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# Students' Worksheet Through Problem Based Learning in Determinant and Inverse Matrix Material

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**Abstract.** Student worksheets are one of the student assignment guidelines that provide students opportunities for developing their mathematical problem-solving abilities. This study aimed to develop the determinant and inverse matrix student worksheets based on problem-based learning. This study was development research using the ADDIE model, consisting of five stages. This study's subjects were SMA X (n=28) in Imogiri, Yogyakarta. Data collection was carried out through questionnaires. The material and media experts assess the validity of student worksheets using a questionnaire. Whereas students assessed the teaching material practicality. The development of students' worksheets based on problem-based learning reached the validity and practical criteria. The validity of the students' worksheets is based on the assessment results by material experts with an average score of 77, which means the students' worksheets belong to the "very good" category. The same result was related to the practicality of the students' worksheets based on media experts' assessment results, which show an average score of 64, which means the students' worksheets belong to the "very good" category. In line with that, the average score of students' responses is 79.83, which indicates that "good" assessment for students' worksheets is used or belonging reaching the practicality criteria of development product.

## INTRODUCTION

Education is a valuable lesson to make people more critical in thinking so as to create a good character [1]. One of the subjects that has an important role in the education unit is mathematics [2, 3]. Mathematics is a formal science that uses symbolic language to study concepts such as numbers, structures, variations, and shapes [4]. Mathematics is a high-power subject because it facilitates the understanding and application of mathematics in human life so it must be studied in schools [5]. Mathematics material in high school consists of many sub-materials, including matrix material. This material must be taught because it is contained in the 2013 Basic Competencies. The study of matrices is very broad, which contains terms in matrices, types of matrices, transpose matrices, matrix equations, and operations on matrices such as matrix addition, matrices, matrices, determinants, inverses, matrix equations, and solving systems of linear equations using both matrices and determinants [6].

Matrix is material that is considered difficult by students, especially because of the many calculations that must be done [7]. From their results of a survey conducted on 120 students of class XII IPA SMAN 4 Surabaya, it shows that as many as 55% of students still get an average score of less than 60 for matrix material. The results of these studies indicate that the matrix material is indeed quite difficult. This is also in accordance with the results of preliminary research conducted which shows that the problem-solving ability of students in solving problems is still relatively low. Whereas one of the important roles of mathematics is problem-solving activities [8]. Problem-solving is an integral part of learning mathematics, so it should not be taken from learning mathematics [9]. Meanwhile, according to Effendi [10], problem solving is one of the abilities that students have to train students to face increasingly complex problems, especially in mathematics. The importance of problem-solving skills in mathematics is also stated by Ruseffendi [11], which states that problem-solving skills are important not only for

those who later will explore or study mathematics but also for those who will apply it in other studies and in everyday life-day. Thus, the important thing in learning mathematics is solving a problem.

However, in reality, the problem-solving ability of students in Indonesia is still very low [12]. This is in accordance with the results of interviews with mathematics teachers in class XI MIPA 1 SMA Negeri 1 Imogiri which showed that the problem-solving abilities of students still tended to be below. In addition, based on the results of observations in the class, it also shows that when students are given math problems, students tend to write only the final answer without including the steps for doing it. This makes it a little difficult for teachers to check students' mathematical problem-solving abilities. Other results obtained from the initial test of problem-solving skills showed that the average number of students who answered correctly for the problem-solving ability aspect was 40.385% on description question 1 and as much as 37,500% on description question 2. From the data analysis, it was found that the problem-solving ability of students was still low. Other results show that students already quite understand the problem and implement plans, but students are still weak in making plans and concluding. So far, the tendency of students to solve a problem is to solve it immediately, but the steps in understanding the problem, planning, and concluding are sometimes not done [13]. Therefore, based on the initial ability test, the average problem-solving ability of students tends to be below.

The low problem-solving ability related to the inability of students to use their knowledge to be poured into a routine or non-routine problem-solving methods or algorithms indicates that students are less trained to deal with mathematical problems. In addition, the low problem-solving ability is also caused by student errors in interpreting the concepts they have acquired during the learning process. Errors in the meaning of this concept will affect the logical thinking of students to understand something [14]. So that students need an approach that makes the problem a starting point in their learning. One learning approach that accommodates this is Problem Based Learning (PBL). According to Zulfah et al. [15], PBL makes problem-giving the beginning of the learning process. Through PBL, students learn from a real problem, are challenged to solve problems, are required to understand the problem and seek answers to existing problems, while the teacher is only a facilitator [16]. The same thing was also stated by Zabit [17], that PBL could teach students to solve problems from the process of understanding to solve problems. So the PBL approach is very suitable to help students solve various types of issues and positively impact students' problem-solving abilities[18].

From the results of the initial research that has been done, the researchers also found that teachers use the Student Worksheet as the primary reference. Student Worksheet is a teaching tool that guides students to understand the material they are working on systematically, through a series of questions and information [19]. This students' worksheet is in the form of a written explanation that guides student activities in learning any topic [20]. students' worksheet contains coherent steps to guide students in carrying out activities to solve problems [21]. Although the students' worksheet used already meets the criteria of one student one book, the students' worksheet still contains a collection of questions and does not make the problem a starting point that can stimulate students' problem-solving abilities. This students' worksheet is because the students' worksheet used by using expository learning begins with a summary of the concept then continues with practice questions so that it does not guide students to start learning with problems. So, it is necessary to develop students' worksheet that can facilitate these things. Therefore, in this research, the students' worksheet developed is considered problem-based learning.

The things that are considered in developing PBL-based students' worksheet are that the use of PBL-based students' worksheet has been shown to improve students' problem-solving abilities [15, 22]. Another study conducted by Kurniati et al. [23] also showed that students' mathematical problem-solving abilities increased using student worksheets based on PISA in the PBL model. Based on the description above, this study aims to determine the effect of using students' worksheet-based Problem Based Learning to improve students' problem-solving abilities in SMA Negeri 1 Imogiri.

## METHODS

This type of research includes research and development (R&D) using the ADDIE model which consists of five stages, namely analysis, design, development, implementation, and evaluation [24]. The subjects in this study were students of class XI SMA Negeri 1 Imogiri. The research instrument used was a questionnaire instrument for assessing the validity of the material and media, as well as student response sheets. In this study, the product produced was teaching materials in the form of Student Worksheets with a PBL approach to improve valid and practical problem-solving skills. The material expert validation questionnaire instrument, the media expert validation questionnaire instrument and the student response instrument were tested for construct validity by

judgment experts. After all the instruments are declared valid by the experts, then these instruments can be used to validate the products developed. Data analysis of expert validation scores and student response scores in the form of a five-scale score was then converted into categories. The analysis of validity and practicality aims to obtain valid and practical qualifications from the developed learning tools. In giving the assessment classification, the results are calculated to get the value of validity with the classification in Table 1 as referred to Widoyoko [25].

**TABLE 1.** Classification of Validity and Practicality Criteria

No	Score	Criteria
1	$\bar{X} > \bar{X}_i + 1,8sb_i$	Very Good
2	$\bar{X}_i + 0,6sb_i < \bar{X} \leq \bar{X}_i + 1,8sb_i$	Good
3	$\bar{X}_i - 0,6sb_i < \bar{X} \leq \bar{X}_i + 0,6sb_i$	Enough
4	$\bar{X}_i - 1,8sb_i < \bar{X} \leq \bar{X}_i - 0,6sb_i$	Not Enough
5	$\bar{X} \leq \bar{X}_i - 1,8sb_i$	Not Less

Description:

$\bar{X}_i$  = Ideal average

$\bar{X}_i$  =  $\frac{1}{2} \times$  (ideal maximum score + ideal minimum score)

$sb_i$  = Ideal standard deviation

$sb_i$  =  $\frac{1}{6} \times$  (ideal maximum score – ideal minimum score)

$\bar{X}$  = Validity score

The ideal maximum score = the number of items of the highest score criteria

The ideal minimum score = the number of items with the lowest score criteria

Based on the analysis of validity and practicality above, the students' worksheet is said to be valid and practical if the average score of the validity assessment meets the minimum criteria of good.

## RESULTS AND DISCUSSION

Researchers developed teaching materials in the form of problem-based learning worksheets using the stages of the ADDIE development model. At the analysis stage, it is done by reviewing the curriculum used and analyzing what competencies must be mastered by students after using the development product. The researcher found that the teachers of SMA Negeri 1 Imogirii used the 2013 curriculum and taught the material on determinants and inverse matrices according to the basic competencies contained in the 2013 curriculum. for 2 meetings per week. The Minimum Completeness Criteria for Mathematics at SMA Negeri 1 Imogiri is 66.6. Another result is that the teacher uses the students' worksheet as the main reference and the students' worksheet is taught using the expository method. The students' worksheet used by the teacher contains a collection of questions and does not make the problem a starting point that can stimulate students' problem-solving abilities. The students' worksheet begins with a summary of the concepts, then continues with the provision of practice questions. Presentation of students' worksheet like this of course cannot guide students to be proficient in solving problems and have their own problem-solving strategies. In addition, the existing students' worksheet also does not contain a stimulus for the ability of students to participate in teams and does not guide students so that they can start learning with problems.

At the design stage, the researcher designs the students' worksheet according to the results of the analysis that has been done. The developed worksheet contains material on determinants and inverse matrices for class XI and includes PBL syntax. In the design stage, the researcher uses a frame of reference that is focused on selecting material according to the characteristics of students and the demands of competence, the learning strategies applied and the evaluation used. So the researchers designed the material for the determinant and the inverse matrix presented with the problem as a starting point for learning. In matrix material, determinants and inverse matrices, the material presented begins with contextual problems and problems that are ill structured. The selection of the type of material presented is based on the results of the analysis of the characteristics of students who are not used to and trained to deal with real problems from mathematical problems as well as types of problems that involve unknown elements, have several concepts and several solutions that require students to express personal opinions. related to his interpersonal activities with the team. From these problems, students are asked to follow each PBL syntax that is designed according to the achievement of certain indicators in problem solving abilities because of the characteristics of students who tend to write only the final answer without including the processing steps. This

means that students need teaching materials that are designed to contain stages of problem solving systematically and rigorously, so that the use of developed teaching materials can improve problem solving abilities.

At the development stage, the researcher carried out the activity of translating the design specifications at the design stage into physical form, so that this activity produced a prototype of a development product in the form of PBL-based worksheets to improve the problem-solving ability of determinants and inverse matrices. Next, the development stage activities are continued by searching and collecting all the sources or references needed for material development, making charts and supporting tables, making illustration pictures, typing, setting layouts and compiling evaluation instruments. In addition to developing the product, the researcher also developed a material expert validity instrument, a media expert validity instrument, a student response questionnaire instrument and a problem-solving ability pretest and posttest instrument. Researchers start the development stage by looking for sources of references used to compile students' worksheet. The references used in the students' worksheet are 1) The researcher's PBL syntax uses Arends' theory [26], 2) The ability to solve problems using theory in the journal belonging to Lubis, et al. [27] 3) The matrix material used refers to the book compiled by the Grasindo Team [28], and 4) Some questions in the students' worksheet refer to Sukardi's website [29].

The references above are used to develop materials designed at the design stage. Next, the researcher realized what was in the design stage into a prototype form of students' worksheet. However, before the students' worksheet prototype was validated by material experts and media experts, researchers developed a validation instrument for media experts and students' worksheet product material experts. The instrument was validated by the validator to produce a valid media expert and material expert validation instrument. The validation was carried out through reviewing each item of the instrument referring to the validation instrument grid for media experts and material experts. The material expert validation sheet instrument was assessed from the feasibility aspect according to the Ministry of National Education [30], namely the feasibility of content, language and presentation feasibility, and suitability with the characteristics of the Problem Based Learning approach according to Santyasa [31]. Meanwhile, the media validation instrument modified according to the needs of the researcher includes the use of fonts, layouts, students' worksheet designs, and illustrations. The student response questionnaire instrument was assessed from three aspects, namely interest, material, presentation, language, and suitability with the Problem Based Learning approach.

The results of the students' worksheet validation that have been developed are then given a qualitative and quantitative assessment. The results of the quantitative assessment use the classification of validity criteria in Table 1. So, the descriptions for the validity scoring for material experts and media experts which are interpreted quantitatively can be seen in Table 2 and Table 3.

**TABLE 2.** Data on the Validity Score of PBL-Based Students' Worksheet Materials Experts

Validator	Total Score	Category
Validator-1Mt	70,5	Very Good
Validator-2Mt	69	Very Good
Total Validator Score		144
Average		77
Category		Very Good

Table 2. shows that each material expert validator gives a score in the "very good" category. These results also show that the average score obtained by material experts on the students' worksheet developed by the researcher is 77 with the "very good" category. In addition, based on Table 3. it also shows that each media expert validator gives a score in the "very good" category.

**TABLE 3.** Data on the Validity Score of PBL-Based Students' Worksheet Media Experts

Validator	Total Score	Category
Validator-1Me	62	Very Good
Validator-2Me	66	Very Good
Total Validator Score		128
Average		64
Category		Very Good

These results also show that the average score obtained by media experts on the students' worksheet developed by the researchers is 64 with the "very good" category. Based on the results of the quantitative assessment of material experts and

media experts, it can be concluded that the worksheets developed based on the assessment of the validity of material experts and media experts show that they are "very valid" to be used for learning mathematics. So, the product design components repaired based on the assessment of the validity of material experts and media experts include:

1. The title page contains an identity that must be filled in by students.
  2. The preface contains the things contained in the students' worksheet, the objectives of the students' worksheet, thanks and requests for constructive criticism and suggestions for improvement of the students' worksheet.
  3. The table of contents contains the sections contained in the students' worksheet and a page is included to make it easier for students to find the section to be addressed.
  4. The symbols contain icon symbols that represent problem based learning syntax and problem solving abilities.
  5. The concept map contains an outline of the material to be achieved. In this case, the material discussed is the determinant of a matrix of order  $2 \times 2$ , determinant of a matrix of order  $3 \times 3$ , inverse matrix of order  $2 \times 2$ , inverse matrix of order  $3 \times 3$ , the properties of the determinant and the properties of the inverse.
  6. The learning activities section contains problems with the Problem Based Learning learning syntax which is structured to achieve problem solving abilities
  7. The determinant and inverse matrix formulas contain formulas that are generally used to obtain the determinant and inverse of matrices of order  $2 \times 2$  and  $3 \times 3$ .
  8. The bibliography contains a list of references used by researchers in compiling students' worksheet.
- Some example of the students' worksheet that developed by the researcher is presented in Figure 1 and Figure 2.

**LEMBAR KERJA PESERTA DIDIK KELAS XI SMA**  
**DETERMINAN & INVERS MATRIKS**  
 Ordo  $2 \times 2$  dan  $3 \times 3$

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$$M = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

$$M^T = \begin{bmatrix} 1 & 0 & 5 \\ 2 & 1 & 6 \\ 3 & 4 & 0 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} ONw & & \\ & ONw & \\ & & ONw \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

$$\det(M) = 1(0 \cdot 24) - 2(0 \cdot 20) + 3(0 \cdot 5) = 1$$

**Nafida Hetty Marhaeni**  
 LKPD Berbasis Problem Based Learning Untuk Meningkatkan Kemampuan Pemecahan Masalah

**Contents :**

- 1. Determinan Matriks Ordo  $2 \times 2$
- 2. Determinan Matriks Ordo  $3 \times 3$
- 3. Invers Matriks Ordo  $2 \times 2$
- 4. Invers Matriks Ordo  $3 \times 3$

**Kompetensi Dasar:**

- Menganalisis sifat-sifat determinan dan invers matriks berordo  $2 \times 2$  dan  $3 \times 3$ .
- Menyelesaikan masalah yang berkaitan dengan determinan dan invers matriks berordo  $2 \times 2$  dan  $3 \times 3$ .

(a)

**DETERMINAN & INVERS MATRIKS**  
 Ordo  $2 \times 2$  dan  $3 \times 3$

**S**alah satu bagian paling penting dari pembahasan materi matriks adalah cara menghitung determinan matriks, invers matriks dan sifat-sifatnya.

LKPD ini disusun dengan pendekatan Problem Based Learning (PBL), dimana proses PBL adalah (1) mengorientasikan peserta didik kepada masalah; (2) mengorganisasikan peserta didik; (3) membimbing penyelidikan individu dan kelompok; (4) mengembangkan dan menyajikan hasil karya; (5) menganalisis dan mengevaluasi proses pemecahan masalah.

Penyusunan LKPD ini bertujuan untuk meningkatkan kemampuan pemecahan masalah matematis siswa. Adapun indikator pemecahan masalah menurut Badan Standar Nasional Pendidikan (BSNP) adalah: (1) menyajikan pemahaman masalah; (2) mengorganisasi data dan menulis informasi yang relevan dalam memecahkan masalah; (3) menyajikan masalah secara matematis dalam berbagai bentuk; (4) memilih pendekatan dan metode pemecahan masalah secara tepat; (5) mengembangkan strategi pemecahan masalah; (6) membuat dan menuliskan model matematika dari suatu masalah; dan (7) menyelesaikan masalah matematika yang tidak rutin. Tahap-tahap pemecahan masalah secara sistematis terdiri atas empat tahap berikut: (1) memahami masalah; (2) membuat rencana penyelesaian; (3) melaksanakan rencana penyelesaian; dan (4) memeriksa kembali atau mengajalinya.

Seluruh menggunakan LKPD ini, kemampuan pemecahan masalah matematika peserta didik diharapkan meningkat. Insipinya pada materi Determinan dan Invers Matriks Ordo  $2 \times 2$  dan  $3 \times 3$ . Materi tersebut termuat pada Peraturan Menteri Pendidikan dan Kebudayaan (Permendikbud) Nomor 37 tahun 2018 dengan Kompetensi Dasar Pengetahuan 3.4 Menganalisis sifat-sifat determinan dan invers matriks berordo  $2 \times 2$  dan  $3 \times 3$ , sedangkan Kompetensi Dasar Keterampilan 4.4 Menyelesaikan masalah yang berkaitan dengan determinan dan invers matriks berordo  $2 \times 2$  dan  $3 \times 3$ .

**Nafida Hetty Marhaeni**, lahir di Temanggung pada 21 Mei 1995. Beliau menyelesaikan S-1 Pendidikan Matematika dari FKIP Universitas Mercu Buana Yogyakarta pada tahun 2017.

Saat ini sedang menyelesaikan tugas akhir di Magister Pendidikan Matematika Universitas Ahmad Dahlan. Adapun judul tugas akhirnya adalah "Pengembangan Lembar Kerja Peserta Didik (LKPD) Berbasis Problem Based Learning (PBL) untuk Meningkatkan Kemampuan Pemecahan Masalah Siswa Kelas XI SMA"

**Magister Pendidikan Matematika UAD**

Design by:  
**Fvri Dwikatno**

(b)

FIGURE 1. (a) Front cover and (b) Back Cover

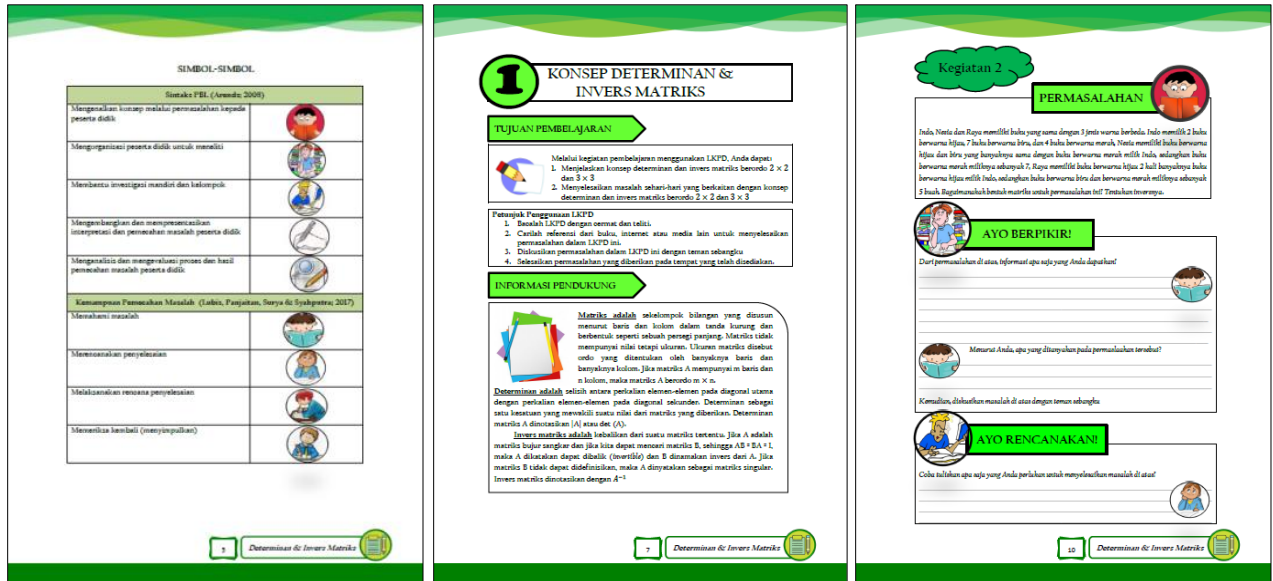


FIGURE 2. Students' Worksheet Content Page

Figure 1 and Figure 2 shows several examples of the display of the students' worksheet developed, namely front and back covers, then the meaning of the symbols in the students' worksheet, an example of students' worksheet activity 1 which contains learning objectives, instructions for use, supporting information, problems in activity 1, and completion steps that must be taken by the student. The research continued to the implementation stage. First, the researcher conducted a trial of the students' worksheet product. Small-scale product trials were carried out to 6 students of class XI MIPA 1. The interpretation of the assessment results was carried out quantitatively using the classification of practicality criteria that have been described in Table 4. and the results can be seen in Table 4.

TABLE 4. Small-Scale Trial

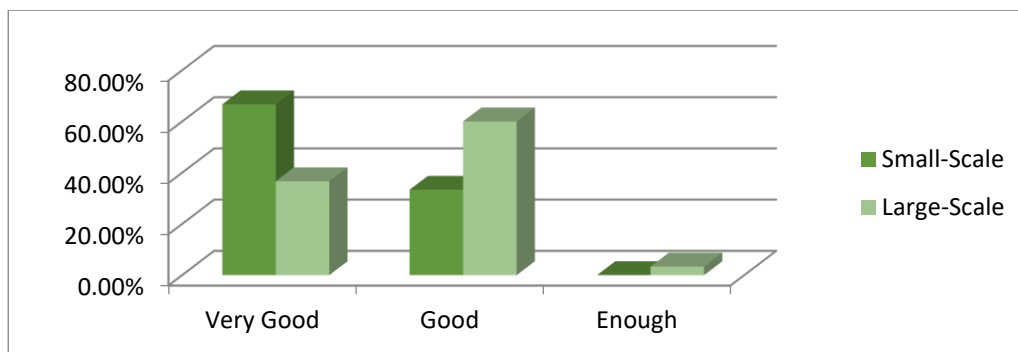
Respondents	Total Score	Category
1	86	Very Good
2	91	Very Good
3	92	Very Good
4	90	Very Good
5	72	Good
6	77	Good
Total	508	
Average	84,67	Very Good

Table 4. shows that the results of the small-scale trial are stated to be good, meaning that the students' worksheet is practically used. The research was continued by implementing the students' worksheet in a large class, namely 30 students of class XI MIPA 3 which was the experimental class in this study. In this case the data used is the assessment of student responses from class XI MIPA 3 as many as 30 people who are the experimental class. The results obtained for large-scale trials are presented in Table 5.

**TABLE 5.** Large-Scale Trial

Respondents	Total Score	Category	Respondents	Total Score	Category
R-1	83	Good	R-16	83	Good
R-2	71	Good	R-17	75	Good
R-3	78	Good	R-18	91	Very Good
R-4	79	Good	R-19	67	Enough
R-5	79	Good	R-20	70	Good
R-6	73	Good	R-21	72	Good
R-7	71	Good	R-22	95	Very Good
R-8	75	Good	R-23	95	Very Good
R-9	72	Good	R-24	87	Very Good
R-10	79	Good	R-25	86	Very Good
R-11	73	Good	R-26	86	Very Good
R-12	70	Good	R-27	88	Very Good
R-13	77	Good	R-28	86	Very Good
R-14	70	Good	R-29	87	Very Good
R-15	89	Very Good	R-30	88	Very Good
Total			2395		
Average			79,83		

Quantitative data on student responses were obtained through conversion into qualitative data to determine the practicality criteria for students' worksheet. Table 5. shows that there is 1 student who considers the students' worksheet "good enough" to be used, 18 students assess the students' worksheet as "good" to use and 11 students assess the students' worksheet as "very good" to use. If you add up the score is 2395 with an average of 79.83 which indicates that the "good" assessment for students' worksheet is used. Based on the practicality assessment, it can be concluded that the students' worksheet developed is declared "practical" because it has met the practicality standards that is meet the minimum good category for the average results of student responses in large-scale trials. The results of the practicality test on a small and large scale are presented in Figure 3.



**FIGURE 3.** Comparison of Small Scale and Large-Scale Practicality Classification

In the Figure 3, it can be seen that there are differences that need to be discussed; namely, there is one student on a large scale who stated that the students' worksheet developed was quite well used. Although on average, the experimental class said that the students' worksheet developed was "good" to use. Based on the results of student response questionnaires, both on a small and large scale, we know that the implementation of problem-based learning-based worksheets has met the feasibility of teaching material in terms of the practicality of product development. The positive response from students as users of teaching materials shows that the use of students' worksheets in learning can help students in solving problems of determinants of matrices and their inverses, which are generally taught through the provision of formulas only. With students' worksheets, learning begins by giving contextual problems related to the determinant of the matrix and its inverse so that students can construct their knowledge about a determinant of the matrix through linking contextual problems with abstract concepts of matrix determinants. From here, students can learn to interpret a problem correctly because students who are trained to



interpret each part of a particular concept with their knowledge of a given problem through inquiry activities will have more complete thinking activities [32].

The completeness of the student's thinking activities can minimize the occurrence of student misconceptions which can be an obstacle in learning students' understanding of concepts correctly[14]. Especially mathematical concepts that have abstract and hierarchical objects of study in learning its material [33]. Therefore, it is necessary for the teacher to design learning tools or teaching materials such as student worksheets according to student characteristics so that learning takes place in a fun and effective way in providing space for students' creativity [34].

The design of student worksheets as an alternative device that supports learning can allow students to develop problem-solving skills, especially student worksheets based on the problem-based learning model. This development can be seen in the result of the implementation stage, which shows increased students' problem skills after using student worksheets. From the results of student work in the student worksheets, it is known that students have achieved the indicators of problem-solving abilities in each assignment contained in the student worksheets with problem-based learning syntax, as shown in Figure 4 (a)-(b).

**AYO BERPIKIRI**

Dari permasalahan di atas, informasi apa saja yang kamu dapatkan?  
 Adi dan Ari mempunyai  $\frac{1}{5}$  bolpoin jenis warna seduh  
 Adi  $\rightarrow$  5 merah & 2 hitam  
 Ari  $\rightarrow$   $\frac{3}{5}$  merah, Adi dan 2x bolpoin hitam

Kemudian,  
 Apa yang ditanyakan pada permasalahan tersebut?  
 Banyak matriks?  
 Invers?

**AYO RENCANAKAN!**

Rumus apa saja yang digunakan untuk menyelesaikan masalah di atas?  
 Hitung  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$   
 Invers  $= \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

From the problems above, what information did you get?  
 Ani and Ari have different colored ballpoint pens.  
 Adi  $\rightarrow$  five red and two black  
 Ani  $\rightarrow$   $\frac{3}{5}$  Adi's red and 2x black ballpoint

What is being asked about the problem?  
 Matrix form?  
 Inverse?

What formula is used to solve the above problem?

	Adi	Ari
Black	$a$	$b$
Red	$c$	$d$

Invers  $= \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

(a) Achievement indicators understand problems and plan the solutions

**AYO SELESAIKAN!**

Jadi, Bagaimana cara Anda menyelesaikan?

Hitan  $\begin{pmatrix} \text{Adi} & \text{Ari} \\ 2 & 4 \end{pmatrix}$  Hitam Ari =  $2 \times \text{Adi}$   
 Merah  $\begin{pmatrix} 3 & 5 \end{pmatrix}$  Merah Ari =  $\frac{3}{5}$  Adi =  $\frac{3}{5} \times 5 = 3$

Bentuk matriks

$$A = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det|A|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{6-20} \begin{pmatrix} 5 & -4 \\ -3 & 2 \end{pmatrix}$$

$$= \frac{1}{-14} \begin{pmatrix} 5 & -4 \\ -3 & 2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -\frac{5}{14} & \frac{2}{7} \\ \frac{3}{14} & -\frac{1}{7} \end{pmatrix}$$

**AYO SIMPULKAN!**

Dari yang sudah Anda kerjakan di atas, dapat disimpulkan bahwa:

a. Bentuk matriksnya adalah...  $\begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$

b. Determinan matriksnya adalah...  $\det|A| = ad - bc = 6 - 20 = -14$

c. Invers matriksnya adalah...  $A^{-1} = \begin{pmatrix} -\frac{5}{14} & \frac{2}{7} \\ \frac{3}{14} & -\frac{1}{7} \end{pmatrix}$

So how do you solve it?

Adi Ari  
 Red  $\begin{pmatrix} 2 & 4 \end{pmatrix}$  Black Ari =  $2 \times \text{Adi}$   
 Black  $\begin{pmatrix} 3 & 5 \end{pmatrix}$  =  $2 \times 2 = 4$   
 Red Ari =  $\frac{3}{5} \times \text{Adi} = \frac{3}{5} \times 5 = 3$

Matriks form:

$$A = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det|A|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{6-20} \begin{pmatrix} 5 & -4 \\ -3 & 2 \end{pmatrix}$$

$$= \frac{1}{-14} \begin{pmatrix} 5 & -4 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -\frac{5}{14} & \frac{2}{7} \\ \frac{3}{14} & -\frac{1}{7} \end{pmatrix}$$


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From what you have done above, it can be concluded:

a. Form of matriks is  $\begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$

b. Matriks determinant is  $\det|A| = ad - bc = 6 - 20 = -14$

c. Matriks invers is  $A^{-1} = \begin{pmatrix} -\frac{5}{14} & \frac{2}{7} \\ \frac{3}{14} & -\frac{1}{7} \end{pmatrix}$

(b) Achievement indicators solve problems and conclude

**FIGURE 4 (a)-(b).** Achievement indicators problem-solving skill

Many previous studies have proven that student worksheets based on problem-based learning can be used to improve problem-solving abilities. According to the research results of Fitriani et al. [35], problem-based learning using student worksheets has proven to be effective in developing problem-solving abilities compared to conventional learning models. This is supported by the results of research by Siagan et al. [22] which showed that the subject matter developed through the problem-based learning model-oriented worksheets was effective in improving students' mathematical problem-solving abilities. Thus, the application of problem-based learning so far can still be said to be effective in improving problem-solving abilities [36].

## CONCLUSIONS

Based on the results and discussion above, it can be concluded that the teaching materials in the form of students' worksheet based on problem-based learning meet the valid and practical criteria of product development. The validity of the learning media is based on the results of the assessment of both material experts and media experts who are categorized as very good. While the practicality of learning media is shown by the very good student response to the use of media during the learning process. Based on these results, the matrix learning media using students' worksheet based on problem-based learning is feasible and can be used as an alternative to teaching determinants and inverse matrices.

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