MODULE

ECONOMETRICS 2

Arranged by:

MAHRUS LUTFI ADI KURNIAWAN

ECONOMICS DEVELOPMENT STUDY PROGRAM
FACULTY OF ECONOMICS AND BUSINESS
AHMAD DAHLAN UNIVERSITY
2022/2023

INTRODUCTION

The preparation of the Econometrics 2 module is basically to help with literature needs for students of the Faculty of Economics and Business. Besides being intended to help students understand Econometrics 2, this module can be used to study other courses related to economics. For this reason, this module explains various materials on Regression with Time Series Data: Stationary Variables and Macroeconomic Forecasting (MFx): Structural VARs.

Hopefully this module can provide broader knowledge to the reader. Although this module has many drawbacks. The author needs constructive criticism and suggestions. Thank You.

Yogyakarta, September 2022

The Writer

CONTENST

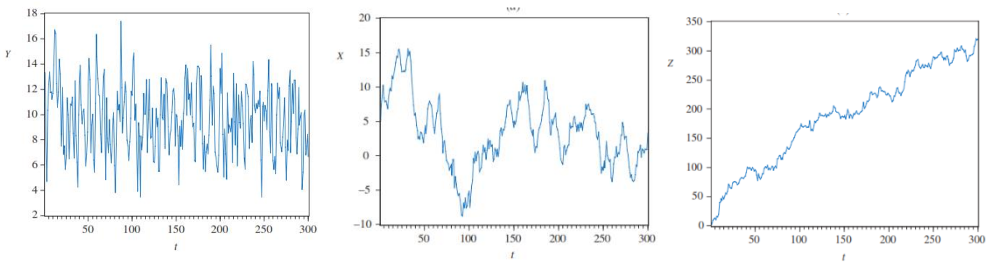
Chapter 1 Regression with Time Series Data: Stationary Variables

A. Introduction

Cross-section observations on a number of economic units at a given time are generated by random sample so uncorrelated, whereas time series is otherwise. The level of income observed in the Smiths' household in one year may be related to the level of income in the year before. One could shuffle the observations and then proceed with estimation without losing any information in cross-section data.

B. Dynamic Nature of Relationships

Dependent variable y is a function of current and past values of an explanatory variable x, e.g., a change in the interest rate now will have an impact on inflation now and in future periods; it takes time for the effect of an interest rate change to fully work → lagged effects: distributed lag model. Lagged dependent variable as one of the explanatory variables, e.g. periods of high inflation will tend to follow periods of high inflation and periods of low. Error term to the continuing impact of change over several periods (serially correlated), e.g. impact of unpredictable shock that feed into the error term will be felt not just in period t, but also in future periods.



Picture 1. *Stationarity*

Stationarity because tends to fluctuate around a constant mean without wandering or trending. The middle graph is non-stationary: slow turning or wandering. The right graph is non-stationary because trending.

C. Finite Distributed Lags

Formula:

$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \ldots + \beta_q x_{t-q} + e_t$

Different lags can be treated in the same way as different explanatory variables. Uses of the model: forecasting and policy analysis. Finite distributed lag model of order *q*: after a finite number of periods *q*, changes in *x* no longer have an impact on *y*.

Assumptions:

y and *x* are typically random, do not know their values prior to sampling, et is independent of all *x*'s in the sample past, current, and future. Conjunction with the other multiple regression assumptions.

Okun's Law:

Change in the unemployment rate from one period to the next depends on the rate of output growth.

$U_t - U_{t-1} = -\Upsilon\,(G_t - G_N)$ $\qquad$ $DU_t = \alpha + \beta_0 G_t - e_t$

$DU_t = \alpha + \beta_0 G_t + \beta_1 G_{t-1} + \beta_2 G_{t-2} + \ldots + \beta_q G_{t-q} + e_t$

Use okun data for exercise.

Table 1. Okun Data

Lag Length $q = 3$				
Variable	**Coefficient**	**Std. Error**	**t-Value**	**p-value**
Constant	0.5810	0.0539	10.781	0.0000
G_t	-0.2021	0.0330	-6.120	0.0000
G_{t-1}	-0.1645	0.0353	-4.459	0.0456
G_{t-2}	-0.0716	0.0353	-2.027	0.0456
G_{t-3}	0.0033	0.0363	0	0.9276
Observations = 95	$R^2 = 0.652$		$\sigma = 0.1743$	
Lag Length $q = 2$				
Variable	**Coefficient**	**Std. Error**	***t*-Value**	***p*-value**
Constant	0.5836	0.0472	12.360	0.0000
G_t	-0.2020	0.0324	-6.238	0.0000
G_{t-1}	-0.1653	0.0335	-4.930	0.0000
G_{t-2}	-0.0700	0.0331	-2.115	0.0371
Observations = 96	$R^2 = 0.654$		$\sigma = 0.1726$	

D. Serial Correlation

1. ARDL model: how a dependent variable can be related to current and past values of an explanatory variable.
2. The effect of a change in the value of an explanatory variable is distributed over a number of future periods.
3. With time-series data, successive observations are likely to be correlated. If unemployment is high in this quarter, it is more likely to be high next quarter → correlation over time.
4. Correlations between a variable and its lags are called autocorrelations.

5. Correlogram shows the correlation between observations that are one period apart, two periods apart, three periods apart, and so on.
6. Philips curve: describing the relationship between inflation and unemployment.

   $INF_t = INF^E_t - \Upsilon\,(U_t - U_{t-1})$

   Falling levels of unemployment reflect excess demand for labour that drives up wages, which in turn drives up prices.

   $INF_t = \beta_1 + \beta_2 DU_t + e_t$

   To examine whether the errors are serially correlated, we first compute the least squares residuals.

   $e_t = INF_t - b_1 - b_2 DU_t$

   There is strong evidence that the residuals are autocorrelated → correlations at lags one through six and at lag eight are all significantly different from zero.

Other test for serially correlated errors:

1. Lagrange Multiplier Test: If Prob. Chi-square is less than 0.05 → reject the null hypothesis of no autocorrelation.
2. Durbin Watson Test: used less frequently today because its critical values are not available in all software packages, and one has to examine upper and lower critical bounds instead.

Estimation with serially correlated errors:

What are the implications of serially correlated errors for least squares estimation?

1. The least squares estimator is still a linear unbiased estimator, but it is no longer best → there exists an alternative estimator with a lower variance → higher probability of obtaining a coefficient estimate close to true value.
2. The formulas for the standard errors are no longer correct → confidence intervals and hypothesis tests may be misleading.
3. Solution: HAC (heteroskedasticity and autocorrelation consistent) standard errors. If HAC standard errors are larger than those from least squares, we will overstate the reliability of the least squares estimates.

Example:

Estimating a more general model:

$\widehat{INF_t} = 0.3336 + 0.5593 INF_{t-1} - 0.6882 DU_t + 0.3200 DU_{t-1}$

(se) (0.0899) (0.0908) (0.2575) (0.2499)

Expectations for inflation in the current quarter are 0.33% plus 0.56 times last quarter's inflation rate. The effect of unemployment is -06882 ($U_t - U_{t-1}$) + 0.3200($U_{t-1} - U_{t-2}$).

DU_{t-1} : significantly different from zero → excluded.

$INF_t = 0.3548 + 0.5282 INF_{t-1} - 0.4909 DU_t$

(se) (0.0876) (0.0851) (0.1921)

Inflationary expectations: $INF^E_t = 0.3548 + 0.5282 INF_{t-1}$

So, 1% rise in the unemployment rate leads to an approximate 0.5% fall in the inflation rate.

E. Autoregressive Distributed Lag Models

An autoregressive distributed lag (ARDL) model is one that contains both lagged *x* and *y*.

ARDL (1,1): $\widehat{INF_t} = 0.3336 + 0.5593\ INF_{t-1} - 0.6882\ DU_t + 0.3200\ DU_{t-1}$

ARDL (1,0): $\widehat{INF_t} = 0.3548 + 0.5282\ INF_{t-1} - 0.4909\ DU_t$

Advantage: capture dynamic effects from lagged *x* and lagged *y* and eliminate serial correlation in the errors.

Criteria for choosing lag:

1. Has serial correlation in the errors been eliminated? If not, biased → user correlogram or LM tests.
2. Are the signs and magnitudes of the estimates consistent from theory?
3. Are the estimates significantly different from 0?
4. What values minimize AIC and SC?

Example:

$\widehat{INF_t} = 0.3548 + 0.5282\ INF_{t-1} - 0.4909\ DU_t$, obs = 90

(se) (0.0876) (0.0851) (0.1921)

1. Check whether errors are serially correlated → correlogram, if not significantly different from 0 → no evidence of serial correlation → further check using LM test.
2. Try ARDL (4,0) → check AIC and SC.
3. Okun's Law: Try ARDL (0,2) → does it contain serially correlated error? What happen if we include DU (-1)?
4. Drop G (-2) since it is no longer significant, check ARDL (1,1) with correlogram, LM test, AIC-SC!

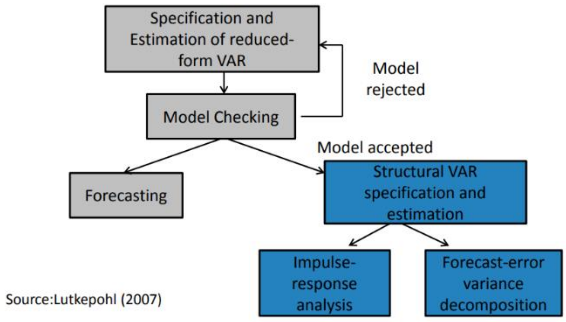
Forecasting:

AR Model: Forecast GDP growth for three quarters ahead based on AR (2). ARDL Model: Forecast future unemployment using ARDL (1,1) → problems: need future values of G.

Chapter 2 Macroeconomic Forecasting (MFx)

A. Introduction: Vector Autoregressive Models (SVAR)

Del Negro and Schorfheide (2011): at first glance, VARs appear to be straightforward multivariate generalizations of univariate autoregressive models. At second sight, they turn out to be one of the key empirical tools in modern macroeconomics. Vector autoregression (VAR) is a statistical model used to capture the relationship between multiple quantities as they change over time. VAR is a type of stochastic process model. VAR models generalize the single-variable (univariate) autoregressive model by allowing for multivariate time series. Vector Autoregressive (VAR) models are widely used in time series research to examine the dynamic relationships that exist between variables that interact with one another. In addition, they are also important forecasting tools that are used by most macroeconomic or policy-making institutions.



Picture 2. ***VARs***

B. Estimation of VARs

Let y_t be a vector with the value of *n* variables at time *t*:

$$\mathbf{y_t = [y_{1,t}\ y_{2,t} \ldots\ y_{n,t}]'}$$

A p-order vector autoregressive process generalizes a one-variable AR(*p*) process to *n* variables:

$$\mathbf{y_t = G_0 + G_1\, y_{t-1} + G_2\, y_{t-2} + \ldots + G_p\, y_{t-p} + e_t}$$

$\mathbf{G_0}$	= (n x 1) vector of constants
$\mathbf{G_j}$	= (n x n) matrix of coefficients
$\mathbf{e_t}$	= (n x 1) vector of white noise innovations
$E\,[\mathbf{e_t}]$	= 0
$E\,[\mathbf{e_t\, e_\pi}']$	$= \begin{cases} \Omega, if\ t = \pi \\ 0\ otherwise \end{cases}$

Example: A VAR (1) in 2 variables

$y_{1,t} = g_{11}\, y_{1,t-1} + g_{12}\, y_{2,t-1} + e_{1,t}$

$y_{2,t} = g_{21}\, y_{1,t-1} + g_{22}\, y_{2,t-1} + e_{2,t}$

In matrix notation:

$\mathbf{y_t} = \mathbf{G_1}\, \mathbf{y_{t-1}} + \mathbf{e_t}$

where,

$\mathbf{y_t} = \binom{y1,t}{y2,t}$, for example: $y_t = \begin{bmatrix} \pi t \\ gdp\ t \end{bmatrix}$

$\mathbf{G_1} = \begin{pmatrix} g11 & g12 \\ g21 & g22 \end{pmatrix}$, $e_t = \binom{e1,t}{e2,t}$

Assumptions about the error terms:

$$E[\mathbf{e_t e_t}'] = \begin{pmatrix} \sigma^2_{e_1} & \sigma_{e_1e_2} \\ \sigma_{e_1e_2} & \sigma^2_{e_2} \end{pmatrix} = \Omega$$

$$E[\mathbf{e_t e_\tau}'] = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \text{for } t \neq \tau$$

General specification choices:

1. Selection of variables to be included: in accordance with economic theory, empirical evidence and/or experience.
2. Exogenous variables can be included: constant, time trends, other additional explanators.
3. Non-stationary level data is often transformed (log levels, log differences, growth rates, etc.)
4. The model should be parsimonious.

C. Stationary VARs

A *p*-the order VAR is said to be covariance-stationary if:

1. The expected value of y_t does not depend on time.

$$E\,[y_t] = E\,[y_{t+j}] = \pi = \begin{bmatrix} \pi 1 \\ \pi\ 2 \\ \pi n \end{bmatrix}$$

2. The covariance matrix of y_t and y_{t+j} depends on the time lapsed *j* and not on the reference period *t.*

$$E[(\mathbf{y_t} - \boldsymbol{\mu})(\mathbf{y_{t+j}} - \boldsymbol{\mu})'] = E[(\mathbf{y_s} - \boldsymbol{\mu})(\mathbf{y_{s+j}} - \boldsymbol{\mu})'] = \boldsymbol{\Gamma}_\mathbf{j}$$

Conditions for stationary:

The conditions for a VAR to be stationary are similar to the conditions for a univariate AR process to be stationary:

$$\mathbf{y}_t = \mathbf{G}_0 + \mathbf{G}_1\mathbf{y}_{t-1} + \mathbf{G}_2\mathbf{y}_{t-2} + \ldots + \mathbf{G}_p\mathbf{y}_{t-p} + \mathbf{e}_t$$

Lag operator

$$\underbrace{(\mathbf{I}_n - \mathbf{G}_1\mathbf{L} - \mathbf{G}_2\mathbf{L}^2 - \ldots - \mathbf{G}_p\mathbf{L}^p)}\mathbf{y}_t = \mathbf{G}_0 + \mathbf{e}_t$$

Lag polynomial

$$\mathbf{G}(\mathbf{L})\mathbf{y}_t = \mathbf{G}_0 + \mathbf{e}_t$$

For y_t to be stationary, the matrix polynomial in the lag operator **G(L)** must be invertible.

A VAR(p) process is stationary (thus invertible) if all the np roots of the characteristic polynomial are (in modulus) outside the unit imaginary circle.

det $(I_n - G_1L - G_2L^2 - \ldots - G_pL^p) = 0$

E-Views calculates the inverse roots of the characteristic AR polynomial, which should then lie within the unit imaginary circle.

Vector moving average representation of a VAR:

If a VAR is stationary, the y_t vector can be expressed as a sum of all of the past white noise shocks e_t (VMA (∞) representation).

$y_t = \pi + G(L)^{-1} e_t$, where $\pi = G(L)^{-1}G_0$

$y_t = \pi + (I_n + \Psi_1 L + \Psi_2 L^2 + \ldots) e_t$

$y_t = \pi + e_t + \Psi_1 e_{t-1} + \Psi_2 e_{t-2} + \ldots$

$y_t = \pi + \sum_{i=0}^{\infty} \Psi_i e_{t-i}$

where Ψ_i is a (n x n) matrix of coefficients, and Ψ_0 is the identity matrix. From the VMA (∞) representation it is possible to obtain impulse response functions.

D. Lag Specification Criteria

VARs are very densely parametrized:

In a VAR (p) we have p matrices of dimension $n \times n$: $G_1, \ldots, G_p$. Assume G_0 is an intercept vector (dimension: $n \times 1$). The number of total coefficients/parameters to be estimated is:

$$n + n \times n \times p = n(1 + n \times p)$$

As for univariate models, one can use multidimensional versions of the:

1. AIC: Akaike information criterion
2. SC: Schwarz information criterion
3. HQ: Hanna-Quinn information criterion

Information-based criteria: trade-off between parsimony and reduction in sum of squares.

E. Forecasting Using VARs

Let Y_{t-1} be a matrix containing all information available up to time *t* (before realizations of e_t are known):

$Y_{t-1} = (y_{t-1}, y_{t-2}, \ldots, y_{t-T})$

Then:

$E\,[y_t \mid Y_{t-1}] = G_0 + G_1 y_{t-1} + G_2 y_{t-2} + \ldots + G_p y_{t-p}$

The forecast error can be decomposed into the sum of et , the unexpected innovation of yt, and the coefficient estimation error:

$y_t - E\,[y_t \mid Y_{t-1}] = e_t + v\,(Y_{t-1})$

If the estimator for the coefficients is consistent and estimates are based on many data observations, the coefficient estimation error tends to be small, and:

$y_t - E\,[y_t \mid Y_{t-1}] \cong e_t$

Iterating one period forward:

$E\,[y_t \mid Y_{t-1}] = G_0 + G_1 E\,[y_t \mid Y_{t-1}] + G_2 y_{t-1} + \ldots + G_p y_{t-p+1}$

Iterating *j* periods forward:

$E\,[y_{t+j} \mid Y_{t-1}] = G_0 + G_1 E\,[y_{t+j-1} \mid Y_{t-1}] + G_2 E\,[y_{t+j-2} \mid Y_{t-1}] + \ldots + G_p y_{t-p+j}$

REFERENCE :

1. Wooldridge, J, M. 2010. Introductory Econometrics: A Modern Approach. WILEY.
2. Dougherty, C. 2001. Introduction to Econometrics, Third Edition. Oxford.
3. Martin, V, L. Hurn, S. dan Harris, D. 2011. Econometric Modelling With Time Series, Specification, Estimation and Testing. WILEY.