**International Journal on Emerging Mathematics Education (IJEME)**
Vol. 5, No. 2, September 2021, pp. 165-180
P-ISSN: 2549-4996, E-ISSN: 2548-5806, DOI: http://dx.doi.org/10.12928/ijeme.v5i2.19456

# Using Context to Learn Concepts of Matrices Meaningfully

**Yoppy Wahyu Purnomo[1], Nurlaela Rahmawati[2], Kadir[3]**
[1]Universitas Negeri Yogyakarta, Yogyakarta, Indonesia
[2]SMAN 6 Kota Tangerang Selatan, Tangerang Selatan, Indonesia
[3]UIN Syarif Hidayatullah, Jakarta, Indonesia
e-mail: yoppy.wahyu@uny.ac.id

**Abstract**

Mathematical concepts will be embedded if they can be imagined and appear in everyday life. Therefore, this study aims to develop Local Instructional Theory in context-based matrices' learning. Design research is adopted to achieve these objectives. The design research has three stages, namely, preliminary experiment, teaching experiment, and retrospective analysis. The study was conducted at a high school in Tangerang Selatan, Province of Banten, Indonesia. A preliminary teaching experiment was carried out in XI MIPA 1, and a teaching experiment was carried out in XI MIPA 2. The data was collected employing tests, observation, interviews, worksheets, and documentation in the form of video recordings, photos, and student journals. The results indicated that students were able to construct their concept of matrices through the context of favorites or hobbies (traveling and sport) as a starting point in compiling and defining matrices and continued with the matrices' multiplication concept (matrix scalar multiplication as repeated addition; terms and products of two matrices).

**Keywords**: Design research, Local instructional theory, Learning trajectory, Realistic mathematics education, Matrices.

***How to Cite***: Purnomo, Y. W., Rahmawati, N., & Kadir, K. (2016). Using Context to Learn Concepts of Matrices Meaningfully. *International Journal on Emerging Mathematics Education, 5*(2), 165-180. http://dx.doi.org/10.12928/ijeme.v5i2.19456.

## INTRODUCTION

Matrices are among the first mathematical topics that students learn at the high school level in Indonesian Curricula. Matrices are important to learn by students at the high school level because as form the basis for further learning of mathematics, such as calculus and transformation geometry and other sciences, such as physics, economics, engineering, and health. Its application can be seen in everyday life, including in the fields of the economy (e.g., sales and production), transportation (e.g., travel routes), and health (e.g., number of patients and drug supplies). Thus, the matrices' position in the mathematics curriculum encourages high school students to succeed in their further studies and careers (Rahmawati & Purnomo, 2020).

In contrast to its urgency, matrices are one of the most challenging and difficult mathematics topics for students at the high school (Aygör & Burhanzade, 2014; Kusuma et al., 2018; Lesmana et al., 2015) and higher education levels (Andrews-Larson et al., 2017; Cárcamo et al., 2019; Carlson, 1993). As the first author's experience as a teacher for 16 years in high school, the pattern of student difficulties in this material is that students have not been able to apply the matrices' concept to solve context-based problems, provide context-based examples, and rarely know the benefits of matrices in everyday life. Horton, Wiegert and Marshall (2008) stated that most students learn the procedure for solving matrices problems but do not know why they are learning it.

*Received January 11, 2021; Revised August 30, 2021; Accepted October 30, 2021*

Rahmawati and Purnomo (2020) argue that the mathematical concept will be embedded if students can imagine it or appear real in daily life. For example, the use of the Mass Rapid Transit (MRT) context to introduce mathematical concepts may be suitable for students in urban areas but not suitable for students in rural areas who do not know or cannot imagine the object. In addition, students should also be allowed to engage in a reinvention of mathematical concepts (Pramudiani et al., 2017; Purnomo et al., 2014; Wijaya, 2008; Wijaya et al., 2015). The process of reinventing mathematical concepts allows students to make connections between concepts and in turn, build a well-established cognitive structure (Kadir, 2004). Therefore, we adopted a realistic mathematics education approach to teaching matrices meaningfully and improve students' conceptual knowledge.

**Realistic Mathematics Education**

Mathematics learning with the Realistic Mathematics Education (RME) approach was first introduced and developed in the Netherlands in 1970 by the Freudenthal Institute (Pramudiani et al., 2017; Wijaya, 2008). RME emphasizes the skills of doing mathematics, discussing, collaborating, and arguing. RME also allows students to understand mathematical concepts by manipulating objects and tools (Arsaythamby & Zubainur, 2014).

RME refers to Freudenthal's opinion that mathematics must be related to reality, and mathematics is a human activity (Hendroanto et al., 2015). This statement contains meaning; the first is that mathematics must be related to reality, meaning that the material provided is essential to be associated with the context or something real (real or has been experienced or known or imagined by students) and, if possible, it is related to real-life everyday situations. The second is that mathematics is a human activity, meaning that RME must provide students with opportunities to carry out mathematical activities and engage themselves in learning; students discuss finding strategies or steps to solve problems. These two meanings are, of course, related to problems that are often encountered in class. That is why the students still have difficulty linking matrices' learning with relevant contexts.

In Gravemeijer (2004), Treffers argues the characteristics of RME such as using context, model, free production, interactive and constructive, and intertwined/integrated. The first one, the use of context, means learning through real problems in daily life or has been experienced by students or come out of their own. This learning bridges mathematical concepts with everyday experiences. The second one, uses model meaning that students make their models in solving problems. The model used is diagrams, tables, sketch pictures, symbols, or students' informal answers. The third one, free production means students are making various ways of solving problems; students are encouraged to reflect on the parts they consider essential in the learning process. Students' informal strategies in the form of contextual problem-solving procedures are sources of inspiration in constructing formal mathematical knowledge. The fourth one, the interactive and constructive, explicitly means the forms of interaction in the form of negotiation, explanation, justification, agreement, disagreement, question, or reflection, are used to achieve formal forms from the informal forms of students. The last one, the meaning of intertwined in RME is the relationship between mathematical concepts with other subjects or materials. In solving a problem, one must know the relationship between the materials. Mathematical applications usually require more complex knowledge of arithmetic, algebra, or geometry, and other fields.

In Revina (2017), Freudenthal states the principles of RME include three things, namely guided reinvention and didactical phenomenology, progressive mathematization, self-develop models. Guided reinvention and didactical phenomenology mean that RME guides students to find their strategies/ways of solving a problem according to their cognitive level because finding themselves will be better understood and long remembered by students. In this case, the learning takes the context problems in real phenomena and is related to mathematical concepts. Progressive mathematization means the formulation of a problem into the language of mathematics in the form of an abstract concept through mathematical modeling. These mathematical problems can be solved formally or informally depending on the students' abilities, then translated back into the original real-world language. Mathematical modeling of problems that are real and can be solved informally is called horizontal mathematics.

Meanwhile, modeling using mathematical real-world, formally resolving it, is called vertical mathematics. Self-develop models mean that students build their mathematical models of the problems presented. In this case, the models used are made by teachers or students who can guide students in solving real (context) problems. These models provide a bridge from students' informal responses to formal forms.

In RME, the emergent model also needs to be considered for sequencing tasks and implementing mathematical activities (Wijaya, 2008; Zandieh & Rasmussen, 2010). The first sequence of mathematics models is a situational model (i.e., real objects for students), then increased to model-of (already in the form of pictures or modeling without objects). Then it shifted to a model-for (in the form of mathematical symbols, such as add, subtract, multiply and divide). The highest level is the model in formal mathematics.

This study used the principles and framework of RME to engage students through guided reinvention activities in context-based learning. This learning emphasizes the skills of the process of doing mathematics, discussion, collaboration, and argumentation. Therefore, students are expected to find their strategies or ways of solving problems, either individually or in groups (Gravemeijer, 2004).

**The Present Study**

Research related to the application of RME at the secondary school level is rare found in the literature. It makes sense because the application of RME in its history started from elementary school (van den Heuvel-Panhuizen & Drijvers, 2014). In addition, some practitioners and even researchers (see more in van den Heuvel-Panhuizen, 2020) have interpreted and criticized that RME is not suitable for education levels that use advanced mathematics (including matrices material) because one of its characteristics is using context, which collides with an abstract mathematical structure. However, van den Heuvel-Panhuizen and Drijvers (2014) stated that the context in RME is anything that students can imagine or have had or experienced. As a foundation and consideration for designing meaningful context-based learning at the advanced mathematics level, this is, of course.

Several previous studies have examined and focused on learning matrices' problems both descriptively (e.g., Kusuma et al., 2018; Lesmana et al., 2015; Suastika, John, & Utami, 2015) as well as with specific interventions (Andrews-Larson et al., 2017; Cárcamo et al., 2019; Parta, 2016). For example, research conducted by Andrew-Larson and colleagues (Andrews-Larson et al., 2017) who designed Hypothetical

Learning Trajectory (HLT) intending to support students in developing flexible ways of reasoning about matrices as linear transformations in the context of introducing linear algebra. Andrews-Larson et al.'s (2017) research was conducted in an introductory class to linear algebra at a public university in the southwestern United States. Similar research was also carried out by Cárcamo et al. (2019), which formulated a Local Instruction Theory (LIT) to support constructing the concepts of spanning set and span in Linear Algebra with first-year engineering students. Both of them use the RME principle to design advanced mathematics learning. Similar to the two, we depart from the same principle, but we apply it in high school, where they learn matrices for the first time at that level. Thus, we want to form a solid foundation for understanding their concepts in learning matrices. Besides, research related to interventions using context-based learning applied to matrices learning in secondary schools is scarce in the literature. This context is used to understand what they are learning and what will be involved in everyday life.

## RESEARCH METHOD

Design research is adopted to achieve research objectives. Design research has three phases, preliminary design, teaching experiment, and retrospective analysis (Gravemeijer, 2004). Furthermore, Gravemeijer (2004) stated that design research aims to develop Local Instruction Theory by collaborating between researchers and educators to improve learning quality.

In the preliminary design phase, researchers looked for literature sources on RME and design research. Researchers began to design the Hypothetical Learning Trajectory (HLT). HLT consists of three components (Putrawangsa, 2018), the learning objectives for students, the planning for learning activities, and hypotheses or assumptions about students' thinking responses to the learning activities applied in the hypothetical learning trajectory. The quality of HLT is then improved by doing a series of validation activities, namely content and construction validation, through expert assessments and limited trials that have been carried out. Mathematical activities are given in the form of problems that follow the rules of the RME approach. Researchers also prepared pretest questions to determine students' initial abilities about the concept of the matrices'.

In the teaching experiment phase, the researcher tested the HLT that had been made and developed it. During this phase, the researcher collected data in classroom observations, interviews, field notes, and students' work results. The learning process is carried out in two cycles; each cycle provides five meetings. Details of the experiment steps activity are in Table 1. The research was conducted at a public senior high school in South Tangerang City, Banten, Indonesia. The school was selected because one of the authors is a mathematics teacher there. The research problem and purpose came from and back to this school.

The first cycle is called the preliminary teaching experiment. This cycle was carried out to test and observe hypotheses (possibility) of students' responses for the method designed and find out the students' possible answers to problems on the activity sheet. A preliminary teaching experiment was carried out in XI MIPA 1. It consists of 36 students; nine students were taken with different abilities (high, medium and low abilities) based on the pretest results. After taking all meetings, they took the posttest.

Table 1. Research procedures and teaching experiment

| Week | Progress | Activities |
|---|---|---|
| | | **Preliminary Design** |
| **Week 1 -4** | Preparation | Literature, designed HLT, made instruments (activity sheets, observation sheets, interview guides), and made research permits. |
| **Week 5** | Observation | 1. Applying for research permission from the Principal of the School. |
| | | 2. Choosing the class to be used for research, namely XI MIPA 1 for preliminary implementation of teaching (Preliminary Teaching Experiment) and XI MIPA 2 for teaching implementation (Teaching Experiment). |
| | | **Preliminary Teaching Experiment** |
| **Week 6** | Pretest | It was attended by 36 students of class XI MIPA 1. The purpose of the pretest was to determine the students' initial abilities regarding the concept of the matrices |
| | Interview | Nine students were selected for in-depth interviews based on the pretest results |
| **Week 7** | The first meeting | Lesson I: My passions (Discussing the concept of defining matrices, matrices elements, size of matrices, and types of matrices) |
| | Second meeting | Lesson II: Reversed position, and what is this? (Discussing Transpose and Similarities of Two Matrices) |
| **Week 8** | Third Meeting | Lesson III: Let's Add and Find the Difference! (Discusses addition and subtraction of matrices) |
| | Fourth Meeting | Lesson IV: Multiplying Real numbers with Matrices |
| **Week 9** | Fifth Meeting | Lesson V: Multiplication of Two Matrices |
| **Week 10** | Postest | Followed by nine students who were selected at the beginning. This is to see the increase in students' understanding of the concept of the matrices |
| | Interview | We are interviewing nine students who took the posttest to find out how to improve understanding of the matrices concept. |
| | | **Teaching Experiment** |
| **Week 11** | Pretest | It was attended by 36 students of class XI MIPA 2. The purpose of the pretest was to determine the students' initial abilities regarding the concept of the matrices |
| | Interview | We are interviewing six students based on the pretest results. |
| **Week 12** | The first meeting | Lesson I: My passions (Discussing the concept of defining matrices, matrices elements, size of matrices, and types of matrices) |
| | Second meeting | Lesson II: Reversed position, and what is this? (Discussing Transpose and Similarities of Two Matrices) |
| **Week 13** | Third Meeting | Lesson III: Let's Add and Find the Difference! (Discusses addition and subtraction of matrices) |
| | Fourth Meeting | Lesson IV: Multiplying Real numbers with Matrices |
| **Week 14** | Fifth Meeting | Lesson V: Multiplication of Two Matrices |
| **Week 15** | Posttest | It was attended by 36 students of class XI MIPA 2. The results will be used to see an increase in students' understanding of the concept of the matrices |
| | Interview | Interviewing several students to determine the increase in understanding the concept of the matrices based on the test results |
| | | **Retrospective Analysis** |
| **Week 16-20** | Retrospective Analysis | Analyzing the data that has been collected during the learning implementation. The HLT that has been made is compared with the learning that has been done and the activities that students have carried out |

Some posttest questions have similarities with the pretest questions to see the progress in understanding the matrices' concept. Some items are different from the pretest to determine how far they understand and can use the matrices' concept. Data were collected from students' writing (pretest and posttest results, worksheets), and videos that show students' activities in learning. Then they were interviewed more deeply. The data were then analyzed to see whether the learning trajectory hypothesis needs to be revised before applying it in the second cycle.

The second cycle is called the teaching experiment, carried out in class XI MIPA 2, consisting of 36 students. The HLT revised in the preliminary teaching experiment was used as a guide. We implemented a group learning setting based on random for each meeting; each group consists of 4 students. There were nine groups involved until the end of this study. The teaching experiment details can be seen in Table 1, which includes the material introduced to the matrices' at the beginning of the meeting and the multiplication between matrices at the end of the meeting.

In the retrospective analysis phase, we analyze the data that has been collected during the learning process. From the HLT that has been made, it is then compared with the learning (including student activities) carried out. If the results still need to be improved, the HLT will be revised. Students' answers on the pretest and posttest were examined to see how the matrices' conceptual knowledge was described. Data from interviews can also deepen answers or confirm the students' works. Qualitative analysis was chosen because this study's focus was not only on the results of students' scores on the pretest and posttest but also on the beginning when they do not understand the concept of the matrices, and they know and can apply the matrices concept through realistic mathematics education (Prahmana, 2017).

## RESULTS AND DISCUSSION

This part presents research findings limited to the teaching experiment phase to focus on what was stated in the research objectives. This phase involved 36 students in class XI of high school. The LIT applied for this phase is a revision of the previously developed HLT and applied to the preliminary teaching experiment phase.

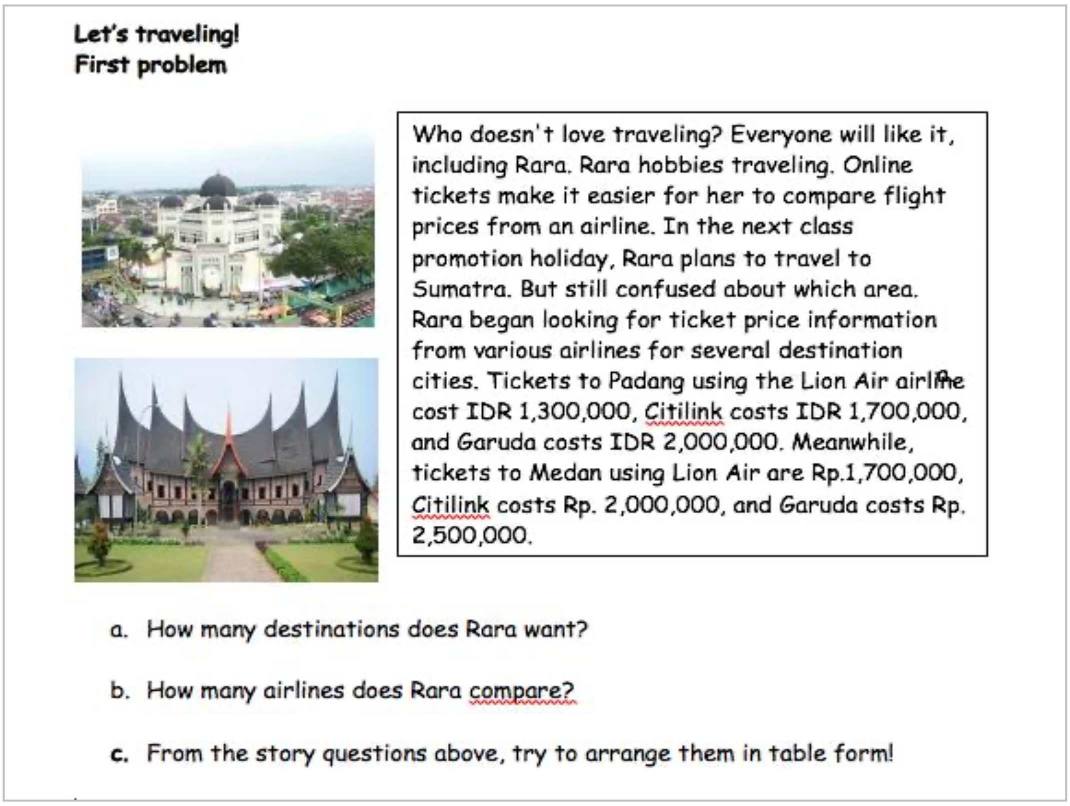
### LIT part 1: The context of hobbies or passions as the starting point in compiling and defining the matrices

*Step 1: Contextual problems that represent the matrices*

The context of the first meeting is about Hobbies Students discuss solving contextual problems on the Student Activity Sheet. Figure 1 below is an introductory sample of matrices material.

Figure 1 presents the first problem regarding traveling hobbies, comparing the tickets of several airlines to Padang and Medan. The next question takes the context of a sports hobby, in this case, football. There is data about the scores of wins, draws and losses from several football clubs in Indonesia.

Of the nine groups, eight groups were able to arrange the matrices from context to table (see Figure 2), and one group gives answers outside of predictions in constructing the matrices from context to table. 'It's means that students were able to do the second stage of modeling, namely the "model of" according to what Freudenthal expressed (Revina, 2017). Freudenthal revealed that the self-model in RME has four steps: the situational model, the model of, the model for, and the formal model (Revina, 2017).



**Let's traveling!**
**First problem**

Who doesn't love traveling? Everyone will like it, including Rara. Rara hobbies traveling. Online tickets make it easier for her to compare flight prices from an airline. In the next class promotion holiday, Rara plans to travel to Sumatra. But still confused about which area. Rara began looking for ticket price information from various airlines for several destination cities. Tickets to Padang using the Lion Air airline cost IDR 1,300,000, Citilink costs IDR 1,700,000, and Garuda costs IDR 2,000,000. Meanwhile, tickets to Medan using Lion Air are Rp.1,700,000, Citilink costs Rp. 2,000,000, and Garuda costs Rp. 2,500,000.

a. How many destinations does Rara want?

b. How many airlines does Rara compare?

c. From the story questions above, try to arrange them in table form!

Figure 1. Contextual Problems as Introduction to Matrices Material

Students have been able to manipulate objects (in this case, the context of the story) to get to the understanding of the matrices concept; according to Arsaythamby and Zubainur (2014) argue that learning realistic mathematics will provide opportunities for students to develop an understanding of mathematical concepts through the manipulation of objects and tools.

| tujuan / maskapai | Padang | Medon |
|---|---|---|
| Citilink | Rp. 1.700.000,- | Rp. 2.000.000,- |
| Lion Air | Rp. 1.300.000,- | Rp. 1.700.000,- |
| Garuda | Rp. 2.000.000,- | Rp. 2.500.000,- |

| | Lion Air | Citilink | Garuda |
|---|---|---|---|
| Padang | Rp 1 300 000 | Rp 1700 000 | Rp 2000 000 |
| Medan | Rp 1700 000 | Rp 2000 000 | Rp 2 500 000 |

Figure 2. Students' Answers in Constructing the Matrices from Context to Table

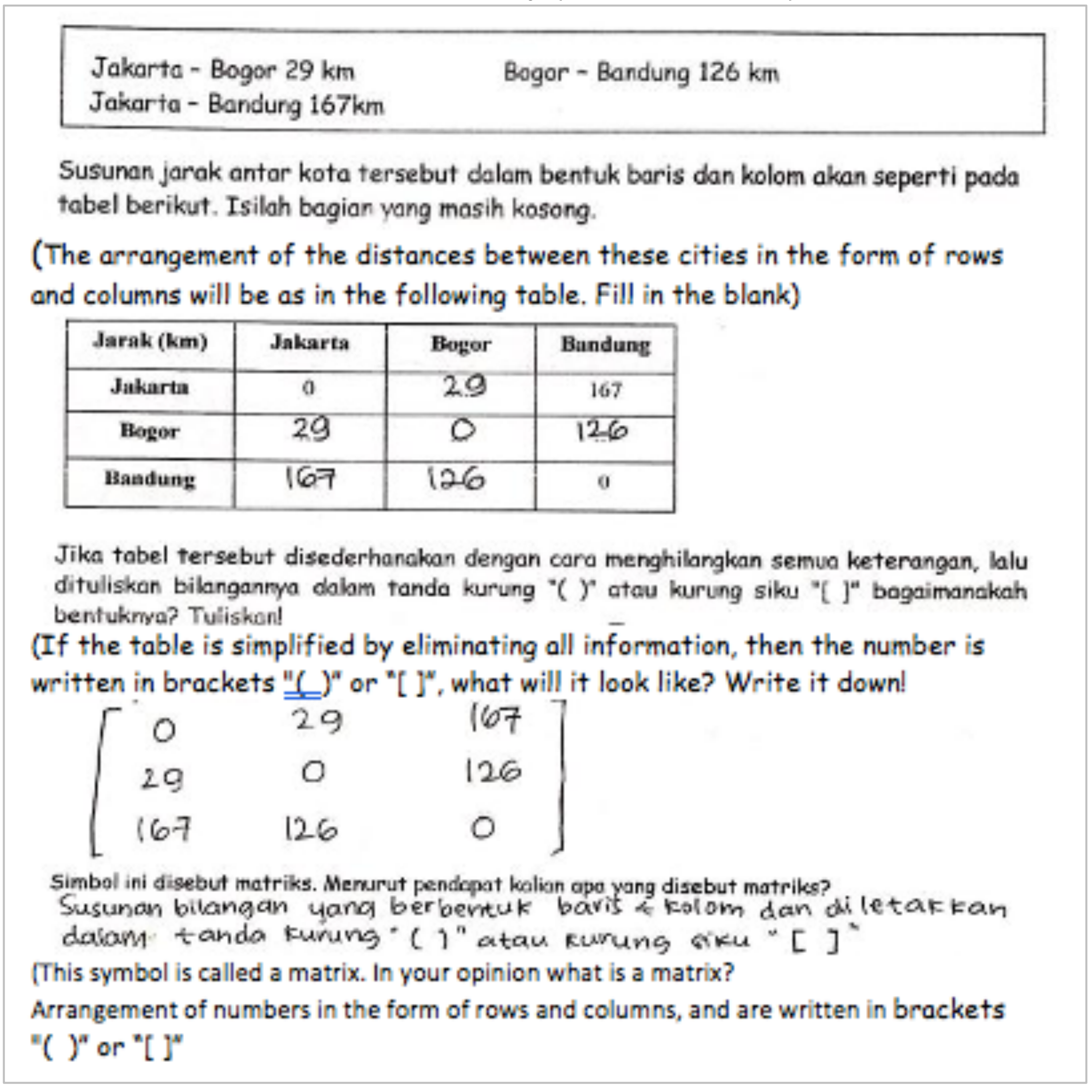
Figure 2 shows the answer as in the student response prediction. Although the answers shown in Figure 2 are different but have the same meaning, they have compared the airline ticket prices in the two destination cities.

| | | |
|---|---|---|
| Padang | Lion Air | Rp 1.300.000 |
| | Citilink | Rp 1.700.000 |
| | Garuda | Rp 2.000.000 |
| Medan | Lion Air | Rp 1.700.000 |
| | Citilink | Rp 2.000.000 |
| | Garuda | Rp 2.500.000 |

Figure 3. Students' Answers Outside of Predictions in Constructing the Matrices from Context to Table

There was an answer beyond the predictions from all of the students' responses (see Figure 3). The group did not compare the ticket prices of several airlines with the same destination. During the discussion session, they learned the difference they made. They arrange a table, as shown in Figure 3; all airline ticket prices are arranged downward, without looking at the same airlines. For example, the Garuda ticket price to Padang is IDR 2,000,000, while to Medan still on the same airline, the price is IDR 2,500,000.

According to Treffers (Gravemeijer, 2004), the use of context is the first characteristic of RME. This learning uses real problems in our life or has been experienced by students, or it is a real situation. This learning leads to mathematical concepts with daily experiences. Some of these problems serve as introductions that will guide students to the next stage of matrices. Context is used as an idea that motivates students to think mathematically (Webb et al., 2011).



Jakarta - Bogor 29 km Bogor - Bandung 126 km
Jakarta - Bandung 167km

Susunan jarak antar kota tersebut dalam bentuk baris dan kolom akan seperti pada tabel berikut. Isilah bagian yang masih kosong.

(The arrangement of the distances between these cities in the form of rows and columns will be as in the following table. Fill in the blank)

| Jarak (km) | Jakarta | Bogor | Bandung |
|---|---|---|---|
| Jakarta | 0 | 29 | 167 |
| Bogor | 29 | 0 | 126 |
| Bandung | 167 | 126 | 0 |

Jika tabel tersebut disederhanakan dengan cara menghilangkan semua keterangan, lalu dituliskan bilangannya dalam tanda kurung "( )" atau kurung siku "[ ]" bagaimanakah bentuknya? Tuliskan!

(If the table is simplified by eliminating all information, then the number is written in brackets "( )" or "[ ]", what will it look like? Write it down!

$$\begin{bmatrix} 0 & 29 & 167 \\ 29 & 0 & 126 \\ 167 & 126 & 0 \end{bmatrix}$$

Simbol ini disebut matriks. Menurut pendapat kalian apa yang disebut matriks?
Susunan bilangan yang berbentuk baris & kolom dan diletakkan dalam tanda kurung "( )" atau kurung siku "[ ]"

(This symbol is called a matrix. In your opinion what is a matrix?
Arrangement of numbers in the form of rows and columns, and are written in brackets "( )" or "[ ]"

Figure 4. Students Compile Matrices from Given Contextual Problems

After compiling the tables, students are asked to arrange them into matrices. Eight of nine groups were able to compose matrices (see Figure 4), define matrices, rows, and columns with their own language based on the matrices they have compiled.

Students also already know about the size of the matrices. The following is dialogue when learning the definition of matrices.

*Teacher: "This symbol is called matrices. What do you know about matrices? "*

*Group 1: "The arrangement of numbers in square brackets which have a pattern."*

*Teacher: "Any other opinions?"*

*Group 3: "Numbers in rows and columns in square brackets."*

*Group 2: "An arrangement of numbers consisting of rows and columns using brackets."*

*Teacher: "So, Is there any difference between table and matrices?"*

*All: "Yes."*

*Teacher: "In matrices, there are what we called rows and columns. Then what is the row?" (Teacher asked group 4)*

*Group 4: "Rows are numbers which are in sideways."*

*Teacher: "Is there any different opinion?"*

*Group 5: "Line is a horizontal arrangement of numbers."*

*Group 1: "Rows are the arrangement of numbers horizontally."*

From the dialogue above, it can be seen that students can define the matrices. Students' definition follows the definition of the matrices expressed by Manulang (2017) that the matrices are an arrangement of numbers consisting of rows and columns using regular brackets. Besides, students can also define rows and columns with their own language based on the matrices they have compiled.

*Step 2: Defining and Constructing the Transpose Matrices*

Figure 5 shows the examples of students' answers to the first problem with transpose matrices. Students have also been able to define the transpose of the matrices. The following is a snippet during the second meeting that discusses the transpose of the matrices.

*Teacher: "Please compare the matrix T and P. How are the elements positioned? Group 6, please. Can you explain it? "*

*Group 6: "Elements in rows turn into columns, and vice versa."*

*Teacher: "The T and P matrices transpose each other. What is the definition of transpose? group 1, now it is your turn to answer."*

*Group 1: "Matrices formed by swapping rows and columns."*

*Teacher: "Any other opinions?"*

*Group 3: "Changing the position of elements from rows to columns and vice versa."*

*Group 5: "Transpose matrices are matrices that are made by swapping between columns and rows."*

*Teacher: "Is it possible if the first row becomes the third column?"*

*Class: "No, it is not."*

*Teacher: "It means if it is in the first row, what would it be?"*

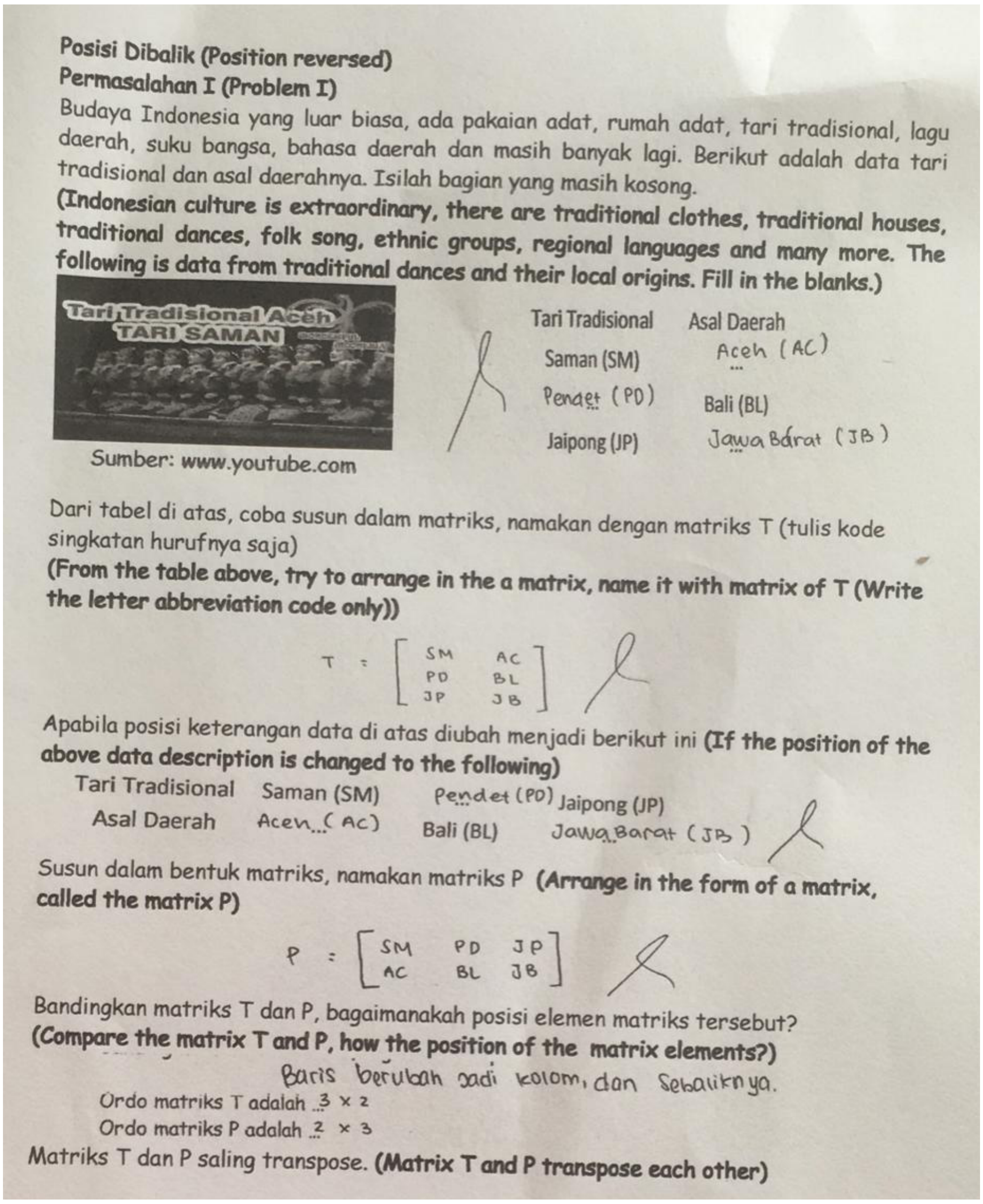
*Class: "It should be in the first column."*

*Teacher: "Good. Now, any other opinion about the definition of transpose matrices? " well, group 3, please. "*

*Group 3: "Transpose matrices is the changing position of the elements of the matrices."*

*Teacher: "All the answers are right, but any of you who may complete the definition, please?"*

*Group 3: "Transpose matrices is the changing position of matrix elements from rows to columns."*



**Posisi Dibalik (Position reversed)**
**Permasalahan I (Problem I)**

Budaya Indonesia yang luar biasa, ada pakaian adat, rumah adat, tari tradisional, lagu daerah, suku bangsa, bahasa daerah dan masih banyak lagi. Berikut adalah data tari tradisional dan asal daerahnya. Isilah bagian yang masih kosong.
**(Indonesian culture is extraordinary, there are traditional clothes, traditional houses, traditional dances, folk song, ethnic groups, regional languages and many more. The following is data from traditional dances and their local origins. Fill in the blanks.)**



Sumber: www.youtube.com

| Tari Tradisional | Asal Daerah |
|---|---|
| Saman (SM) | Aceh (AC) |
| Pendet (PD) | Bali (BL) |
| Jaipong (JP) | Jawa Barat (JB) |

Dari tabel di atas, coba susun dalam matriks, namakan dengan matriks T (tulis kode singkatan hurufnya saja)
**(From the table above, try to arrange in the a matrix, name it with matrix of T (Write the letter abbreviation code only))**

$$T = \begin{bmatrix} SM & AC \\ PD & BL \\ JP & JB \end{bmatrix}$$

Apabila posisi keterangan data di atas diubah menjadi berikut ini **(If the position of the above data description is changed to the following)**

| Tari Tradisional | Saman (SM) | Pendet (PD) | Jaipong (JP) |
|---|---|---|---|
| Asal Daerah | Aceh (AC) | Bali (BL) | Jawa Barat (JB) |

Susun dalam bentuk matriks, namakan matriks P **(Arrange in the form of a matrix, called the matrix P)**

$$P = \begin{bmatrix} SM & PD & JP \\ AC & BL & JB \end{bmatrix}$$

Bandingkan matriks T dan P, bagaimanakah posisi elemen matriks tersebut?
**(Compare the matrix T and P, how the position of the matrix elements?)**
Baris berubah jadi kolom, dan sebaliknya.

Ordo matriks T adalah $3 \times 2$
Ordo matriks P adalah $2 \times 3$

Matriks T dan P saling transpose. **(Matrix T and P transpose each other)**

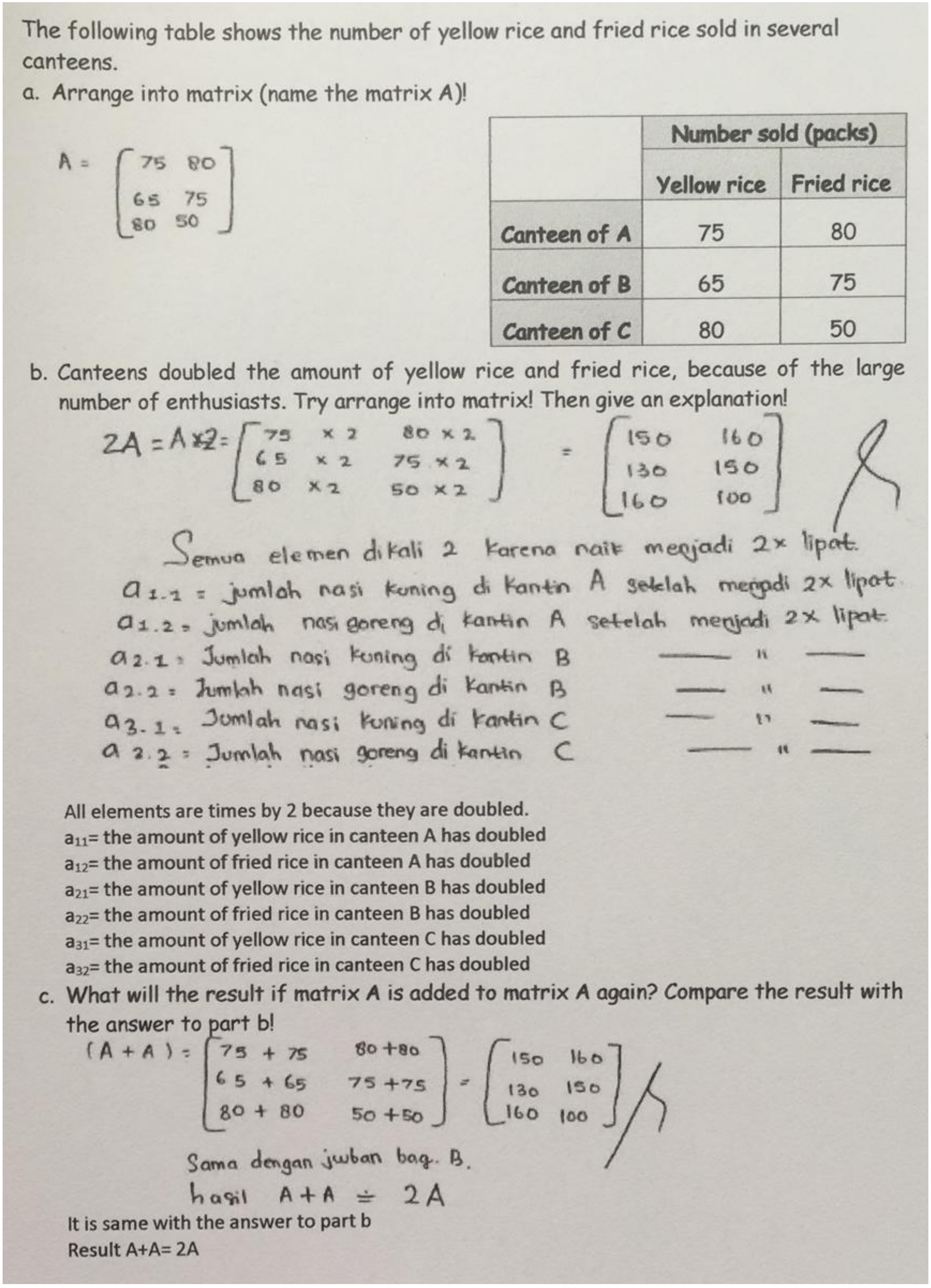
Figure 5. Students' Answers about the Transpose Matrices

From the dialogue passage during the lesson above, students can arrange the transpose of the matrices, distinguish the initial matrices from the transpose matrices, and define the matrices' transpose.

**LIT part 2: Matrices multiplication concept (matrix scalar multiplication as repeated addition; terms and product of two matrices**

*Step 1: Multiply the matrices by the scalar as repeated addition.*

In this session, students work on matrices and scalar multiplication. All students have been able to finish the multiplication matrices by a scalar, see Figure 6.



The following table shows the number of yellow rice and fried rice sold in several canteens.

a. Arrange into matrix (name the matrix A)!

$$A = \begin{bmatrix} 75 & 80 \\ 65 & 75 \\ 80 & 50 \end{bmatrix}$$

| | Number sold (packs) | |
|---|---|---|
| | Yellow rice | Fried rice |
| Canteen of A | 75 | 80 |
| Canteen of B | 65 | 75 |
| Canteen of C | 80 | 50 |

b. Canteens doubled the amount of yellow rice and fried rice, because of the large number of enthusiasts. Try arrange into matrix! Then give an explanation!

$$2A = A \times 2 = \begin{bmatrix} 75 \times 2 & 80 \times 2 \\ 65 \times 2 & 75 \times 2 \\ 80 \times 2 & 50 \times 2 \end{bmatrix} = \begin{bmatrix} 150 & 160 \\ 130 & 150 \\ 160 & 100 \end{bmatrix}$$

Semua elemen dikali 2 karena naik menjadi 2x lipat.

$a_{1.1}$ = jumlah nasi kuning di kantin A setelah menjadi 2x lipat.

$a_{1.2}$ = jumlah nasi goreng di kantin A setelah menjadi 2x lipat.

$a_{2.1}$ = Jumlah nasi kuning di kantin B — " —

$a_{2.2}$ = Jumlah nasi goreng di kantin B — " —

$a_{3.1}$ = Jumlah nasi kuning di kantin C — " —

$a_{3.2}$ = Jumlah nasi goreng di kantin C — " —

All elements are times by 2 because they are doubled.

$a_{11}$= the amount of yellow rice in canteen A has doubled

$a_{12}$= the amount of fried rice in canteen A has doubled

$a_{21}$= the amount of yellow rice in canteen B has doubled

$a_{22}$= the amount of fried rice in canteen B has doubled

$a_{31}$= the amount of yellow rice in canteen C has doubled

$a_{32}$= the amount of fried rice in canteen C has doubled

c. What will the result if matrix A is added to matrix A again? Compare the result with the answer to part b!

$$(A + A) = \begin{bmatrix} 75 + 75 & 80 + 80 \\ 65 + 65 & 75 + 75 \\ 80 + 80 & 50 + 50 \end{bmatrix} = \begin{bmatrix} 150 & 160 \\ 130 & 150 \\ 160 & 100 \end{bmatrix}$$

Sama dengan jwban bag. B.

hasil $A + A \doteq 2A$

It is same with the answer to part b

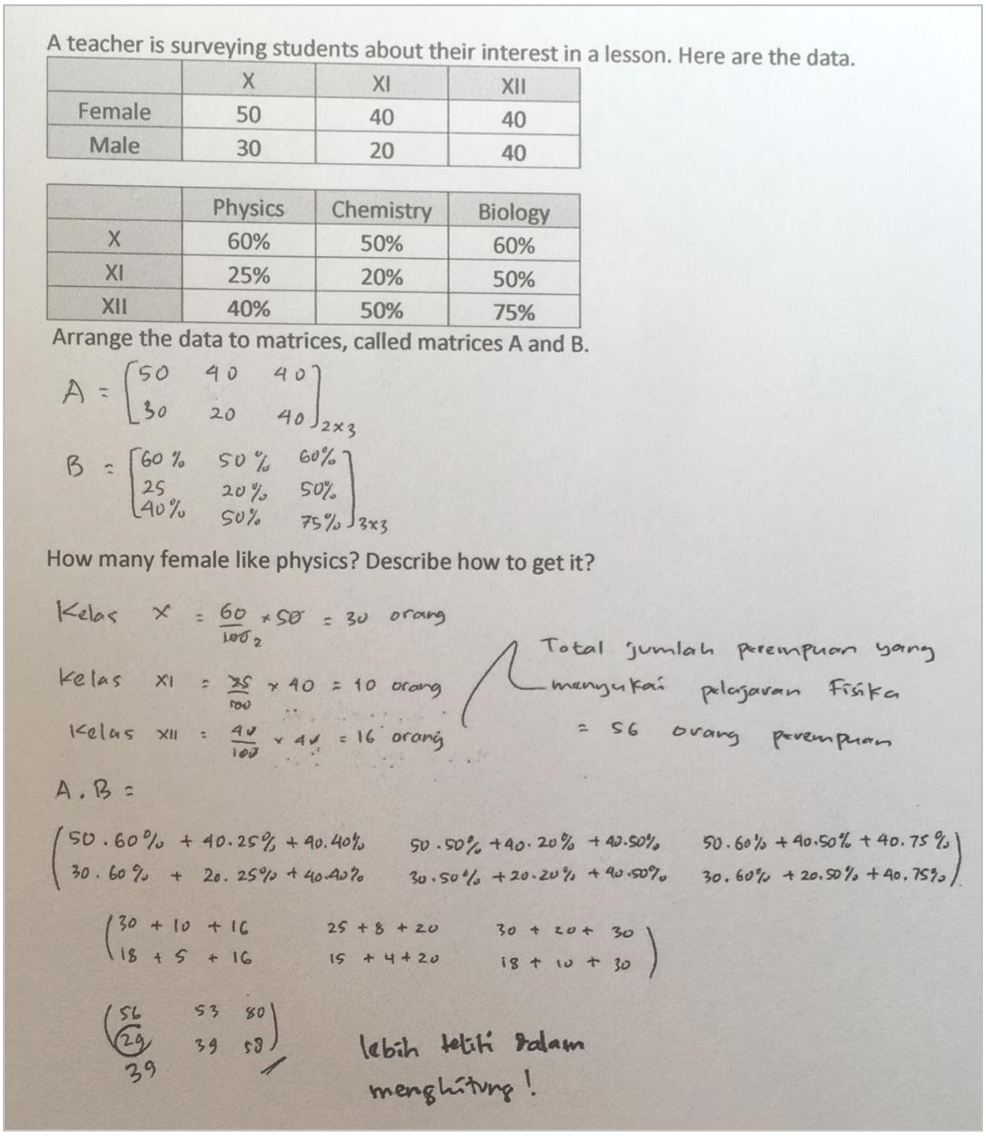
Result A+A= 2A

Figure 6. The example of Student Answers on Multiplication of a Matrix with a Scalar as a Repeated Addition

Figure 6 shows the multiplication of matrix A with scalar 2 (question b), students explain for each element. For question c, students do repeated addition (A+A), it the same answer with question b. Students can conclude that multiplication matrices will be the same as the repeated addition from the questions given. The result of this research indicates that students have carried out the third realistic mathematics education principle according to Freudenthal's theory (Revina, 2017) (2 x A = A + A), namely the self-develop model, where students have been at the formal model stage, students can formulate multiplication matrices with a scalar as a repeated addition after doing discussion based on the contextual questions given.

*Step 2: Terms and Results of Two Matrix Multiplications*

Using context can also visualize the matrices' concept of several objects, which then naturally leads students to discover matrices multiplication. Students who are usually given routine multiplication of two matrices today are given contextual questions about a survey result of student interest in a specific subject.


A teacher is surveying students about their interest in a lesson. Here are the data.

| | X | XI | XII |
|---|---|---|---|
| Female | 50 | 40 | 40 |
| Male | 30 | 20 | 40 |

| | Physics | Chemistry | Biology |
|---|---|---|---|
| X | 60% | 50% | 60% |
| XI | 25% | 20% | 50% |
| XII | 40% | 50% | 75% |

Arrange the data to matrices, called matrices A and B.

$$A = \begin{bmatrix} 50 & 40 & 40 \\ 30 & 20 & 40 \end{bmatrix}_{2\times 3}$$

$$B = \begin{bmatrix} 60\% & 50\% & 60\% \\ 25 & 20\% & 50\% \\ 40\% & 50\% & 75\% \end{bmatrix}_{3\times 3}$$

How many female like physics? Describe how to get it?

Kelas X : $\frac{60}{100} \times 50 = 30$ orang

Kelas XI : $\frac{25}{100} \times 40 = 10$ orang

Kelas XII : $\frac{40}{100} \times 40 = 16$ orang

Total jumlah perempuan yang menyukai pelajaran Fisika = 56 orang perempuan

A . B =

$$\begin{pmatrix} 50 \cdot 60\% + 40 \cdot 25\% + 40 \cdot 40\% & 50 \cdot 50\% + 40 \cdot 20\% + 40 \cdot 50\% & 50 \cdot 60\% + 40 \cdot 50\% + 40 \cdot 75\% \\ 30 \cdot 60\% + 20 \cdot 25\% + 40 \cdot 40\% & 30 \cdot 50\% + 20 \cdot 20\% + 40 \cdot 50\% & 30 \cdot 60\% + 20 \cdot 50\% + 40 \cdot 75\% \end{pmatrix}$$

$$\begin{pmatrix} 30 + 10 + 16 & 25 + 8 + 20 & 30 + 20 + 30 \\ 18 + 5 + 16 & 15 + 4 + 20 & 18 + 10 + 30 \end{pmatrix}$$

$$\begin{pmatrix} 56 & 53 & 80 \\ 29 & 39 & 58 \end{pmatrix}$$

39

lebih teliti dalam menghitung!

Figure 7. Contextual problems lead students to make reinvention matrices multiplication concept

The question about the data from male and female students who like certain subjects (Physics, chemistry, and biology). The students were confused about the relationship between the questions given and the matrices multiplication material. After discussing with their group and the teacher then gives questions that lead students to find the multiplication matrices concept of the contextual problem (see Figure 7); it shows that students from group two can reinvent the multiplication of two matrices formulation. The following is a part of the dialog from the discussion at the fifth meeting about matrices multiplication.

*Teacher: "Do you think Matrices A and B can be multiplied?"*

*Group 2: "Sure."*

*Teacher: "that is right. Can you explain why it can be?"*

*Group 2: "Because the number of column A is the same as the number of rows B."*

*Teacher: "Good, next question; how to multiply it?"*

*Group 2: "This one multiplies with that one" (shows rows and columns)*

*Teacher: "So, we can match row with….?"*

*Group 2: "The first row with column one, first row multiply column two, the first row multiply with column three."*

*Teacher: "So, the size of matrices from the multiplication result of the matrices will be?"*

*Group 2: "2x3"*

*Teacher: "What is 2x3 in the first-row showing?"*

*Group 2: "Female"*

*Teacher: "That is right; then what does each column show?"*

*Group 2: "Woman who likes physics."*

*Teacher: "What about the second column?"*

*Group 2: "Females who like chemistry."*

*Teacher: "What about in column 3?"*

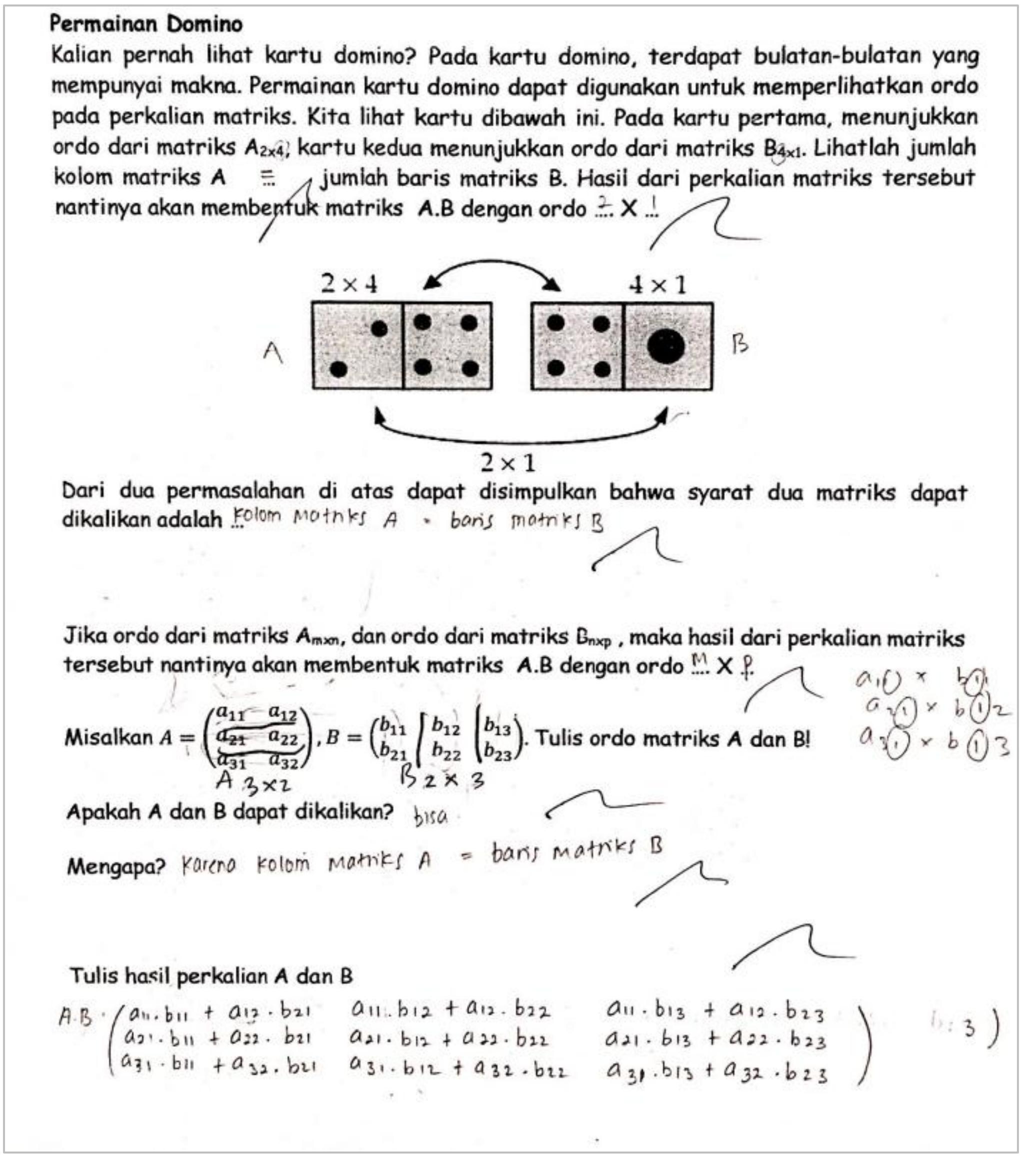
*Group 2: "Biology"*

*Teacher: "Ok, those who like physics in which element are they?"*

*Group 2: "they are in element 1."*

From the above discussion, it can be seen that students know the terms of the matrices can be multiplied, the size of matrices from the multiplication of two matrices, and the meaning of each element based on the problem being asked.

Students have also been able to write down the matrices' multiplication requirements and write down the multiplication result of two matrices (see Figure 8). The result of this research indicates that students have carried out the third realistic mathematics education principle according to Freudenthal's theory (Revina, 2017), namely the self-develop model, where students are already at the formal model stage, students can formulate a method of multiplying two matrices from the discussion results based on the contextual questions given.



**Permainan Domino**

Kalian pernah lihat kartu domino? Pada kartu domino, terdapat bulatan-bulatan yang mempunyai makna. Permainan kartu domino dapat digunakan untuk memperlihatkan ordo pada perkalian matriks. Kita lihat kartu dibawah ini. Pada kartu pertama, menunjukkan ordo dari matriks $A_{2\times 4}$; kartu kedua menunjukkan ordo dari matriks $B_{4\times 1}$. Lihatlah jumlah kolom matriks A = jumlah baris matriks B. Hasil dari perkalian matriks tersebut nantinya akan membentuk matriks A.B dengan ordo 2 X 1



Dari dua permasalahan di atas dapat disimpulkan bahwa syarat dua matriks dapat dikalikan adalah kolom matriks A = baris matriks B

Jika ordo dari matriks $A_{m\times n}$, dan ordo dari matriks $B_{n\times p}$, maka hasil dari perkalian matriks tersebut nantinya akan membentuk matriks A.B dengan ordo M X P

Misalkan $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$, $B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}$. Tulis ordo matriks A dan B!

$A_{3\times 2}$ $B_{2\times 3}$

$a_1(1) \times b(1)1$
$a_2(1) \times b(1)2$
$a_3(1) \times b(1)3$

Apakah A dan B dapat dikalikan? bisa

Mengapa? karena kolom matriks A = baris matriks B

Tulis hasil perkalian A dan B

$$A.B = \begin{pmatrix} a_{11}\cdot b_{11} + a_{12}\cdot b_{21} & a_{11}\cdot b_{12} + a_{12}\cdot b_{22} & a_{11}\cdot b_{13} + a_{12}\cdot b_{23} \\ a_{21}\cdot b_{11} + a_{22}\cdot b_{21} & a_{21}\cdot b_{12} + a_{22}\cdot b_{22} & a_{21}\cdot b_{13} + a_{22}\cdot b_{23} \\ a_{31}\cdot b_{11} + a_{32}\cdot b_{21} & a_{31}\cdot b_{12} + a_{32}\cdot b_{22} & a_{31}\cdot b_{13} + a_{32}\cdot b_{23} \end{pmatrix}$$

b : 3)

Figure 8. Students' answers about matrices multiplication requirements and results

From all learning activities, realistic mathematics education can be used to develop an understanding of the multiplication matrices concept in senior high school students. The steps starting from context help learn matrices multiplication operations from informal to more formal ways. This is in line with Hidayat's research that with realistic mathematics education, students' conceptual understanding becomes better (Hidayat & Iksan, 2015). The point of conceptual knowledge (Purnomo et al., 2019) is that students can understand the connections between mathematical concepts and integration for contextual situations that have also been achieved.

## CONCLUSION AND LIMITATION

This study aims to develop LIT to understand the matrices concept for senior high school students. This study indicates that the context presented is constructive for students to compile and define the matrices and reinvention the multiplication matrices formula. The arranged LIT includes two important points. First, using the context of a favorite or a hobby as a starting point in compiling and defining the matrices. Second, using the scalar multiplication concept of the matrices as an iterative addition. Students have a good foundation for understanding the matrices and their application in everyday life from these two points.

This study's results are satisfying, but we still face burdens, especially for the time needed in the learning process. Students need much time to interpret the concept from the given context and reinvention the multiplication matrices formula, while the duration in the learning process is only 90 minutes for one meeting.

**REFERENCES**

Andrews-Larson, C., Wawro, M., & Zandieh, M. (2017). A hypothetical learning trajectory for conceptualizing matrices as linear transformations. *International Journal of Mathematical Education in Science and Technology*, *48*(6), 809–829.

Arsaythamby, V., & Zubainur, C. M. (2014). How a Realistic Mathematics Educational Approach Affect Students' Activities in Primary School. *Procedia-Social and Behavioral Science*, *159*, 309–313.

Aygör, N., & Burhanzade, H. (2014). The Comparative Analyzes of the Student's Performance about Matrix in Student Selection Exam (ÖSS) and the Approved Lecture Books of Ministry for the National Education (MEB). *Procedia - Social and Behavioral Sciences*.

Cárcamo, A., Fuentealba, C., & Garzón, D. (2019). Local Instruction Theories at the University Level: An Example in a Linear Algebra Course. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(12).

Carlson, D. (1993). Teaching Linear Algebra: Must the Fog Always Roll in? *The College Mathematics Journal*, *24*(1), 29–40.

Gravemeijer, K. (2004). Local Instruction Theories as Means of Support for Teachers in Reform Mathematics Education. *Mathematical Thinking and Learning*, *6*(2), 105–128.

Hendroanto, A., Budayasa, I. K., Galen, F. Van, & Eerde, D. Van. (2015). *Supporting Students ' Spatial Ability in Understanding Three-Dimensional Representations*. Universitas Negeri Surabaya.

Hidayat, R., & Iksan, Z. H. (2015). The Effect of Realistic Mathematic Education on Students' Conceptual Understanding of Linear Progamming. *Creative Education*.

Horton, R. M., Wiegert, E. M., & Marshall, J. C. (2008). Squaring Matrices: Connecting Mathematics and Science. *The Mathematics Teacher*, *102*(2), 102–106.

Kadir. (2004). Efektivitas Strategi Peta Konsep dalam Pembelajaran Sains dan Matematika (Meta-Analisis Penelitian Eksperimen Psikologi dan Pendidikan). *Jurnal Pendidikan Dan Kebudayaan*, *10*(051), 761–781.

Kusuma, N. F., Subanti, S., & Usodo, B. (2018). Students' misconception on equal sign. *Journal of Physics: Conference Series*, *1008*(1).

Lesmana, H., Yusmin, E., & Sayu, S. (2015). Pendeskripsian Pemahaman Konseptual Siswa Menyelesaikan Soal-Soal Operasi Matriks Kelas X SMKN 3 Pontianak. *Jurnal Pendidikan Dan Pembelajaran*, *IV*(12), 1–10.

Manulang, S. (2017). *Matematika SMA Kelas X* (D. Agung Lukito (ed.); Revisi). Kementrian Pendidikan dan Kebudayaan.

Parta, K. F. dan I. N. (2016). Pengembangan Lembar Kerja Siswa Materi Perkalian

Matriks Bercirikan Penemuan Terbimbing untuk Siswa SMK Kelas X. *Jurnal Pendidikan Matematika*, *I*(9), 1721–1729.

Prahmana, R. C. I. (2017). *Design Research (Teori dan Implementasinya: Suatu Pengantar)*. PT Rajagrafindo Perkasa.

Pramudiani, P., Oktafiani, A. F., Aziz, T. A., & Purnomo, Y. W. (2017). Enhancing Conceptual Knowledge about Shape through Realistic Mathematics Education. *Jurnal Inovasi Pendidikan Dasar*, *3*(1), 31–38.

Purnomo, Y. W., Kowiyah, Alyani, F., & Assiti, S. S. (2014). Assessing number sense performance of Indonesian elementary school students. *International Education Studies*, *7*(8), 74–84.

Purnomo, Y. W., Widowati, C., & Ulfah, S. (2019). Incomprehension of the Indonesian Elementary School Students on Fraction Division Problem. *Infinity Journal*, *8*(1), 57–74.

Putrawangsa, S. (2018). *Desain Pembelajaran Design Research sebagai Pendekatan Desain Pembelajaran*. CV. Reka Karya Amerta (Rekarta).

Rahmawati, N., & Purnomo, Y. W. (2020). Analysis of Student Learning Barriers in Solving Context Problems Related to the Matrix. *The 2nd International Conference of Education on Science, Technology, Engineering, and Mathematics (ICE-STEM 2020)*, 56–61.

Revina, S. (2017). *Influence of Culture on the Adaptation of Realistic Mathematics Education in Indonesia* (Issue June). University of Hong Kong.

Suastika, John, T., & Utami, T. (2015). Penelusuran Miskonsepsi Mahasiswa tentang Matriks Menggunakan Certainty of Response Index. *Prosiding Seminar Nasional Fisika SNF 2015*, *IV*(Oktober).

van den Heuvel-Panhuizen, M. (2020). *Seen Through Other Eyes—Opening Up New Vistas in Realistic Mathematics Education Through Visions and Experiences from Other Countries BT - International Reflections on the Netherlands Didactics of Mathematics : Visions on and Experiences with Realistic Mathematics Education* (M. van den Heuvel-Panhuizen (ed.); pp. 1–20). Springer International Publishing.

van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 521–525). Springer Netherlands.

Webb, D. C., Kooij, H. Van Der, & Geist, M. R. (2011). Design Research in the Netherlands: Introducing Logarithms Using Realistic Mathematics Education. *Journal of Mathematics Education at Teacher College*, *2*, 47–52.

Wijaya, A. (2008). *Design Research in Mathematics Education: Indonesian Traditional Games as Means to Support Second Graders Learning of Linear Measurement*. Universiteit Utrecht.

Wijaya, A., van den Heuvel-Panhuizen, M., & Doorman, M. (2015). Teachers' teaching practices and beliefs regarding context-based tasks and their relation with students' difficulties in solving these tasks. *Mathematics Education Research Journal*, *27*(4), 637–662.

Zandieh, M., & Rasmussen, C. (2010). Defining as a mathematical activity: A framework for characterizing progress from informal to more formal ways of reasoning. *Journal of Mathematical Behavior*, *29*(2), 57–75.