{1}{T}\sum_{t-1}^{T} R_{it} \quad (4)$$

where

r_i : *Expected Return* of share i in the time of t

R_{it} : *Return* of share i in the time of t

T : Time period

The risk between shares can be defined using covariant matrix in which covariant σ_{ij} is written in following formula:

$$\sigma_{ij} = \frac{1}{T}\sum_{t-1}^{T}(R_{it} - r_i)(R_{jt} - r_j) \quad (5)$$

where

r_i : *Expected Return* of share i in the time of t

R_{it} : *Return* of share i in the time of t

r_j : *Expected Return* of share j in the time of t

R_{jt} : *Return* of share j in the time of t

T : Time period

The portfolio optimization model of *bi-objective* based on the work frame of mean-variance, which Markowitz previously proposed, models are able, at the same time, to maximize the portfolio *return* $(f_1(x))$ and minimalize portfolio risk $(f_2(x))$,which is written is written in the following formula:

$$\max f_1(x) = \sum_{i=1}^{n} r_i x_i \tag{6}$$

$$\max f_2(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j$$

with constraint function:

$$\sum_{i=1}^{n} x_i = 1$$

$x_i \geq 0,\ i = 1,2,\cdots,\ n, j = 1,2,\cdots,\ n$

In which r_i is *expected return* and σ_{ij} is covariant between asset i and j.

The linear membership function is often used because of its simplicity and can be defined as repairing by using two points, and those are upper and lower acceptability. The continued explanation of the linear membership function is written below:

1. *The Membership function* of the objective function of the *return* portfolio can be defined as:

$$\mu_{f_1}(x) = \begin{cases} 1 & if \quad f_1(x) \geq f_1^R \\ \dfrac{f_1(x) - f_1^L}{f_1^R - f_1^L} & if \ f_1^L < f_1(x) < f_1^R \\ 0 & if \quad f_1(x) \leq f_1^L \end{cases}$$

In which f_1^L is the worst lower limit (low aspiration level) and f_1^R is the best upper limit (high aspiration level) from *the return* portfolio. The said explanation is illustrated in the following graph:

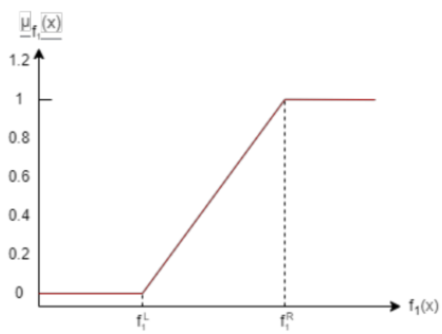

Figure 1. Membership function of objective function return portofolio

2. Membership function of the objective function of the risk portfolio can be defined as:

$$\mu_{f_2}(x) = \begin{cases} 1 & if \quad f_2(x) \leq f_2^R \\ \dfrac{f_2^R - f_2(x)}{f_2^R - f_2^L} & if \ f_2^L < f_2(x) < f_2^R \\ 0 & if \quad f_2(x) \geq f_2^L \end{cases}$$

Where f_2^L is the best lower limit (low aspiration level) and f_2^R is the worst upper limit (high aspiration level) from the risk portfolio. The explanation is illustrated in the following graph:

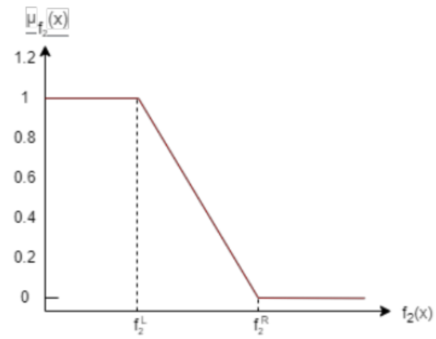

Figure 2. Membership function of objective function risk portofolio

The optimization model of *fuzzy bi-objective* in solving portfolio problems can be written in the following formula:

$$\max \lambda \tag{7}$$

that depends on

$$\lambda \leq \mu_{f_1}(x),$$
$$\lambda \leq \mu_{f_2}(x),$$
$$\sum_{i=1}^{n} x_i = 1,$$
$$x \geq 0, i = 1,2,3,\cdots,n,$$
$$0 \leq \lambda \leq 1$$

where λ is an additional variable that represents *membership function*.

Solving the lambda model by interactive *fuzzy* approach consists of several following steps:

1. Forming mathematic model in equation 6
2. Solving equation 6 as the objective single problem in relation to objective function of *return* and risk. Mathematically,
   a. Objective function of *return*
   $$\max f_1(x) \text{ with constraint } \sum_{i=1}^{n} x_i = 1 \text{ and } x_i \geq 0, \quad i = 1,2,3,\cdots,n$$
   b. Objective function of *return*
   $$\max f_2(x) \text{ with constraint } \sum_{i=1}^{n} x_i = 1 \text{ and } x_i \geq 0, \quad i = 1,2,3,\cdots,n$$
   Assuming x^1 and x^2 are defined as the optimum solution by solving a single objective problem with an objective function of *return* and risk. If both solutions are defined as the same $x^1 = x^2 = (x_1, x_2, \cdots, x_n)$, an efficient solution can be obtained. If it doesn't stop, it's continued into the next step, Step 3.
3. Evaluation of obtained objective function

   Determining the worst lower limit (f_1^L) and the best upper limit (f_1^R) are done for the objective function of the return, while the best lower limit (f_2^L) and the worst upper limit (f_2^R) are done for the objective function of risk. The obtained limits are obtained and written in the following formulas:

$$f_1^R = f_1(x^1)$$
$$f_1^L = f_1(x^2)$$
$$f_2^L = f_2(x^2)$$

$$f_2^R = f_2(x^1)$$

4. Determining membership function for *return* and risk
5. Developing mathematic model in equation 7 and solving it. Giving the solution to the investor [22].

2.3 Procedures

This research was conducted with the following procedures:

1. Studying literature that is related to portfolio, stock, and portfolio optimization with fuzzy.
2. Accessing research data through website https://finance.yahoo.com.
3. Choosing 20 company share that is available in LQ45 market.
4. Calculating the *return* of each share and the *expected return*.
5. Determining which share that is going to be used in portfolio selection where the chosen share is the share that shows positive *expected return*.
6. Calculating the risk between the chosen shares.
7. Forming mathematic model, model of *bi-objective* optimization portfolio.
8. Solving the problem of model of *bi-objective* optimization portfolio as objective singular problem in relation of *return* objective function and risk. Mathematically it can be written as,
   a. Objective function of *return*

$$\max f_1(x) \text{ with constraint } \sum_{i=1}^{n} x_i = 1 \ and \ x_i \geq 0, \qquad i = 1,2,3,\cdots,n$$

   b. Objective function of *return*

$$\max f_2(x) \text{ with constraint } \sum_{i=1}^{n} x_i = 1 \ and \ x_i \geq 0, \qquad i = 1,2,3,\cdots,n$$

9. Evaluating the objective function over the obtained result

   Determining the worst lower limit (f_1^L) and the best upper limit (f_1^R) for objective function of *return* and the best lower limit (f_2^L) and the worst upper limit (f_2^R) for objective function of risk. The formula of obtained limits are:

$$f_1^R = f_1(x^1)$$
$$f_1^L = f_1(x^2)$$
$$f_2^L = f_2(x^2)$$
$$f_2^R = f_2(x^1)$$

10. Determining the *membership function* of *return* and risk.
11. Determining new model of the obtained *membership function*.
12. Doing an optimization on the new model and obtaining the result of portfolio optimization.

3. RESULTS AND DISCUSSION

The share that is used for optimization in this study is extracted from the calculation of *the expected return* from 20 shares in the span of 1 January 2020 until 1 January 2021, whether it has a positive value, negative value, or even zero value. A negative value on expected return means loss. Table 1 is the data of 10 shares that fulfill the criteria, which it has a positive *expected return*.

Table 1. Recapitulation of Positive *Expected Return*

Share Core	Minimum
ADRO	0.005059047
ANTM	0.125256193
ASII	0.007389624
BBCA	0.006712295
BBRI	0.003012295
BBTN	0.023017824
BRPT	0.006903961
BSDE	0.013594574
ERAA	0.069381350
INCO	0.055436168

The results in Table 1. above are obtained using equation 4. The portfolio will be formed based on the result of positive e*xpected returns,* which are the companies with share codes of ADRO, ANTM, ASII, BBCA, BBRI, BBTN, BRPT, BSDE, ERAA, INCO. The *expected return* of mentioned share codes will be used as an objective function of *return* in portfolio optimization.

The table above shows that the value of positive expected return from each company can be referred to as the amount of return value. The portfolio will be formed according to the result of the positive expected return value, which is shown in companies whose share codes are ADRO, ANTM, ASII, BBCA, BBRI, BBTN, BRPT, BSDE, ERAA, and INCO. The expected return value of share code ACES is calculated by adding ACES' share return value from February 2020 until January 2021 before being divided by the number of periods from February 2020 until January 2021. The expected return from the said share code will be used in portfolio optimization as the expected purpose function.

The risk of share portfolio optimization is calculated by the covariant matrix σ_{ij}. The risk calculation for each share code will be done to the total of 10 shares that have been selected using the mathematical model in equation 5. Table 2 is a covariant matrix between the chosen ten shares.

Table 2. Covariant Matrix

	ADRO	ANTM	ASII	⋯	INCO
ADRO	0.014027	0.003895	0.007622	⋯	0.005608
ANTM	0.003895	0.063823	0.026913	⋯	0.020877
ASII	0.007622	0.026913	0.020617	⋯	0.011754
⋮	⋮	⋮	⋮	⋱	⋮
INCO	0.005608	0.020877	0.11754	⋯	0.015859

The portfolio will be formed by using the result obtained from the covariant matrix and later will be used as an objective function of risk on portfolio optimization.

The data above is the calculation result of 10 companies in which the risks between companies are being identified, and then the data is used to form an optimization model for the risk purpose function. The portfolio then will be formed by using the obtained result from the covariant matrix and later will be used as a risk purpose function in portfolio optimization. For example, identifying the asset risk between ANTM and ADRO can be found by subtracting ANTM return in every period and multiplying it with ADRO's expected to return in every period before adding it with the result of subtraction between ADRO in each period with ADRO's expected return. The overall result then will be divided by the expected return.

According to the mathematical model in equation 6, a bi-objective portfolio optimization model will be formed by using the expected return value and the following covariant matrix. In which x_i stood for the company's assets as written in the table.

Table 3. Forming Optimization Model

Perusahaan	Simbol
ADRO	x_1

ANTM	x_2
ASII	x_3
BBCA	x_4
BBRI	x_5
BBTN	x_6
BRPT	x_7
BSDE	x_8
ERAA	x_9
INCO	x_{10}

$$\max f_1(x) = 0.005059047x_1+0.125256193x_2+0.0073896241x_3+0.006712202x_4+0.003012295x_5 +0.023017824x_6+0.006903961x_7+0.013594574x_8+0.069381350x_9+0.055436168x_{10}$$

$$\min f_2(x) = 0.014027x_1x_1+0.063823x_2x_2+0.020617x_3x_3+0.006214x_4x_4+0.015724x_5x_5 +0.072675x_6x_6+0.107596x_7x_7+0.0239071x_8x_8+0.039401x_9x_9+0.015859x_{10}x_{10} +0.003895x_1x_2+0.007622x_1x_3+0.003014x_1x_4+0.009273x_1x_5+0.002771x_1x_6 +0.002688xx_1x_7+0.005300x_1x_8+0.000201x_1x_9+0.005608x_1x_{10}+0.026913x_2x_3 +0.014066x_2x_4+0.015750x_2x_5+0.02986x_2x_6+0.021001x_2x_7+0.026859x_2x_8 +0.032864x_2x_9+0.020877x_2x_{10}+0.007577x_3x_4+0.012476x_3x_5+0.016134x_3x_6 +0.008705x_3x_7+0.012245x_3x_8+0.016975x_3x_{10}+0.011754x_3x_{10}+0.007677x_4x_5 +0.015393x_4x_6-0.001939x_4x_7+0.008068x_4x_8+0.007249x_4x_9+0.005356x_4x_{10} +0.022054x_5x_6+0.001801x_5x_7+0.014135x_5x_8+0.011427x_5x_9+0.00913x_5x_{10} +0.014710x_6x_7+0.034540x_6x_8+0.023911x_6x_9+0.013107x_6x_{10}+0.01912x_7x_8 +0.031895x_7x_9+0.020655x_7x_{10}+0.017318x_8x_9+0.010113x_8x_{10}+0.016136x_9x_{10}$$

with constraint function written as:

$$\sum_{i=1}^{n} x_i = 1,$$

In which variables $x_1, x_2, \ldots\ldots, x_{10}$ shows the amount of the proportional investment value of happening x_i is one of the happening is going to happen and no other possibilities left.

$$x_i \geq 0, \;\; i = 1,2,3,4,5,6,7,8,9,10.$$

The formula is obtained by changing *the double-objective* optimization model into *a single-objective* optimization model to determine the boundaries that are used to form the membership function. In determining the lower and upper limit, the *bi-objective* portfolio optimization model is formed into the following *single-objective* optimization model:

1. Objective Function of *Return*

$$\max f_1(x) = 0.005059047x_1+0.125256193x_2+0.0073896241x_3+0.006712202x_4 +0.003012295x_5+0.023017824x_6+0.006903961x_7+0.013594574x_8 +0.069381350x_9+0.055436168x_{10}$$

with constraint function:

$$\sum_{i=1}^{n} x_i = 1,$$

$$x_i \geq 0, \;\; i = 1,2,3,4,5,6,7,8,9,10.$$

\

2. Objective Function of Risk

$$\min f_2(x) = 0.014027x_1x_1+0.063823x_2x_2+0.020617x_3x_3+0.006214x_4x_4+0.015724x_5x_5 +0.072675x_6x_6+0.107596x_7x_7+0.0239071x_8x_8+0.039401x_9x_9+0.015859x_{10}x_{10} +0.003895x_1x_2+0.007622x_1x_3+0.003014x_1x_4+0.009273x_1x_5+0.002771x_1x_6$$

$$+0.002688xx_1x_7+0.005300x_1x_8+0.000201x_1x_9+0.005608x_1x_{10}+0.026913x_2x_3$$
$$+0.014066x_2x_4+0.015750x_2x_5+0.02986x_2x_6+0.021001x_2x_7+0.026859x_2x_8$$
$$+0.032864x_2x_9+0.020877x_2x_{10}+0.007577x_3x_4+0.012476x_3x_5+0.016134x_3x_6$$
$$+0.008705x_3x_7+0.012245x_3x_8+0.016975x_3x_{10}+0.011754x_3x_{10}+0.007677x_4x_5$$
$$+0.015393x_4x_6-0.001939x_4x_7+0.008068x_4x_8+0.007249x_4x_9+0.005356x_4x_{10}$$
$$+0.022054x_5x_6+0.001801x_5x_7+0.014135x_5x_8+0.011427x_5x_9+0.00913x_5x_{10}$$
$$+0.014710x_6x_7+0.034540x_6x_8+0.023911x_6x_9+0.013107x_6x_{10}+0.01912x_7x_8$$
$$+0.031895x_7x_9+0.020655x_7x_{10}+0.017318x_8x_9+0.010113x_8x_{10}+0.016136x_9x_{10}$$

with constraint function:

$$\sum_{i=1}^{n} x_i = 1,$$
$$x_i \geq 0,\ \ i = 1,2,3,4,5,6,7,8,9,10$$

After doing optimization for each objective function, whether it's maximizing or minimalizing the objective function of risk and *return*, the following is the obtained result:

Table 4. The Optimization Result of Objective Function of *Return* and Risk

	x^1	x^2
Return $(f_1(x))$	0.125256	0.00301229
Risk $(f_2(x))$	0.063823	0.00445511

with

$$f_1^R = f_1(x^1)$$
$$f_1^L = f_1(x^2)$$
$$f_2^L = f_2(x^2)$$
$$f_2^R = f_2(x^1)$$

so the *membership function* for objective function of *expected return* and portfolio risk are:

$$\mu_{f_1}(x) = \begin{cases} 1 & if\ f_1(x) \geq 0.125256 \\ \frac{f_1(x)-0.00301229}{0.125256-00301229} & if\ 0.00301229\ < f_1(x) < 0.125256 \\ 0 & if\ f_1(x) \leq 0.00301229 \end{cases}$$

and

$$\mu_{f_2}(x) = \begin{cases} 1 & if\ f_2(x) \leq 0.00445511 \\ \dfrac{0.063823 - f_2(x)}{0.063823 - 0.00445511} & if\ 0.00445511\ < f_2(x) < 0.063823 \\ 0 & if\ f_2(x) \geq 0.063823 \end{cases}$$

After *the membership function* is obtained for the objective function of *expected return* and risk, according to the mathematical model in equation 7, a new model of *bi-objective fuzzy* optimization can be formed to solve the portfolio selection problem and can be written as the following formula:

$$Max\quad \lambda$$

In which λ is an additional variable that represents membership function.
with constraint function:

$$0.005059047x_1+0.125256193x_2+0.0073896241x_3+0.006712202x_4+0.003012295x_5$$
$$+0.023017824x_6+0.006903961x_7+0.013594574x_8+0.069381350x_9+0.055436168x_{10}$$
$$-0.12224371\lambda \geq 0.00201299,$$
$$0.014027x_1x_1+0.063823x_2x_2+0.020617x_3x_3+0.006214x_4x_4+0.015724x_5x_5$$
$$+0.072675x_6x_6+0.107596x_7x_7+0.0239071x_8x_8+0.039401x_9x_9+0.015859x_{10}x_{10}$$

$$+0.003895x_1x_2+0.007622x_1x_3+0.003014x_1x_4+0.009273x_1x_5+0.002771x_1x_6+0.002688\mathrm{x}x_1x_7$$
$$+x_7 0.005300x_1x_8+0.000201x_1x_9+0.005608x_1x_{10}+0.026913x_2x_3+0.014066x_2x_4$$
$$+0.015750x_2x_5+0.02986x_2x_6+0.021001x_2x_7+0.026859x_2x_8+0.032864x_2x_9+0.020877x_2x_{10}$$
$$+0.007577x_3x_4+0.012476x_3x_5+0.016134x_3x_6+0.008705x_3x_7+0.012245x_3x_8$$
$$+0.016975x_3x_9+0.011754x_3x_{10}+0.007677x_4x_5+0.015393x_4x_6-0.001939x_4x_7$$
$$+0.008068x_4x_8+0.007249x_4x_9+0.005356x_4x_{10}+0.022054x_5x_6+0.001801x_5x_7$$
$$+0.014135x_5x_8+0.011427x_5x_9+0.00913x_5x_{10}+0.014710x_6x_7+0.034540x_6x_8$$
$$+0.023911x_6x_9+0.013107x_6x_{10}+0.01912x_7x_8+0.031895x_7x_9+0.020655x_7x_{10}$$
$$+0.017318x_8x_9+0.010113x_8x_{10}+0.016136x_9x_{10}+0.5936788\lambda \leq 0.063823,$$

with constraint function:

$$\sum_{i=1}^{n} x_i = 1,$$
$$x_i \geq 0, \quad i = 1,2,3,4,5,6,7,8,9,10.$$

The above model is optimized thus, there will be an obtained result of *expected return*, risk, and budget proportion that will be allocated from the portfolio, which is written as follows:

Table 5. The Result of *Return* and Risk

λ	*Return*	Risk
0.694328	0.694328	0.0226022117

Table 6. Budget Proportion of Optimization Result

Share Code	Minimum
ADRO	0.1149577×10^{-7}
ANTM	0.4328280
ASII	0.2856066×10^{-8}
BBCA	0.7728622×10^{-8}
BBRI	0.4587149×10^{-8}
BBTN	0.6810672×10^{-8}
BRPT	0.3030953×10^{-8}
BSDE	0.4877755×10^{-8}
ERAA	0.1601465
INCO	0.4070255

Table 5 is the result of the new model optimization with variable λ, where the obtained value of optimized *return* is 0.0878895207 while the risk is 0.0226022117. Table 6 is the result of new model optimization with variable λ, where the budget proportion that is allocated for share code ANTM is 43.3%, and for the other share codes like ADRO, ASII, BBCA, BBRI, BBTN, and BRPT is close to zero, so the budget proportion that is allocated is 0%, on the other side the share code of ERAA has a value of 16%, and INCO is 40.7%.

4. CONCLUSIONS

The study explains the method of choosing a share that will be optimized according to criteria that have been decided by the writer. The chosen share then will be formed into *a bi-objective* optimization model using ten shares that have fulfilled the criteria. *The bi-objective* optimization model then will be changed into *a single-objective* with the purpose of finding the limit that will be used in the making of *the fuzzy* optimization model. The optimized *fuzzy* optimization model results in an optimal result where the *return* of the optimized portfolio is 8.8%, and the risk of the optimized portfolio is 2.3%.

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