Some diagnostics learning problems on basic arithmetic skills of junior high school students

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Abstract. The main obstacle in mathematics learning at junior high school level is the lack of students' skills in performing basic operations of integers and fractions. This skill should be impeccable on elementary school level, nonetheless a big number of junior high school students experience impediment to follow the learning activities because they did not have enough pre-requisites on basic arithmetic. The similar circumstances are most probably also taken place at senior high school level. This study investigated students' skills on basic arithmetic and then presents some analyses of diagnostic learning problems. The crucial findings in this research are various facts that might be ignored by teachers over the years, but they were alleged to be roots of the leading causes of the low student skills and understanding in basic operations of numbers. The diagnostic results will be worthwhile to improve the method and approach of mathematics teaching not only at junior high school but also at elementary school. Accordingly, the article offers several approaches to overcome the problems found in the study.

1. Introduction

It is commonly accepted that mathematics originated with the practical problems of counting and recording numbers [1]. Numbers have been the most familiar entity to human life since pre-civilization until now and forever. Although the birth of the idea of number is so hidden behind the veil of countless ages, that is convincing philosophically there were numbers before Big-Bang and it seems like there should always be numbers, even if there isn't a universe [2]. From human awareness point of view, a consciousness to numbers as the embryo of number sense is believed to have been owned by humans since they were toddlers. This fact is indicated by the ability of toddlers, for example in recognizing the number of their toys even though they have not been introduced with numeral symbols or other letter symbols. This is the reason why toddlers can actually be taught to count before they can read or recognize numbers, just as they learn to talk before they can read or write [3].

In everyday life, humans are always in contact with numbers used for various purposes such as counting, enumerating, measuring, comparing, or even labelling. Counting is basically a fun activity for children. Starting from the enumeration activity, children will be introduced with various numerical symbols. More often, the numbers concept is taught to children as an abstract and isolated concept from concrete objects as they used in learning to count in the previous phase. Basic arithmetic operations such as addition, subtraction, multiplication, and division are more obtrusive taught as doctrines, namely belief in truth, memorization, and applied to solve problems. Learning activities are commonly dominated by memorizing facts and applying methods procedurally without needing to know why the methods are valid. Demands for learning outcomes in mathematics are still limited to students' skill to get the final answers. Depth understanding has not been a major concern in the learning outcomes evaluation.

Many sad news describing our students' literacy on mathematics. Based on data from the 2016 Indonesian National Assessment Program (INAP) of the Ministry of Education and Culture, it was above 77% of Indonesian students' ability on math were lacking and only about 2% were categorized as good. This result was similar to assessment conducted by PISA an international assessment that measures 15-year-old students' literacy on reading, mathematics, and science where Indonesia on 2018 placed the bottom 10 position among 70 countries involved in the assessment program [4]. This position was slightly better than previous 2015 occupied the bottom 5 position. This bottom ranking was not much different from another assessment conducted by TIMSS in 2015. Recently, how very shocking the news was from the Research on Improvement of System Education (RISE) 2018 that the ability of students to solve simple math problems did not differ significantly between students entering elementary school (SD) and those who have graduated from high school (SMA). With this result, Indonesia is classified as country with "mathematical emergency".

The effort to lift the position on PISA or TIMSS even to medium level within the next decade seems very hard unless there will be a miracle of exponential growing in the quality improvement of mathematics education in Indonesia. While the mathematics learning in developed countries has applied a high-level reasoning and creativity approach, we still focus on fixing student basic arithmetic skills even at a very low level, particularly the addition and subtraction of integers. It is not surprising when it was revealed that majority of students have not a good "number sense" [5, 6]. It indicated students did not learn mathematics with making sense, because the number sense grows from meaningful learning experiences [7].

Knowledge (understanding), skill, and the meaning of numbers as well as the number representation ability should be instilled to students in an integrated way. Students with poor skills in basic arithmetic certainly do not have the knowledge and meaning of numbers. The furthermore impact, student will get many difficulties to learn any mathematical topics, e.q. "algebraic forms" involving variables as abstractions of numbers. The poor perform in mathematics also found among many university students. Poor student's mathematical background was conveyed by Khouyibaba [8] who succeeded in identifying three contributing factors, namely the lack of conceptual understanding of mathematical principles, the consequences of using a calculator too early, and the negative impact of learning given by private mathematics teachers in middle school.

Students actually do not need the correct answers of the math problems for their life because only a few students take carrier (job) relating to math contents directly, but the skills of reasoning and critical thinking that should be the focus of math courses will always be needed. However, skill on basic arithmetic are must not only as requisites for learning math and other subject but also as basic skill for life. Effort to fix up the problem of basic arithmetic skills are usually carried out through a series of drills by providing repetitive arithmetic exercises, yet without understanding the process of steps. The various strategies offered to overcome arithmetic basic skills problems did not consider the diagnosis results of learning difficulties, student learning experiences, and the underlying concept [9-11]. Teachers frequently take shortcuts by asking students to memorize patterns, for instance using the statement "minus met minus is plus", without explaining the concept behind the statement. Taking such shortcuts with no regards of thought process can be misleading from the goal of learning mathematics itself. In addition, skills that was acquired through the fast method are usually only stored in short-term memory. This is the alleged cause of children easy to forget their prior knowledge and skills although not long ago.

This article describes the basic arithmetic skills of grade 1 junior high school students equipped with diagnosis of learning problems, in particular the addition and subtraction of integers including negative numbers. Even though it seems trivial, it was found that negative number is caught sight by many students as an abstract concept so that difficult to understand. Here, the diagnostic is performed to know the root of such problem. Diagnostic here refers to Leuders [12] that

> *Diagnostic activities comprise the gathering and interpretation of information on the learning conditions, the learning process or the learning outcome, either by formal testing, by observation, by evaluating students' writings or by conducting interviews with students.*

Data used in diagnostic are based on the results of pre-test, midterm test, interviews with students and teachers, classroom-observations, and students' textbook. Some crucial concepts on the student textbooks are reviewed and criticized. Eventually, some alternatives to improve learning approaches are proposed in the last section.

2. Description of basic arithmetic skills of students and some diagnostics

Formally, the numbers have been introduced to pupils since they were at elementary school grade 1 ranging from the presentation of whole numbers up to 99, explaining the meaning, sorting, and solving problems of daily life related to the addition and subtraction of these numbers, up to solving problems related to addition, subtraction, multiplication, and division involving negative integers, fractions, and decimal numbers in grade 6. One of the graduate competencies standard (SKL) in mathematics elementary school is understanding the concepts of integers and fractions, arithmetic operations and their properties, and using them in solving problems of everyday life.

Referring to this standard, the elementary school graduates should already be good on basic arithmetic skills. These skills are obligatory as prerequisites to learn mathematics in junior high school. Considering that it is very necessary to learn mathematics, the similar topic is repeated at the beginning of middle school by two standards. Standard 3.1: explain and determine the ordering on integers (positive and negative), and fraction (ordinary, mix, decimal, percent), and standard 3.2: explain and perform arithmetic operations on integers and fractions by utilizing various operation properties. It was very surprising, the diagnostic test confirmed their skills are far from expectation. Following table shows the results of diagnostic test on four main indicators.

The test was conducted on 4th week for SMP X and 2nd week for SMP Y on the first semester academic year 2019/2020. The participants had learned this topic at elementary school and repeated when test was being carried over. Taking notice learning experience, SMP X is longer so that it should be better than SMP Y in the score achievement. But in reality, the skills of students in the two schools did not differ significantly. The scores presented in Table 1 exhibit the basic arithmetic skills of students in both schools. The interesting findings are they are not only very low but also similar pattern of their deficiencies.

Table 1. The basic arithmetic skills of students in both schools

Indicators	Test Problems	Percentage students with correct answers		Weighted mean
		SMP X[a]	SMP Y[b]	
Addition and subtraction of integers	$-17+15=$	68%	62%	67%
	$-12-15=$	24%	13%	22%
Multiplication and division of integers	$(-4)\times 13=$	60%	62%	60%
	$(-21)\div(-7)=$	61%	72%	63%
Addition and subtraction of fractions	$1/3-1/6=$	51%	49%	51%
	$(-4/5)+(1/5)=$	24%	26%	24%
Mixed operation on fractions	$\frac{3}{4}-\frac{1}{3}\times\frac{6}{5}=$	15%	15%	15%
Fraction Ordering	$1/3\ \cdots\ 2/5$	68%	75%	69%
	$-1/2\ \cdots\ -1/4$	36%	40%	37%

[a]SMP X is SMP Negeri 5 Ponorogo with 272 students,
[b]SMP Y is SMP Azmania Ponorogo with 47 students

The such unexpected circumstance must be resolved as soon as possible in order that students can follow further topics in mathematics. For instance, the algebra form and the linear equation of one variable are the next topics that highly required students' skill on basic arithmetic. Otherwise, students will be tortured, frustrated, and ultimately failed in learning mathematics. A suitable remedial program is required to cope the mathematics learning accident occurred in elementary school. For this purpose, some diagnostics of previous learning problems performed through current student understanding. Based on diagnostic test and interview with students we obtain the following root of problems.

2.1 Subtraction of fractions
It should be easier that integers subtraction $-12 - 15$ than subtraction of fraction $1/3 - 1/6$, but in the fact it was opposite where the percentage students with correct answer is 22% compared to 51%. The diagnostic results for this problem are the following.

The procedure to solve $1/3 - 1/6$ has been standard and simply to follow by students, firstly to equal both denominators, then subtract the first nominator by the second. It was 51% students with correct answer close to double from another one that only 22%. But, when they are asked to explain why this procedure does work they did not understand at all. They believe this method valid was from their teachers when they were in elementary school, meanwhile teachers never explained the reason behind. For teachers, it does not matter as long as students can apply it to find the correct answer. This phenomenon was confirmed when author visited elementary school around and took classroom observations and teacher interview. It is interesting that for addition of fraction when the first number is negative, i.e. $(-4/5) + (1/5)$, there only around 24% of students with correct answer. Theoretically, the addition of fractions $(-4/5) + \left(\frac{1}{5}\right)$ should be easier than $1/3 - 1/6$ since their denominators were already equal. The presence of negative fraction $(-4/5)$ made this problem more difficult for students.

Likewise, the subtraction of integers $-12 - 15$ requires the concept of negative numbers which is more abstract than positive numbers. In this case, students were not able to distinguish between minus sign "$-$" as either a subtraction operation or an unseparable part of negative number. Here (-12) is a negative number so that minus sign here is not an operation. More often, students pronoun the expression $-12 - 15$ by "min (minus) twelve min (minus) fifteen". They seemingly did not understand that subtraction as inverse of addition. This supposition is supported by 64% of 266 students gave the correct answer on problem $-17 + (-6)$, while only 46% with correct answer on problem $-17 - 6$, even though both problems produce the same answer.

2.2 Multiplication and division
The students' skill on multiplication and division of integers are good enough since more than 60% students can answer correctly. Students applied the standard algorithm and memorizing to solve this problem without making sense.

When students asked to out sign less than "<" or greater than ">" on $1/3 \cdots 2/5$, it was obtained almost 70% students answer correctly. Furthermore in an interview some students were requested to explain the method, they applied the "cross product" strategy: $\frac{1}{3} \cdots \frac{2}{5} \rightarrow 1 \times 5 \cdots 2 \times 3 \rightarrow 5 \cdots 6 \rightarrow 5 < 6 \rightarrow \frac{1}{3} < \frac{2}{5}$. They did not understand this method actually adopt and adapt the same procedure when do addition or subtraction of two fractions. The skills without knowledge they obtain when learn mathematics in elementary school. Even though they can put inequality sign exactly, but they were not able to put both numbers position on the numbers line. On other hand, there were only 37% students with correct answer for problem $-1/2 \cdots -1/4$, this signifies they can not apply the same strategy to negative numbers. Thus, the students did not properly understand the concept of negative numbers.

2.3 Mixed operation
The worst situation was found on mixing operation of fraction, there were only 15% students with correct answer. It requires multiple skills in ordering, multiplying, and subtracting fractions.

In addition to previous findings, some strange findings are uncovered through the midterm exam. Many students solve like this $(-7) + (-26) = 33$ as well as $8 + (-15) = -23$. They applied the priciple of "negative-positive meeting" to adddition even though it only does work on multiplication and subtraction. Students with very low score on midterm test are treated in remedial program. Most of them provided the same answer to problem $13 - 25$, that is 12. Again, the interview revealed that they did not have a clear understanding on negative integers. It was no problem when they asked to answer $3 - 1, 3 - 2$ until $3 - 3$, even though generally rather slow. Furthermore, when they asked to determine the result of $3 - 4$, $3 - 5$, many students seemed confused, others replied it cannot be subtracted. This means students had a perception in concrete realm, where "subtraction" is understood as taking objects from the collection of objects. In case there are only 3 objects will be taken 4, it is not possible to be performed. This situation generally occurred among laggard students those who responses very slow messages delivered by lesson [13]. Sadly, more than half of students are indicated as laggards.

3. Subject matter of numbers in students' textbook: Some reviews and critical notes

Since the curriculum 2013 was imposed by government, the old books were not valid anymore and have to be replaced by new books involving scientific approaches. By this approaches, any concept in mathematics must be derived scientifically. The direct application of formulas and follow the procedure blindly are not recommended. On the following, some reviews and critical noted are addressed to the mathematics textbook for SMP Grade 1 Revised Edition published officially by Ministry of Education and Culture, Republic of Indonesia [14].

3.1 Integers' position in number line

Integers are introduced through their position in the numbers line, namely, negative integers, zero, and positive integers. How do three kinds of those integers correspond was not reviewed, included how do we understand the such order of number line. The next discussion is to compare two big numbers without being preceded by the concept of place value system (base-10).

3.2 Addition concept

The concept of addition is introduced by considering 3 dolls owned by Mia and then 4 more dolls obtained as her birthday gift. The number of dolls that Mia has now is represented by $3 + 4$. The meaning of $3 + 4$ is ilustrated on numbers line as follows: "We depart from origin 0 moving 3 units to the right. Then, because of getting 4 more dolls it means keep moving 4 units to the right, finally arrive at 7." At first glance there is no oddity with this explanation, but the logic path is lost when illustrating the number of dolls as a dimensionless quantity by distance of length dimension. The remaining question is how to examine the logical relationship between the number of dolls, the origin, the distance and the direction of movement, as presented in Figure 1.

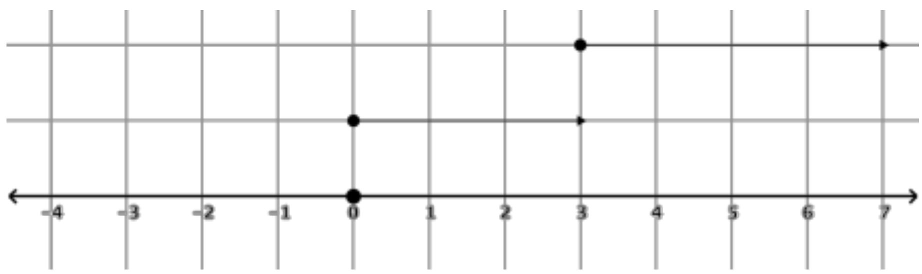

Figure 1. The meaning of $3 + 4$

3.3 Subtraction

In the book, the subtraction of integers is also demonstrated on the numbers lines as presented in Figure 2.

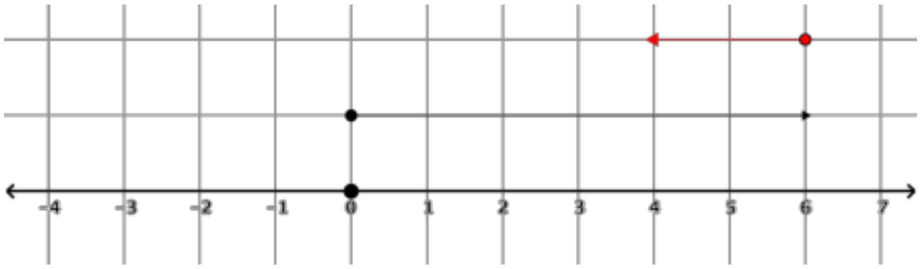

Figure 2. Subtraction of $6 - 2$ on the numbers line

There is a narration explaining this exhibition, "consider the subtraction of $6 - 2$ is equal to addition of $6 + (-2)$", while the meaning of $6 + (-2)$ has no shown yet on the numbers line. It is also explained that "left arrow indicates either subtraction by positive or addition by negative". The presence of two interpretations for one arrow can make students confused. In addition, there is no logical reasons for connecting both interpretations.

3.4 Subtraction of two negatives

The next illustration is the subtraction of two negatives $(-2) - (-5)$, as presented in Figure 3.

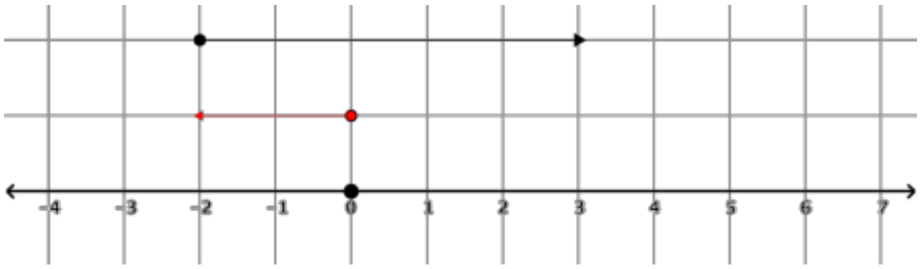

Figure 3. Subtraction of $-2 - (-5)$ on the numbers line

Here the right arrow shows the subtraction of negative, previously this direction for addition of positive (see Figure 1). As book with a scientific approach, this book has guided students to ask questions related to anything being study. Since students are not accustomed to think critically, it is most probably that students accept this illustration as fact to memorize. Suppose that a smart student proposes the question as "why each arrow direction has two interpretations, the left arrow indicates either subtraction by positive or addition by positive? Specifically, can the fact that $6 - 2$ is equal to $6 + (-2)$ be verified by this illustration, or vise versa. If we start from defining of the meaning of arrow direction to conclude the truth $6 - 2 = 6 + (-2)$ then it obscures the scientific value of argument. Similar problem to $2 - (-5) = 2 + 5$.

The other facts that usually memorizing but meaningless mathematically is the signs of number as the result of multiplication or division, namely $(+) \times (+) = (+)$, $(+) \times (-) = (-)$, $(-) \times (+) = (-)$, and $(-) \times (-) = (+)$. The fact $(-) \times (-) = (+)$ most probably used as underlying principle "negative met negative is negative" as well as "positive met negative is negative". Unfortunately, as mentioned before, some students did apply incorrect where the principles applied on addition and subtraction.

Basically this book has attempted to involve scientific approaches, but readers (teachers and students) regard the exposition is circling and too long with book thickness reaches more than 300 pages just for 1 semester. Perhaps because the book is considered too intricate by teachers and students, the acceptance of book is not so encouraging. Instead of following the approaches in the book, teachers just take examples and exercises to be discussed by conventional approach.

4. The alternative approach to teach concept of integers and its arithmetic

Mathematics is a subject that uses a deductive approach, but it is very open to apply some inductive approach for bridging the abstract nature of mathematics and students' ability in the stage of thinking concretely. Nevertheless, students should be directed to be accustomed with abstract concepts. In facts, mathematics is often taught by practical and pragmatic approaches. The students' skills to answer questions in exam, especially the national exam is the main goal of learning mathematics in school regardless of whether students understand or not what they are doing.

In terms of content, mathematics is actually not so important for most students because only a small number of them will take fields or carriers that are directly related to mathematical content. But the skills of reasoning and critical thinking that should be the focus of math courses will always be needed in the 21st century [15]. Mathematics should be used to teach useful mental method that can be applied to the real world [3].

This section proposes several alternative approaches for teaching the concepts of numbers, especially integers. It integrates the abstract concepts and common perception of students every day.

4.1 Introduction and construction of integers

Considering the numbers was born from counting activities, the first numbers set that needs to be introduced to children is the natural numbers 1, 2, 3, $\cdots$. This number is used to count, usually associated with the number of objects such as number of dolls, quantity of marbles, number of students, and some others. Next, the formal construction of integers is given as follows.

At beginning there were only two integers, namely 0 and 1 together with two binary operations addition $(+)$ and multiplication $(\times)$. The rest of natural numbers 2, 3, 4, $\cdots$ are governed by a single number 1, where 2 = 1 + 1, 3 = 1 + 1 + 1, and so on. Based on this construction, it is reasonable to say 2 is greater than 1, 3 is greater than 2, 4 is greater than 3 and so forth. 1 is called the unit and 0 is the zero element. Properties of unit and zero element:

4.1.1 *Property zero element*. It states that any number is added to zero, the result does not change, that is $a + 0 = a \textit{ and } 0 + a = a$.

4.1.2 *Negative element*. Each natural number has a counterpart which is called the negative element in which if both numbers are added results zero. For examples,

- 1 has negative (-1) and $1 + (-1) = 0$ and $(-1) + 1 = 0$ holds.
- 2 has negative (-2) and $2 + (-2) = 0$ and $(-2) + 2 = 0$ holds.

In general, every number a has negative $(-a)$ such that $a + (-a) = (-a) + a = 0$. In this case $(-a)$ is also a number, reads “negative a”. The negative sign “$-$“ here is not the division operation, but an integrated part of a.

4.1.3 *Unit element*. Property of unit element declares that any number when is multiplied to 1, the result does not change, that is a $\times$ 1 = a and 1 $\times$ a = a.

Integers numbers constitutes a set consisting of natural numbers (positive integers), zero, and negative integers. Here are integers: $\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots$. The whole numbers is natural numbers which including 0, i.e. $0, 1, 2, 3, \cdots$. The negative element property below is very important in developing the concept of addition and subtraction.

$$a + (-a) = (-a) + a = 0 \qquad (1)$$

In the next stage, students are introduced to the numbers line. Intuitively, order properties of numbers can be described through this numbers line. Commonly, the numbers line is used to implant concept of addition and subtraction at once, but crashing with its original concepts as be mentioned in previous section. This proposal will introduce the concept of addition and subtraction in accordance with daily students' perception in which the addition relates to activity of giving meanwhile the subtraction corresponds to activity of taking. On early stage students imagine addition operation will increase the quantity and subtraction will reduce the quantity.

4.2 The meaning of addition: Natural law or axiom?
The meaning of addition can be described in Figure 4.

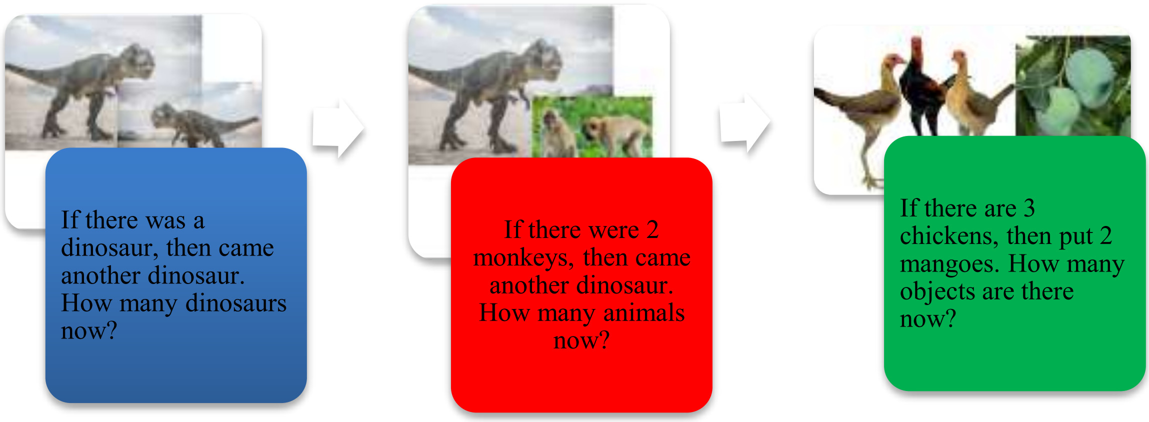

Figure 4. Natural law of addition

The left panel describes $1 + 1 = 2$ where 1 and 2 represents the number of dinosaurus. The middle panel ilustrates $2 + 1 = 3$ where 2, 1, and 3 represents the number of animals. The right panel exhibites $3 + 2 = 5$ where 3, 2, and 5 signifies the number of general objects. The sum results here are the same as the results obtained when defining the natural numbers before. This means that the role of addition is both natural law and axiom.

4.3 Addition and Subtraction using squares model: A concrete approach
The model consists of a collection of squares in two types, namely "plus-square" and "minus-square". A plus-square reflects 1 and a minus-square represents (-1). We may use coloring for this model. Figure 5, 6, and 7 describes an initial consensus.

Figure 5. Represents 1

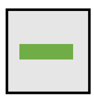

Figure 6. Represents –1

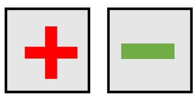

Figure 7. Denotes $1 + (-1) = (-1) + 1 = 0$

For example: Solve $3 - 5$ using square model. To solve the problem, here the steps for both integers are positive.

- Prepare 3 plus-squares to represent 3 (see Figure 8).

Figure 8. Represents 3

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- The subtraction "$3-5$" is interpreted as taking 5 plus-squares from 3 already plus-squares. Of course, this is not possible because there are only 3 plus-squares available. To make this possible we merge 2 more plus-squares together with its pairs 2 minus-squares so that the value is still 3 (see Figure 9).

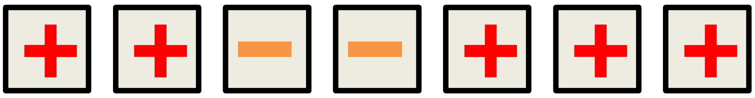


Figure 9. Adding 2 positives and 2 negatives

- To solve $3-5$, we take 5 plus-squares and we have 2 minus-squares left. Figure 10 shows that -2 is the result.

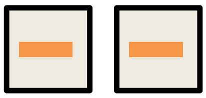


Figure 10. Adding 2 positives and 2 negatives

Another example is a problem of $-3-2$. The steps are:

- For $-3-2$, it means we take 2 plus-squares from a set of 3 minus-squares. Certainly, it cannot be performed since there are only 3 minus-squares. For the purpose, add 2 more plus-squares together with their pairs so that we have now 5 minus-square and 2 plus-squares. After being taken 2 plus-squares, there are 5 minus-squares left, this is nothing but -5 as the result of $-3-2$.
- On case $2+(-3)$ we add 3 minus-squares into 2 plus-squares. Consider 2 plus-squares and 2 minus-squares results zero, we discard these pairs so that there is only 1 minus-square. This final situation corresponds to -1 as the result of $2+(-3)$.
- For $-2-(-1)$, taking directly 1 minus-square from 2 minus-square, clearly left 1 minus-square indicating -1 as the result of $-2-(-1)$.
- How about $-2+1$? Here, we add 1 plus-square into collection of 2 minus-squares. Since the pair 1 minus-square and 1 plus-square produces nol, then the left 1 minus-square represents -1, the result of $-2+1$. In this simulation we can establish the fact that $-2—1 = -2+1$.

Through the above demonstration, we can guide students to a conclusion that for every integers a and b the following identity holds:

$$a - b \coloneqq a + (-b) \tag{2}$$

Then, for examples,

- $4-3 = 4+(-3) = 1$.
- $2-(-1) = 2+\big(-(-1)\big) = 2+1 = 3$.

We could read,

- $2-(-1) = 3$ is read "two" minus (is subtracted by) "negative one" is equal to "three".
- $(-5)-2 = -7$ is read "negative five" minus (is subtracted by) "two" is equal to "negative seven".

This media had been applied on teaching addition and subtraction of integers numbers including negative integers. The effect was very good to help students understand the concept of addition and subtraction involving negative integers, in particular students who categorized as laggards. For large numbers, students can imagine a set of such squares. For example, the problem $(-11) - 15$, students are directed to change this subtraction into the equivalent additon, i.e. $(-11) - 15 = (-11) + (-15)$. The last form tells us to merge 2 set of minus-squares each consisting of 11 and 15, so that there are 26 minus-squares in total. This means $(-11) + (-15) = -26$.

The other advantage of this using model in teaching of numbers is to stimulate students' *number sense* as well as their ability in number representation. When students are confronted with the number 6, students are not only imagined 6 objects, for example 6 plus-squares but also they are able to present 6 into various representations, for example, $6 = 1 + 5 = 2 + 4 = 4 + 2 = 3 + 3$, and others. Similarly, when learning multiplication. When students are asked to solve $-4 + 6$, they are expected able to apply property of negative element, for example, $-4 + 6 = -4 + 4 + 2 = 0 + 2 = 2$. This strategy conceptually will be useful when working with large numbers.

It is important to understand that every number has a negative pair. The following figure shows the integer and its negative pair. Since negative of 1 is (-1) then negative of (-1) is 1. This pattern is applicable for any number. In general, for any real number a we have:

$$-(-a) = a \tag{3}$$

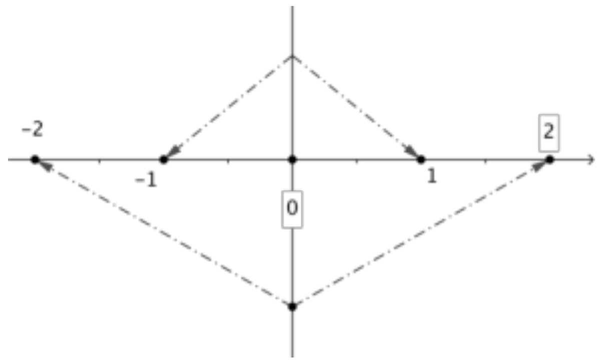


Figure 11. Diagram of negative pairing

In Figure 11, it describes that two numbers mutually negative each other are equidistant from zero.

4.4 Addition and subtraction by the numbers line

In order to avoid any ambiguous, each arrow direction is set into single interpretation. For this purpose, the numbers line should be taken only for addition in which the right direction interprets addition with a positive and left direction for addition with a negative. Figure 12 illustrates $(-3) + 2$.

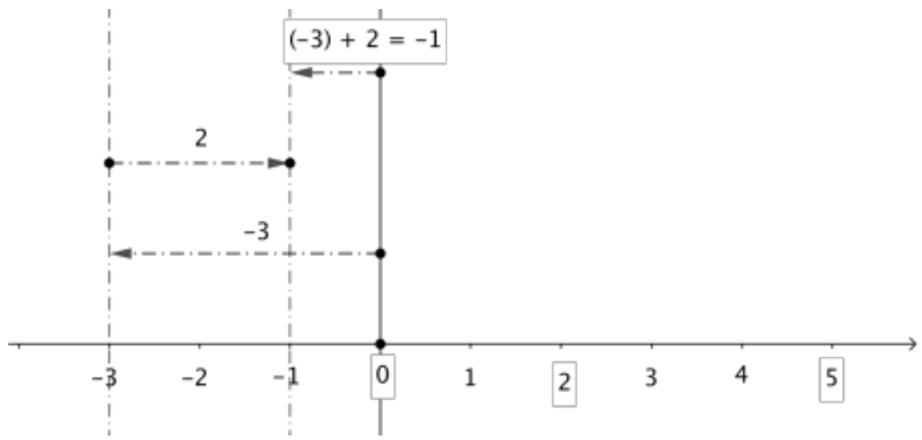


Figure 12. Diagram of addition on the numbers line

For subtraction operation, the problem is transformed first into equivalent form by $a - b \coloneqq a + (-b)$, then the previous rules is applied. Furthermore, the commutative and associative properties can be established using the numbers line by the same interpretation.

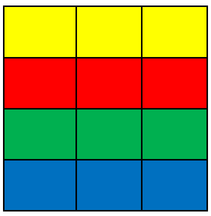
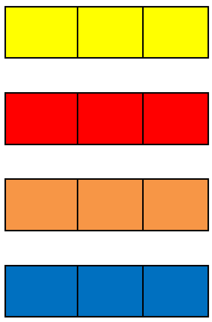
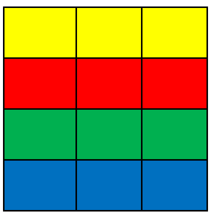
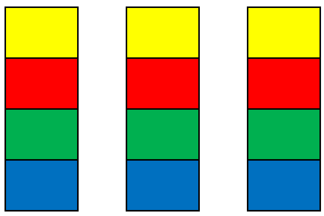
4.5 Multiplication and division

The multiplication concept is proposed by using the area model to build a formal definition. A controversial whether 3×4 is equal to 4×3 will be easy explained through this model visualization. So far the concept of multiplication does not a matter, instead the concept of division is still understood as the opposite of multiplication. The relationship between multiplication and division is given as follows:

$$a \div b = c \text{ if and only if } a = b \times c \quad (4)$$

Students have understanding that $\frac{12}{3} = 4$ since $12 = 3 \times 4$. How numbers 12,3, and 4 constitute relationship visually might not be understood by students. The following Table 2 are two kinds concept of division.

Table 2. Two kinds concept of division

Division	Initial Nominator	After Division	Justification
Concept 1: $\frac{12}{3} = 4$	 12 squares are collected into groups of 3 squares.	 It results 4 gropus, each contains 3 squares.	Relevant to definition: $12 = 3 + 3 + 3 + 3$ $= 4 \times 3.$
Concept 2: $\frac{12}{3} = 4$	 12 squares are evenly distributed into 3 groups.	 It results that each group contains 4 squares.	Relevant to definition: $12 = 4 + 4 + 4$ $= 3 \times 4.$

In teaching abstract concepts, teachers are required to visualize these concepts so that easily understood by students. To be careful that mathematics language and language in everyday life is not always in accordance. As a mathematics teachers mentor, author was asked by a mathematics teacher, "how to explain students logically the fact that 2 divided by ½ results 4". In everyday language whatever is divided, the results are reduced. In this case divided even get larger. Students seemed was interpreting the division as "break up into smaller" regardless the divisor, this is relevant to concept 1. I passed an idea to the teacher by imagining 2 liters of water, poured into cups of 1/2 liter then there will be 4 cups full of water. In this case, division refers to concept 2, i.e. to collect in certain amount (divisor).

Synchronization of mathematical language and its interpretation in everyday life is an attempt of mathematical humanization. Mathematics itself is basically humanistic. Hersh stated that mathematics must be understood as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context [2]. Furthermore, Boaler emphasized the importance of mathematics learning called 5C's of mathematics engagement, namely curiosity, connection making, challenge, creativity, and collaboration [16]. Meaningful learning can only be achieved if students enjoy and understand the subject matter. The use of cuisenaire rod and area models in teaching numbers is strongly suggested since it is very helpful for realizing these 5 aspects, especially to train student creativity.

So far, the students' creativity has not been touched in mathematics learning because mathematics learning is attentive more on how to get the final answer than thinking process and student creativities. A creative student will calculate $21 - 6$ through $20 - 5$ since easier. In calculating area of given rectangle figure with sides 18 and 5, creative students will make various models but easier to handle, either making partition or extension. Some possibilities are $5 \times 18 = 5 \times (9 + 9) = 45 + 45 = 0$, $5 \times 18 = 5 \times (10 + 8) = 50 + 40 = 90$, atau $5 \times 18 = 5 \times (20 - 2) = 100 - 10 = 90$. The skill of expressing numbers into easier forms is the result of creativity. The author once asked students to calculate multiplication 499×25. As be guessed, students worked with the standard multi-layered method. A creative student will apply trick $499 \times 25 = (500 - 1) \times 25 = 500 \times 25 - 1 \times 25 = 100 \times 5 \times 25 - 25 = 12500 - 25 = 12475$.

5. Concluding Remarks

Based on the results of this study, it was found that the basic arithmetic skills of mostly students of junior high school grade 1 were classified as very poor. This condition certainly did not meet the prerequisite to learn mathematics in junior high school. This problem is strongly suspected as the impact of student learning experiences at the previous elementary school.

Even though the new curriculum has been implemented for more than 5 years, the scientific approach in mathematics learning at junior high school has not been running well. The competencies in knowledge, skills, and attitudes as a learning outcome of this curriculum was not achieved by majority of primary school graduates. In particular, the lack of students' skills on basic arithmetic is the main obstacle in teaching and learning mathematics at junior high school. In addition, unattractive textbooks and lack of teacher innovation also aggravate to this situation.

Students are still forced to learn mathematics even though they do not have enough prerequisites on basic arithmetic. Students will suffer and learning will definitely fail. Efforts to overcome this problem have been made without considering the root cause of the problem. The learning problem academically of each student should be analyzed through diagnostics test. Various evaluation instruments can be modified to be diagnostic test. A structured remedial program based on diagnostic test must be conducted as early as possible.

Ideally the problems of students' skills on basic arithmetic should be completely finished in elementary school. Too much mathematics contents that students must learn in elementary school makes teachers not focus on this basic skills, whilst these skills are very important not only for students continuing education level but also for their life skills.

Teaching numbers in primary school must take as a priority, not only on preparing basic arithmetic skills but also reaching aspects number sense of students. Other topics could be set as optional, except topics required for learning numbers such as the concepts of length and area. Provides students' skills and understanding of numbers are already good in primary school, the other topics can be learned easily when they enter in junior high school. A few topics with deep understanding is better than many but superficial.

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